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EQUATIONS OF MOTION OF THE RESTRICTED THREE-BODY PROBLEM WITH NON-ISOTROPICALLY VARIABLE MASSES WITH REACTIVE FORCES

Abstract. During the formation of planetary systems, especially at the non-stationary stage, the gravitational field of the central protostar (e.g., protosun) and the most massive protoplanets (e.g., protojupiter) often dominate. In this regard, we consider the restricted three-body problem with variable masses changing non-isotropically at different rates, as a celestial-mechanical model of small-body motion in a non-stationary protoplanet system. Based on the generalized Meshchersky equation, the differential equations of the restricted three-body problem are derived in the absolute coordinate system and in the presence of a reactive forces. At the same time it is assumed that the masses of the bodies are increasing due to accreting particles from the outer space, and decreasing due to the particle loss. Based on the equation of motion obtained in the absolute coordinate system, the equations of motion in a relative coordinate system whose origin is at the center of the protostar are derived in the presence of a reactive forces. Special cases of the obtained differential equations of motion of a non-stationary dynamic system in a relative coordinate system are discussed.

Key words: restricted three-body problem, non-isotropically variable mass, reactive forces.

1. Introduction. One of the actual problems in modern astronomy is the origin and evolution of small bodies (asteroids, comets) in planetary systems. Studying the movement and evolution of small bodies, we will try to understand what the planetary system was in the past and how the planetary system will be in the future. Small bodies collide with planets every day, in particular with the Earth in the Solar system.

In the case of the Earth, in most cases they are small and burn in the atmosphere of the Earth, before falling to the surface [1]. Nevertheless, even if we talk about less than 10 meters in size asteroids, there were many of them in the Earth history. Only in the last century were two such events (at least) - the Tungussky and the Chelyabinsk meteorites. This is one of the important aspects of the small bodies' dynamics in planetary systems.

Now and in most cases, the dynamics of small bodies studies based on the Keplerian motion of the two-body problem with constant masses [2]. However, such an important parameter of small bodies as a mass is variable. Consequences of the small bodies' mass variability, especially during the stage of non-stationarity of the gravitating system, have been little studied [3-7]. We have studied the motion of an infinitesimal body in protoplanetary systems based on a restricted three-body problem with variable mass in the presence of reactive forces. Here we got the differential equations of motion for the restricted three-body problem with non-isotropically with variable mass in the absolute coordinate system and in the relative coordinate system with the origin at the protostar center. The obtained equations of motion describe the dynamic evolution of the considered non-stationary system of gravitating bodies with non-isotropically changing masses in the presence of reactive forces.

2. Problem statement and the equation of motion in the absolute coordinate system

2.1 Problem statement. Let's consider gravitating system consisting of three spherical celestial bodies with variable masses. We assume that the bodies are bodies with spherical mass distributions or points. Suppose, T_0 - the central protostar (more massive body), T_1 - protoplanet (less massive body), T_2 - a body with a small mass. Accordingly, we denote the masses, which are functions of time.

$$m_0 = m_0(t),\ m_1 = m_1(t),\ m_2 = m_2(t). \tag{2.1}$$

Suppose that the masses of the bodies are increasing due to accreting particles, and decreasing due to the particle loss. In this case, the relative speed of the particles separating from the body is different than the relative speed of the accreting particles to the body. Consider the general case when the masses of bodies change not isotropically at different rates [8, 9, 10]

$$\frac{\dot m_0}{m_0} \neq \frac{\dot m_1}{m_1}, \qquad \frac{\dot m_0}{m_0} \neq \frac{\dot m_2}{m_2}, \qquad \frac{\dot m_1}{m_1} \neq \frac{\dot m_2}{m_2}. \tag{2.2}$$

We assume that an infinitesimal body with mass m_2, does not affect the motion of the two massive bodies with the masses m_0 and m_1, this is a restricted formulation of the three bodies problem with variable mass in the presence of reactive forces [11-13]

$$m_0 \gg m_2, \quad m_1 \gg m_2. \tag{2.3}$$

Required description of the dynamical evolution of three gravitating bodies with variable mass in the presence of reactive forces in this formulation.

