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NUMERICAL IMPLEMENTATION OF SOLVING A BOUNDARY VALUE PROBLEM FOR A SYSTEM OF LOADED DIFFERENTIAL EQUATIONS WITH PARAMETER

Abstract. A linear two-point boundary value problem for the loaded differential equations with parameter is considered. This problem is investigated by parameterization method. We offer algorithm for solving to boundary value problem for the system of loaded differential equations with parameter. In first, original problem is reduced to equivalent problem consisting the Cauchy problems for system of ordinary differential equations with parameters in subintervals and functional relations with respect to introduced additional parameters. At fixed values of parameters the Cauchy problem for system of ordinary differential equations in subinterval has a unique solution. This solution is represented with fundamental matrix of system. Using these representations we compile a system of linear algebraic equations with respect to parameters. We proposed algorithm for finding of numerical solution to the equivalent problem. This algorithm includes of the numerical solving of the Cauchy problems for system of the ordinary differential equations and solving of the linear system of algebraic equations. For numerical solving of the Cauchy problem we apply the Runge–Kutta method of 4th order. The proposed numerical implementation is illustrated by example.

Key words: boundary value problem with parameter, loaded differential equation, numerical method, algorithm.

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As well-known, the problem of constructing effective models finds its solution in many areas of science and technology. The active development of computer technology in recent decades, the emergence of new software tools designed to automate professional activity, has significantly affected the methods for solving the problems of identification of parameters. The application of software tools specialized in the field of scientific, technical and engineering calculations provides an opportunity for a deeper study of the investigated area, transferring the main burden of solving the problems from the development, debugging of algorithms and programs to the study of qualitative and numerical characteristics of the problem. Therefore, a modern approach in the theory of control and identification of parameters should be directed to the development of new constructive methods and modifications of known methods for solving boundary value problems for loaded differential equations with parameters. The theory of boundary value problems for the loaded differential equations with parameters is rapidly developing and is used in various fields of applied mathematics, biophysics, biomedicine, chemistry, etc. [1-10]. In spite of this, the questions of finding the effective criterions of unique solvability and constructing the numerical algorithms for finding the solutions of boundary value problems for the system of loaded differential equations with parameters still remain open. One of the constructive methods for investigating and solving the boundary value problems with parameters for the system of ordinary differential equations is the parameterization method [11]. The parameterization method was developed for the investigating and solving the boundary value problems for the system of ordinary differential equations. On the basis of this method, coefficient criteria for the unique solvability of linear boundary value problems for the system of ordinary differential equations were obtained. Algorithms for finding the approximate solutions were also proposed and their convergence to the exact solution of the problem studied was established. Later, the parameterization method was developed for the two-point boundary value problems for the Fredholm integro-differential equations [12-16]. Necessary and sufficient conditions for the solvability and unique solvability are established, the algorithms for finding the approximate solutions of the problems considered are constructed. In [17], methods for investigating and solving the linear boundary value problems for the linear Fredholm integro-differential equation on the basis of new algorithms of parameterization method are constructed. In [18] this algorithm is used for solve boundary value problem for system of ordinary differential equations with parameter.

In present paper the proposed new algorithms of parameterization method are extended to boundary value problem for loaded differential equations with parameters. We offer the numerical implementation of these algorithms to solve boundary value problem for the loaded differential equations with parameters.

So, we consider the linear boundary value problem for the loaded differential equations with parameter

$$\frac{dx}{dt} = A(t)x + \sum_{j=0}^{N} K_j(t)x(\theta_j) + A_0(t)\mu + f(t), \quad x \in R^n, \quad \mu \in R^m, \ t \in (0,T), \tag{1}$$

$$Bx(0) + Cx(T) + B_0\mu = d, \quad d \in R^{n+m}, \tag{2}$$

where the $(n \times n)$-matrices $A(t), K_j(t), \ j = \overline{0,N}$, $(n \times m)$-matrix $A_0(t)$, and n-vector-function $f(t)$ are continuous on $[0,T]$, the $((n+m)\times n)$ - matrices B, C, the $((n+m)\times m)$ - matrix B_0 are constants.