2.2 Differential equations of motion in the absolute coordinate system. Based on the generalized equations of Meshchersky [12-14], in the presence of reactive forces we get

$$m_0 \ddot{\vec R}_0 = grad_{\vec R_0} U_{01} + \dot m_{01} \vec V_{01} + \dot m_{02} \vec V_{02}, \tag{2.4}$$

$$U_{01} = f\left(\frac{m_0 m_1}{R_{01}}\right), \tag{2.5}$$

$$\vec V_{01} = \vec u_{01} - \dot{\vec R}_0, \quad \vec V_{02} = \vec u_{02} - \dot{\vec R}_0, \tag{2.6}$$

$$m_0 = m_0(t_0) - |m_{01}| + m_{02} = m_0(t_0) - \int_{t_0}^{t} (|\dot m_{01}|) dt + \int_{t_0}^{t} (\dot m_{02}) dt. \tag{2.7}$$

$m_0(t_0) = const$ mass of the body T_0 at the initial time t_0, m_{01} - the mass of particles separated from the body T_0 in time t, m_{02} - the mass of particles accreting to the body T_0 in time t.

$$m_1 \ddot{\vec R}_1 = grad_{\vec R_1} U_{10} + \dot m_{11} \vec V_{11} + \dot m_{12} \vec V_{12}, \tag{2.8}$$

$$U_{10} = f\left(\frac{m_1 m_0}{R_{10}}\right), \tag{2.9}$$

$$\vec V_{11} = \vec u_{11} - \dot{\vec R}_1, \qquad \vec V_{12} = \vec u_{12} - \dot{\vec R}_1, \tag{2.10}$$

$$m_1 = m_1(t_0) - |m_{11}| + m_{12} = m_1(t_0) - \int_{t_0}^{t} (|\dot m_{11}|) dt + \int_{t_0}^{t} (\dot m_{12}) dt. \tag{2.11}$$

$m_1\left(t_0\right)=const$ mass of the body T_1 at the initial time t_0, m_{11} - mass of particles separated from the body T_1 in time t ,m_{12} - the mass of particles accreting to the body T_1 in time t .

$$m_2\ddot{\vec{R}}_2 = grad_{\vec{R}_2}\tilde{U} + \dot{m}_{21}\vec{V}_{21} + \dot{m}_{22}\vec{V}_{22}, \tag{2.12}$$

$$\tilde{U} = fm_2\left(\frac{m_0}{R_{20}} + \frac{m_1}{R_{21}}\right), \tag{2.13}$$

$$\vec{V}_{21} = \vec{u}_{21} - \dot{\vec{R}}_2, \qquad \vec{V}_{22} = \vec{u}_{22} - \dot{\vec{R}}_2, \tag{2.14}$$

$$m_2 = m_2\left(t_0\right) - \left|m_{21}\right| + m_{22} = m_2\left(t_0\right) - \int_{t_0}^{t}\left(\left|\dot{m}_{21}\right|\right)dt + \int_{t_0}^{t}\left(\dot{m}_{22}\right)dt\,. \tag{2.15}$$

$m_2\left(t_0\right)=const$ mass of the body T_2 at the initial time t_0, m_{21} - mass of particles separated from the body T_2 in time t ,m_{22} - the mass of particles accreting to the body T_2 in time t .

In equations (2.4) - (2.15) $\vec{u}_{01}$, $\vec{u}_{11}$, $\vec{u}_{21}$ - are the absolute velocities of the separating particles,

$$\vec{V}_{01} = \vec{u}_{01} - \dot{\vec{R}}_0,\ \vec{V}_{11} = \vec{u}_{11} - \dot{\vec{R}}_1,\ \vec{V}_{21} = \vec{u}_{21} - \dot{\vec{R}}_2. \tag{2.16}$$

relative velocities of separating particles.

Accordingly, $\vec{u}_{02}$, $\vec{u}_{12}$, $\vec{u}_{22}$ - absolute velocities of the accreting particles,

$$\vec{V}_{02} = \vec{u}_{02} - \dot{\vec{R}}_0, \vec{V}_{12} = \vec{u}_{12} - \dot{\vec{R}}_1, \vec{V}_{22} = \vec{u}_{22} - \dot{\vec{R}}_1. \tag{2.17}$$

relative velocities of accreting particles. Also marked, that $\vec{R}_j$ - the radius vector of the center of mass bodies in the absolute coordinate system$\left(j=0,1,2\right)$, $\vec{R}_{ij}$ – the mutual distances of the mass bodies center $\left(j,i=0,1,2,\ j\neq i\right)$, f - gravitational constant.

From equations (2.4) - (2.5), (2.8) - (2.9), (2.12) - (2.13) follows

$$\ddot{\vec{R}}_0 = grad_{\vec{R}_0}U_{01} + \frac{1}{m_0}\left(\dot{m}_{01}\vec{V}_{01} + \dot{m}_{02}\vec{V}_{02}\right) \tag{2.18}$$

$$U_{01} = f\left(\frac{m_1}{R_{01}}\right), \tag{2.19}$$

$$\ddot{\vec{R}}_1 = grad_{\vec{R}_1}U_{10} + \frac{1}{m_1}\left(\dot{m}_{11}\vec{V}_{11} + \dot{m}_{12}\vec{V}_{12}\right) \tag{2.20}$$

$$U_{10} = f\left(\frac{m_0}{R_{10}}\right), \tag{2.21}$$

$$\ddot{\vec{R}}_2 = grad_{\vec{R}_2}\tilde{U}^* + \frac{1}{m_2}\left(\dot{m}_{21}\vec{V}_{21} + \dot{m}_{22}\vec{V}_{22}\right) \tag{2.22}$$

$$\tilde{U}^* = f\left(\frac{m_0}{R_{20}} + \frac{m_1}{R_{21}}\right). \tag{2.23}$$

Equations (2.18) - (2.19), (2.20) - (2.21) define the problem of two bodies with variable masses in the presence of reactive forces for the absolute coordinate system.

Equations (2.22) - (2.23) define a restricted three body problem with variable masses in the presence of reactive forces for the absolute coordinate system.

Following L.G. Lukyanov [15] we assume that the reactive forces is applied to the inertia center of the corresponding spherical celestial bodies.

3. Equations of motion of bodies in a relative coordinate system

We introduce the relative coordinate system with the origin at the center of the central protostar T_0, whose axes are parallel to the corresponding axes of the absolute coordinate system. Denote

$$\vec{R}_{01} = \vec{R}_1 - \vec{R}_0\,,\ \vec{R}_{02} = \vec{R}_2 - \vec{R}_0\,. \tag{3.1}$$

Then in relative coordinates equations of motion for the two primary bodies problem has the form

$$\ddot{\vec{R}}_{01} = grad_{\vec{R}_{01}}\tilde{U}_{01} + \frac{1}{m_1}\left(\dot{m}_{11}\vec{V}_{11} + \dot{m}_{12}\vec{V}_{12}\right) - \frac{1}{m_0}\left(\dot{m}_{01}\vec{V}_{01} + \dot{m}_{02}\vec{V}_{02}\right), \tag{3.2}$$

$$\tilde{U}_{01} = f\frac{m_0 + m_1}{R_{01}}\ . \tag{3.3}$$

In relative coordinates, the equations of motion of a body with a small mass, in the attraction field of two primary bodies, can be written as

$$\ddot{\vec{R}}_{02} = grad_{\vec{R}_{02}}\tilde{U}^* + \frac{1}{m_2}\left(\dot{m}_{21}\vec{V}_{21} + \dot{m}_{22}\vec{V}_{22}\right) - \frac{1}{m_0}\left(\dot{m}_{01}\vec{V}_{01} + \dot{m}_{02}\vec{V}_{02}\right). \tag{3.4}$$