Let $C([0,T],R^n)$ denote the space of continuous functions $x:[0,T] \to R^n$ with the norm $\|x\|_1 = \max_{t\in[0,T]} \|x(t)\|$. A solution to problem (1), (2) is a pair $(\mu^*, x^*(t))$, with $x^*(t) \in C([0,T],R^n)$, $\mu^*(t) \in R^m$, where the function $x^*(t)$ is continuously differentiable on $(0,T)$ and satisfies the loaded differential equation (1) and boundary condition (2) with $\mu = \mu^*$.

Given the points: $\theta_0 = 0 < \theta_1 < \theta_2 < \ldots < \theta_{N-1} < \theta_N = T$, and let $\Delta_N(\theta)$ be the partition of interval $[0,T)$ into N subintervals: $[0,T) = \bigcup_{r=1}^{N} [\theta_{r-1}, \theta_r)$.

By $C([0,T],\Delta_N, R^{nN})$ we denote the space of function systems $x[t] = (x_1(t), x_2(t), \ldots, x_N(t))$, where $x_r : [\theta_{r-1}, \theta_r) \to R^n$ are continuous and have finite left-hand limits $\lim_{t\to\theta_r - 0} x_r(t)$ for all $r = \overline{1,N}$, with the norm $\|x\|_2 = \max_{r=\overline{1,N}} \sup_{t\in[\theta_{r-1},\theta_r)} \|x_r(t)\|$.

Denote by $x_r(t)$ the restriction of function $x(t)$ to the r-th interval $[\theta_{r-1}, \theta_r)$ and reduce problem (1), (2) to the equivalent multipoint boundary value problem with parameter for the loaded differential equations

$$\frac{dx_r}{dt} = A(t)x_r + \sum_{j=0}^{N-1} K_j(t)x_{j+1}(\theta_j) + K_N(t)x(\theta_N) + A_0(t)\mu + f(t), \tag{3}$$

$$t \in (\theta_{r-1}, \theta_r), r = \overline{1,N},$$

$$Bx_1(0) + Cx_N(T) + B_0\mu = d, \tag{4}$$

$$\lim_{t\to\theta_s-0} x_s(t) = x_{s+1}(\theta_s), \quad s=\overline{1,N-1}, \tag{5}$$

$$\lim_{t\to\theta_N-0} x_N(t) = x(\theta_N), \tag{6}$$

where (5), (6) are conditions for matching the solution at the interior points of partition $\Delta_N(\theta)$ and at the point $t=\theta_N$.

The solution of problem (3) - (6) is the triple $(\mu^*, x^*(\theta_N), x^*[t])$ with elements $\mu^*\in R^m$, $x^*(\theta_N)\in R^n$, $x^*[t]=(x_1^*(t), x_2^*(t),\dots, x_N^*(t))\in C([0,T],\Delta_N, R^{nN})$, where functions $x_r^*(t)$, $r=\overline{1,N}$, are continuously differentiable on $[\theta_{r-1},\theta_r)$, which satisfies system of loaded differential equations (3) and boundary condition (4) with $\mu=\mu^*$ and continuity conditions (5), (6).

We introduce additional parameters $\lambda_r = x_r(\theta_{r-1})$, $r=\overline{1,N}$, and $\lambda_{N+1}=x(\theta_N)$, $\lambda_{N+2}=\mu.$ Making the substitution $u_r(t)=x_r(t)-\lambda_r$, on every $r-$th interval $[\theta_{r-1},\theta_r)$, $r=\overline{1,N}$, we obtain multipoint boundary value problem with parameters

$$\frac{du_r}{dt} = A(t)(u_r+\lambda_r) + \sum_{j=0}^{N} K_j(t)\lambda_{j+1} + A_0(t)\lambda_{N+2} + f(t), \quad t\in[\theta_{r-1},\theta_r), \quad p=\overline{1,N}, \tag{7}$$

$$u_r(\theta_{r-1})=0, \; p=\overline{1,N}, \tag{8}$$

$$B\lambda_1 + C\lambda_{N+1} + B_0\lambda_{N+2} = d, \tag{9}$$

$$\lambda_s + \lim_{t\to\theta_s-0} u_s(t) = \lambda_{s+1}, \quad s=\overline{1,N}. \tag{10}$$

A pair $(\lambda^*, u^*[t])$ with elements $\lambda^*=(\lambda_1^*,\lambda_2^*,\dots,\lambda_{N+1}^*,\lambda_{N+2}^*)\in R^{nN+m}$, $u^*[t]=(u_1^*(t),u_2^*(t),\dots,u_N^*(t))\in C([0,T],\Delta_N,R^{nN})$, is said to be a solution to problem (7)-(10) if the functions $u_r^*(t)$, $r=\overline{1,N}$ are continuously differentiable on $[\theta_{r-1},\theta_r)$, and satisfy (7) and additional conditions (9), (10) with $\lambda_j=\lambda_j^*$, $j=\overline{1,N+2}$, and initial conditions (8).