Denote the reactive forces (per unit mass)

$$\vec{\Phi}_{11} = \frac{1}{m_1}\left(\dot{m}_{11}\vec{V}_{11}\right) - \frac{1}{m_0}\left(\dot{m}_{01}\vec{V}_{01}\right),\ \vec{\Phi}_{12} = \frac{1}{m_1}\left(\dot{m}_{12}\vec{V}_{12}\right) - \frac{1}{m_0}\left(\dot{m}_{02}\vec{V}_{02}\right), \tag{3.5}$$

$$\vec{F}_1 = \vec{F}_1(t) = \vec{\Phi}_{11} + \vec{\Phi}_{12}\,. \tag{3.6}$$

$$\vec{\Phi}_{21} = \frac{1}{m_2}\left(\dot{m}_{21}\vec{V}_{21}\right) - \frac{1}{m_0}\left(\dot{m}_{01}\vec{V}_{01}\right),\ \vec{\Phi}_{22} = \frac{1}{m_2}\left(\dot{m}_{22}\vec{V}_{22}\right) - \frac{1}{m_0}\left(\dot{m}_{02}\vec{V}_{02}\right), \tag{3.7}$$

$$\vec{F}_2 = \vec{F}_2(t) = \vec{\Phi}_{21} + \vec{\Phi}_{22}\,. \tag{3.8}$$

Note that the reactive forces (per unit mass) are due to separating particles denoted by

$$\vec{\Phi}_{11} = \frac{1}{m_1}\left(\dot{m}_{11}\vec{V}_{11}\right) - \frac{1}{m_0}\left(\dot{m}_{01}\vec{V}_{01}\right),\ \vec{\Phi}_{21} = \frac{1}{m_2}\left(\dot{m}_{21}\vec{V}_{21}\right) - \frac{1}{m_0}\left(\dot{m}_{01}\vec{V}_{01}\right). \tag{3.9}$$

Similarly, reactive forces (per unit mass) are due to accreting particles indicated by the formulas

$$\vec{\Phi}_{12} = \frac{1}{m_1}\left(\dot{m}_{12}\vec{V}_{12}\right) - \frac{1}{m_0}\left(\dot{m}_{02}\vec{V}_{02}\right),\ \vec{\Phi}_{22} = \frac{1}{m_2}\left(\dot{m}_{22}\vec{V}_{22}\right) - \frac{1}{m_0}\left(\dot{m}_{02}\vec{V}_{02}\right). \tag{3.10}$$

In the notation (3.5) - (3.8) formulas (3.2) (3.4) has the form

$$\ddot{\vec{R}}_{01} = grad_{\vec{R}_{01}}\tilde{U}_{01} + \vec{F}_1, \tag{3.11}$$

$$\ddot{\vec{R}}_{02} = grad_{\vec{R}_{02}}\tilde{U}^* + \vec{F}_2\,. \tag{3.12}$$

We introduce the following notation.

$$\vec{R}_{01}=\vec{R}_1-\vec{R}_0=\vec{r}_1\left(x_1,y_1,z_1\right),\ \vec{R}_{02}=\vec{R}_2-\vec{R}_0=\vec{r}_2\left(x_2,y_2,z_2\right),\ \vec{r}_1-\vec{r}_2=\vec{r}_{21}. \quad (3.13)$$

The equations of motion (3.11) - two primary bodies problems with variable masses in the presence of reactive forces (3.6) we can write in the form

$$\ddot{\vec{r}}_1+f\left(m_0+m_1\right)\frac{\vec{r}_1}{r_1^3}=\vec{F}_1, \quad (3.14)$$

$$r_1=\sqrt{x_1^{\ 2}+y_1^{\ 2}+z_1^{\ 2}}\ , \quad (3.15)$$

$$\vec{F}_1=\vec{F}_1\left(t\right)=\vec{\Phi}_{11}+\vec{\Phi}_{12}. \quad (3.16)$$