Problem (1), (2) is equivalent to problem (7)-(10) in the following sense. If a pair $(\lambda^*, u^*[t])$ is a solution to problem (7)-(10), then the pair $(x^*[t],\mu^*)$ with function $x^*(t)$ defined by the equalities $x^*(t)=\lambda_r^*+u_r^*(t)$, $t\in[\theta_{r-1},\theta_r)$, $r=\overline{1,N}$, $x^*(T)=\lambda_{N+1}^*$, $\mu^*=\lambda_{N+2}^*$, is a solution to problem (1), (2). Conversely, if a pair $(\widetilde{x}(t),\widetilde{\mu})$ is a solution to problem (1), (2) and $\widetilde{\lambda}_r=\widetilde{x}(\theta_{r-1})$, $\widetilde{\lambda}_{N+1}=\widetilde{x}(\theta_N)$, $\widetilde{\lambda}_{N+2}=\widetilde{\mu}$, $\widetilde{u}_r(t)=\widetilde{x}(t)-\widetilde{x}(\theta_{r-1})$, $t\in[\theta_{r-1},\theta_r)$, $r=\overline{1,N}$, then the pair $(\widetilde{\lambda},\widetilde{u}[t])$ with $\widetilde{\lambda}=(\widetilde{\lambda}_1,\widetilde{\lambda}_2\dots,\widetilde{\lambda}_{N+1},\widetilde{\lambda}_{N+2})\in R^{nN+m}$, and $\widetilde{u}[t]=(\widetilde{u}_1(t),\widetilde{u}_2(t),\dots,\widetilde{u}_N(t))$, is a solution to problem (7)- (10).

Let $X(t)$ be a fundamental matrix to the differential equation $\dfrac{dx}{dt}=A(t)x$ on $[\theta_{r-1},\theta_r)$, $r=\overline{1,N}.$

Then the unique solution to the Cauchy problem for the system of ordinary differential equations (7), (8) at the fixed values $\lambda=(\lambda_1,\lambda_2\dots,\lambda_{N+1},\lambda_{N+2})$ has the following form

$$u_r(t)=X(t)\int_{\theta_{r-1}}^{t}X^{-1}(\tau)A(\tau)d\tau\cdot\lambda_r+X(t)\int_{\theta_{r-1}}^{t}X^{-1}(\tau)\sum_{j=0}^{N}K_j(t)d\tau\lambda_{j+1}+$$

$$+X(t)\int_{\theta_{r-1}}^{t}X^{-1}(\tau)A_0(\tau)d\tau\lambda_{N+2}+X(t)\int_{\theta_{r-1}}^{t}X^{-1}(\tau)f(\tau)d\tau,\ t\in(\theta_{r-1},\theta_r),\ r=\overline{1,N}. \quad (11)$$

Substituting (11) into continuity conditions (10) and taking into account (9), we get the system of algebraic equations with respect to unknown parameters $\lambda=(\lambda_1,\lambda_2,\ldots,\lambda_{N+1},\lambda_{N+2})\in R^{nN+m}$:

$$B\lambda_1+C\lambda_{N+1}+B_0\lambda_{N+2}=d, \quad (12)$$

$$\lambda_s+X(\theta_s)\int_{\theta_{s-1}}^{\theta_s}X^{-1}(\tau)A(\tau)d\tau\cdot\lambda_r+X(\theta_s)\int_{\theta_{s-1}}^{\theta_s}X^{-1}(\tau)\sum_{j=0}^{N}K_j(\tau)d\tau\lambda_{j+1}-\lambda_{s+1}+$$

$$+X(\theta_s)\int_{\theta_{s-1}}^{\theta_s}X^{-1}(\tau)A_0(\tau)d\tau\cdot\lambda_{N+2}+X(\theta_s)\int_{\theta_{s-1}}^{\theta_s}X^{-1}(\tau)f(\tau)d\tau=0,\quad s=\overline{1,N}. \quad (13)$$