Accordingly, the equations of motion of a body with a small mass (3.12), in the attraction field of two primary bodies, can be written as

$$\ddot{\vec{r}}_2=grad_{\vec{r}_2}\tilde{U}^*+\vec{F}_2.$$

The last equation can be written as

$$\ddot{\vec{r}}_2+f\left(m_0+m_2\right)\frac{\vec{r}_2}{r_2^3}=grad_{\vec{r}_2}U+\vec{F}_2, \quad (3.17)$$

$$U=fm_1\left(\frac{1}{r_{21}}-\frac{x_1x_2+y_1y_2+z_1z_2}{r_1^3}\right), \quad (3.18)$$

$$r_{21}=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2+\left(z_1-z_2\right)^2}\ , \quad (3.19)$$

$$r_1=\sqrt{x_1^{\ 2}+y_1^{\ 2}+z_1^{\ 2}}\ ,\ r_2=\sqrt{x_2^{\ 2}+y_2^{\ 2}+z_2^{\ 2}}\ , \quad (3.20)$$

$$\vec{F}_2=\vec{F}_2\left(t\right)=\vec{\Phi}_{21}+\vec{\Phi}_{22}. \quad (3.21)$$

The obtained equations of relative motion (3.17)-(3.21), a restricted three-body problem with variable masses in the presence of reactive forces, more adequately describe the dynamic evolution of infinitesimal body motion with variable mass in non-stationary gravitating multiple systems than the isotropic change in the masses of bodies [4, 11, 15, 16]. In this case, the relative motion of the two primary bodies is described by equations (3.14)-(3.16). Note that in some exoplanetary systems [17-19], where nonstationary processes, apparently, in still dominates in the formation of planetary systems then the equations obtained can be effectively used.

In orbital coordinate systems, reactive forces (3.16), (3.21) can be written as

$$\vec{F}_2=\vec{F}_2\left(t\right)=\vec{F}_{2r}+\vec{F}_{2\tau}+\vec{F}_{2n}=\vec{F}_2\left(F_{2r}\left(t\right),F_{2\tau}\left(t\right),F_{2n}\left(t\right)\right), \quad (3.22)$$

$$\vec{F}_1=\vec{F}_1\left(t\right)=\vec{F}_{1r}+\vec{F}_{1\tau}+\vec{F}_{1n}=\vec{F}_1\left(F_{1r}\left(t\right),F_{1\tau}\left(t\right),F_{1n}\left(t\right)\right), \quad (3.23)$$

where denoted by radial $\vec{F}_{ir}$, transversal $\vec{F}_{i\tau}$ and normal components $\vec{F}_{in}$ of reactive forces.

4. Particular cases of the equation of motion (3.14), (3.17). The obtained equations of motion in the relative coordinate system, with the origin at the body center, describe the bodies dynamics in the considered formulation of the problem in the general case. There is interest to study the problem under consideration in various particular assumptions regarding in the bodies mass change. The possible combination of three bodies mass change for different cases is quite a lot. A full analysis of various particular cases will be performed in another work. In this paper we consider, as an example, one interesting special case.

The case when the masses of three bodies are constant, but with variable composition. Let the changes in the mass of each body be characterized by the fact that the second mass of the accreting particles of a particular body is equal to the second mass of the same body discarded particles. Then we have

$$-\dot{m}_{01} = \dot{m}_{02} = \dot{m}_0^*, \quad -\dot{m}_{11} = \dot{m}_{12} = \dot{m}_1^*, \quad -\dot{m}_{21} = \dot{m}_{22} = \dot{m}_2^*. \tag{4.1}$$

From formulas (2.7), (2.11), (2.15) it follows that the masses of the bodies are constant, but with variable composition

$$\begin{aligned} m_0 &= m_0(t_0) = m_0^* = const, \\ m_1 &= m_1(t_0) = m_1^* = const, \\ m_2 &= m_2(t_0) = m_2^* = const. \end{aligned} \tag{4.2}$$

From formulas (3.6), (3.5) and (4.1), taking into account (2.6), (2.10), (2.14), we get

$$\begin{aligned} \vec{F}_1 &= \vec{F}_1^* = \vec{\Phi}_{11}^* + \vec{\Phi}_{12}^* = \frac{\dot{m}_{11}}{m_1}\vec{V}_{11} - \frac{\dot{m}_{01}}{m_0}\vec{V}_{01} + \frac{\dot{m}_{12}}{m_1}\vec{V}_{12} - \frac{\dot{m}_{02}}{m_0}\vec{V}_{02} = \\ &= \frac{1}{m_1}\left(\dot{m}_{11}\vec{V}_{11} + \dot{m}_{12}\vec{V}_{12}\right) - \frac{1}{m_0}\left(\dot{m}_{01}\vec{V}_{01} + \dot{m}_{02}\vec{V}_{02}\right) = \\ &= \frac{\dot{m}_1^*}{m_1}\left(-\vec{V}_{11}^* + \vec{V}_{12}^*\right) - \frac{\dot{m}_0^*}{m_0}\left(-\vec{V}_{01}^* + \vec{V}_{02}^*\right) = \\ &= \frac{\dot{m}_1^*}{m_1}\left(-\vec{u}_{11}^* + \vec{u}_{12}^*\right) - \frac{\dot{m}_0^*}{m_0}\left(-\vec{u}_{01}^* + \vec{u}_{02}^*\right). \end{aligned} \tag{4.3}$$

Similarly, taking into account formulas (3.8), (3.7), (4.1) and considering these formulas (2.6), (2.10), (2.14), we get

$$\begin{aligned} \vec{F}_2 &= \vec{F}_2^* = \vec{\Phi}_{21}^* + \vec{\Phi}_{22}^* = \frac{\dot{m}_{21}}{m_2}\vec{V}_{21} - \frac{\dot{m}_{01}}{m_0}\vec{V}_{01} + \frac{\dot{m}_{22}}{m_2}\vec{V}_{22} - \frac{\dot{m}_{02}}{m_0}\vec{V}_{02} = \\ &= \frac{1}{m_2}\left(\dot{m}_{21}\vec{V}_{21} + \dot{m}_{22}\vec{V}_{22}\right) - \frac{1}{m_0}\left(\dot{m}_{01}\vec{V}_{01} + \dot{m}_{02}\vec{V}_{02}\right) = \\ &= \frac{\dot{m}_2^*}{m_2}\left(-\vec{V}_{21}^* + \vec{V}_{22}^*\right) - \frac{\dot{m}_0^*}{m_0}\left(-\vec{V}_{01}^* + \vec{V}_{02}^*\right) = \frac{\dot{m}_2^*}{m_2}\left(-\vec{u}_{21}^* + \vec{u}_{22}^*\right) - \frac{\dot{m}_0^*}{m_0}\left(-\vec{u}_{01}^* + \vec{u}_{02}^*\right). \end{aligned} \tag{4.4}$$

As a result, we can write

$$\vec{F}_1 = \vec{F}_1^* = \frac{\dot{m}_1^*}{m_1}\left(-\vec{V}_{11}^* + \vec{V}_{12}^*\right) - \frac{\dot{m}_0^*}{m_0}\left(-\vec{V}_{01}^* + \vec{V}_{02}^*\right) = \frac{\dot{m}_1^*}{m_1}\left(-\vec{u}_{11}^* + \vec{u}_{12}^*\right) - \frac{\dot{m}_0^*}{m_0}\left(-\vec{u}_{01}^* + \vec{u}_{02}^*\right). \tag{4.5}$$

$$\vec{F}_2 = \vec{F}_2^* = \frac{\dot{m}_2^*}{m_2}\left(-\vec{V}_{21}^* + \vec{V}_{22}^*\right) - \frac{\dot{m}_0^*}{m_0}\left(-\vec{V}_{01}^* + \vec{V}_{02}^*\right) = \frac{\dot{m}_2^*}{m_2}\left(-\vec{u}_{21}^* + \vec{u}_{22}^*\right) - \frac{\dot{m}_0^*}{m_0}\left(-\vec{u}_{01}^* + \vec{u}_{02}^*\right). \tag{4.6}$$

Thus, in the particular case of (4.1), in the three-body gravitating system under consideration, each body has a constant mass. But at the same time, the composition of each body changes, which may affect the chemical structure of these bodies [20]. The relative coordinate system in the equation of motion (3.14), (3.17) contains reactive forces according to (4.5) - (4.6).

The obtained equations of motion of the problem under consideration are rather complicated, therefore, in the future they will be investigate by perturbation theory methods [8].

5. Conclusion

The work investigated the movement of an infinitesimal body in the gravitational field of two massive bodies in the framework of a restricted three-body problem with masses that changing at different rates non-isotropically in the presence of reactive forces.

Based on the generalized Meshchersky equation, the differential equations of the restricted three-body problem with masses varying at different rates non-isotropically are derived, in the absolute coordinate system with the presence of reactive forces. At the same time, it is assumed that the masses of the bodies are increasing due to accreting particles from the outer space, and decreasing due to the particle loss. Also obtained equations of the relative motion for a restricted three-body problem with variable masses, varying at non-isotropically different rates, in a relative coordinate system, in the presence of reactive forces, with the origin at the central protostar center.

Discussed particular cases of the obtained differential equations of motion, considered for a nonstationary dynamic system in a relative coordinate system. Considered one case, when the mass of each body is constant, but with variable composition.

The resulting equations of motion of the restricted three-body problem with non-isotropically varying masses in the presence of reactive forces more adequately describe the dynamic evolution of a small-body motion with a variable mass in non-stationary gravitating multiple systems than isotropic changing masses of the bodies.

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РЕАКТИВТІ КҮШ ЕСКЕРІЛГЕН МАССАЛАРЫ ИЗОТРОПТЫ ЕМЕС ӨЗГЕРЕТІН ШЕКТЕЛГЕН ҮШ ДЕНЕ ЕСЕБІНІҢ ҚОЗҒАЛЫС ТЕҢДЕУІ

Аннотация. Планеталық жүйелердің қалыптасуы кезінде, әсіресе стационар емес кезеңде орталық протожұлдыздың (мысалы, протокүн) және ең массивті протопланетаның (мысалы, протоюпитер) гравитациялық өрісі басым. Осыған орай стационар емес протопланеталық жүйелердегі кіші дене қозғалысының негізгі аспан-механикалық моделі ретінде әртүрлі қарқында изотропты емес өзгеретін массасы айнымалы шектелген екі дене есебі қарастырылады. Мещерскийдің жалпылама теңдеуін негізге ала отырып реактивті күштері бар абсолютті координат жүйесіндегі шектелген үш дене есебінің дифференциалдық теңдеулері алынды. Сонымен, ғарыштық ортадан қосылатын бөлшектердің әсерінен массаның өсуі, сондай-ақ лақтырылатын бөлшектердің есебінен массаның азаюы байқалуы мүмкін деген болжам жасалуда. Абсолютті координат жүйесінде алынған қозғалыс теңдеуін ескере отырып салыстырмалы координат жүйесінде орталық протожұлдыздың центрі координаттар басы болып табылатын реактивті күш ескерілген қозғалыс теңдеуі алынды. Салыстырмалы координат жүйесіндегі стационар емес динамикалық жүйеде қарастырылған дифференциалдық теңдеулердің дербес жағдайы талқыланды.

Түйін сөздер: шектелген үш дене есебі, массаның изотропты емес өзгеруі, реактивті күштер.

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УРАВНЕНИЯ ДВИЖЕНИЯ ОГРАНИЧЕННОЙ ЗАДАЧИ ТРЕХ ТЕЛ С НЕИЗОТРОПНО ИЗМЕНЯЮЩИМИСЯ МАССАМИ ПРИ НАЛИЧИИ РЕАКТИВНЫХ СИЛ

Аннотация. В ходе образования планетных систем, особенно в этапе нестационарности, часто доминирует гравитационное поле центральной протозвезды (например, протосолнце) и самой массивной протопланеты (например, протоюпитер). В связи с этим, рассматривается ограниченная задача трех тел с переменными массами, изменяющимися не изотропно в различных темпах, как исходная небесно-механическая модель движений малого тела в нестационарных протопланетных системах. Исходя из обобщенного уравнения Мещерского выведены дифференциальные уравнения ограниченной задачи трех тел в абсолютной системе координат, при наличии реактивных сил. При этом предполагается одновременно рост массы тел из-за присоединяющихся (налипания) частиц из космической среды, а также уменьшение массы тел за счет отбрасываемых частиц. Исходя из уравнения движения, полученные в абсолютной системе координат, выведены уравнения движения в относительной системе координат с началом в центре центральной протозвезды, при наличии реактивных сил. Обсуждается частные случаи полученных дифференциальных уравнения движения, рассмотренной нестационарной динамической системы, в относительной системе координат.

Ключевые слова: ограниченная задача трех тел, неизотропное изменения масс, реактивные силы.

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REFERENCES

[1] Shustova B.M. Ryhlova L.V. Asteroidno-kometnaja opasnost': vchera, segodnja, zavtra M.: Fizmatlit, 2010 (in Russian)

[2] Engelhardt T. et. al. An Observational Upper Limit on the Interstellar Number Density of Asteroids and Comets. The Astronomical Journal. 2017. V.153. Iss.3, 11 pp. (in Eng.)

[3] Omarov T.B. (Editor) Non-Stationary Dynamical Problems in Astronomy. New-York: Nova Science Publ. Inc., 2002. - 260 p. (in Eng.)

[4] Eggleton P. Evolutionary processes in binary and multiple stars, Cambridge University Press, New York, 2006, 322 p. (in Eng.)

[5] Cherepawuk A.M. Tesnye dvojnye zvezdy. Chast' II.-M.: Fizmatlit, 2013. 572 p. (in Russian)

[6] Veras D., Hadjidemetriou J.D., Tout C.A. An Exoplanet’s Response to Anisotropic Stellar Mass-Loss During Birth and Death. - MNRAS. 2013. DOI: 10.1093/mnras/stt1451 (in Eng.)

[7] Surdin V.G. Solnechnaja sistema. M.: Fizmatlit, 2017. 460 p. (in Russian)

[8] Minglibaev M.Zh. Dinamika gravitirujuwih tel s peremennymi massami i razmerami. Postupatel'noe i postupatel'no-vrawatel'noe dvizhenie. LAP LAMBERT Academic Publishing, Germanija, 2012, 229 p. (in Russian)

[9] Minglibayev M.Zh. A.N. Prokopenya, M. Mayemerova, Zh.U. Imanova Three-body problem with variable masses that change anisotropically at different rates. Mathematics in Computer Science, 2017, Vol.11, (3-4). P. 383-391 SJR 2016-0.288 (in Eng.)

[10] Jewitt D. The Active Asteroids, The Astronomical Journal. 2012. V.143. Iss.3. 14 pp. (in Eng.)

[11] Poljahova E.N. Nebesno-mehanicheskie aspekty zadach dvuh i treh tel s peremennymi massami // Uchenye zapiski LGU, Ser. matem. nauk. 1989. T.42, №424, vyp. 64. P. 104-143. (in Russian)

[12] Luk'janov L.G., Shirmin G.I. Lekcii po nebesnoj mehanike. Almaty, 2009. 227 p. (in Russian)

[13] Mewerskij I.V. Raboty po mehanike tel peremennoj massy. M.: Gosudarstvennnoe izdatel'stvo tehniko-teoreticheskoj literatury, 1952. 281 p. (in Russian)

[14] Kosmodem'janskij A.A. Kurs teoreticheskoj mehaniki. 2-oj tom. M.:Prosvewenie, 1966. 402 p. (in Russian)

[15] Luk'janov L.G. Dinamicheskaja jevoljucija orbit zvezd v tesnyh dvojnyh sistemah s konservativnym obmenom mass // Astron.zhurn. 2008. T.85, №8. P.755-768. (in Russian)

[16] Bekov A.A., Omarov T.B. The Theory of Orbits in Non-Stationary Stellar Systems. // Astronomical and Astrophysical Transactions. 2003. V.22. P. 145-153. (in Eng.)

[17] http://spacetimes.ru/exoplanets

[18] http://exoplanetarchive.ipac.caltech.edu

[19] http://exoplanet.eu

[20] Busarev V.V. New Notions of Asteroids of Primitive Types. // Modern Star Astronomy. Vol. 1, Astronomy-2018 (XIII Congress of the International Public Organization "Astronomical Society"). Conference Abstracts, Moscow: IZMIRAN, 2018. P. 9-18. (in Eng.).