Denoting by $Q_{\bullet}(\Delta_N)$ the matrix corresponding to the left-hand side of system (12), (13) which is consist of the coefficients at the parameters λ_r, $r=\overline{1,N+2},$ and then introducing the vector

$$F_*(\Delta_N)=\left(-d,X(\theta_1)\int_{\theta_0}^{\theta_1}X^{-1}(\tau)f(\tau)d\tau,\ldots,X(\theta_N)\int_{\theta_{N-1}}^{\theta_N}X^{-1}(\tau)f(\tau)d\tau\right),$$

we write the system (12), (13) as:

$$Q_*(\Delta_N)\lambda=-F_*(\Delta_N),\quad \lambda\in R^{nN+m}. \quad (14)$$

The boundary value problem (1), (2) is solved by the following algorithm:

As can be seen from the equations (12), (13), the coefficients and right-hand side of the system (14) are composed of solutions to the Cauchy problems

$$\frac{dz}{dt}=A(t)z+A(t),\quad z(\theta_{r-1})=0,\quad r=\overline{1,N}, \quad (15)$$

$$\frac{dz}{dt}=A(t)z+K_j(t),\quad z(\theta_{r-1})=0,\quad r=\overline{1,N}, \quad (16)$$

$$\frac{dz}{dt}=A(t)z+A_0(t),\quad z(\theta_{r-1})=0,\quad r=\overline{1,N}, \quad (17)$$

$$\frac{dz}{dt}=A(t)z+f(t),\quad z(\theta_{r-1})=0,\quad r=\overline{1,N}. \quad (18)$$

Construct the following algorithm for the numerical solving of two-point boundary value problem for the systems of loaded differential equations with parameter by applying the Runge–Kutta method of 4th order for numerical solving of the Cauchy problem (15) – (18).

Suppose we have a partition $\theta_0=0<\theta_1<\ldots<\theta_{N-1}<\theta_N=T$. Divide each subinterval $[\theta_{i-1},\theta_i),\ i=\overline{1,N}$, into N_i parts. Define the approximate values of coefficients and right-hand side of (14) via solutions to the Cauchy matrix and vector problems obtained using the Runge–Kutta method of 4th order with step $h_i=(\theta_i-\theta_{i-1})/N_i,\ i=\overline{1,N}$. Then we obtain the following approximate system of algebraic equations with respect to parameters λ:

$$Q_*^{\tilde{h}}(\Delta_N)\lambda=-F_*^{\tilde{h}}(\Delta_N),\quad \lambda\in R^{nN+m}. \quad (19)$$

Solving the system of linear algebraic equations (19) we find $\lambda^{\tilde{h}} \in R^{nN+m}$.

As noted above, $\lambda^{\tilde{h}} = \left(\lambda_1^{\tilde{h}}, \lambda_2^{\tilde{h}}, \ldots, \lambda_{N+2}^{\tilde{h}}\right) \in R^{nN+m}$ components are the values of approximate solution to problem (1), (2) at the initial points of subintervals:

$$x^{\tilde{h}_r}(\theta_0) = \lambda_1^{\tilde{h}},\ x^{\tilde{h}_r}(\theta_1) = \lambda_2^{\tilde{h}},\ \ldots,\ x^{\tilde{h}_r}(\theta_N) = \lambda_{N+1}^{\tilde{h}},\ \ \mu^{\tilde{h}_r} = \lambda_{N+2}^{\tilde{h}}.$$

Applying the Runge–Kutta method of 4th order for numerical solving of Cauchy problem

$$\frac{dx}{dt} = A(t)x + \sum_{j=0}^{N} K_j(t)\lambda_{j+1}^{\tilde{h}} + A_0\lambda_{N+2}^{\tilde{h}} + f(t),$$

$$x(\theta_{r-1}) = \lambda^{\tilde{h}},\ t \in [\theta_{r-1}, \theta_r), \qquad r = \overline{1, N},$$

we determine the numerical solution to problem (1), (2). To illustrate the proposed approach of the numerical solving of two-point boundary value problem for systems of loaded differential equations with parameter (1), (2) based on the parametrization method, let us consider the following example

Example. Consider on $[0, T]$ the linear two-point boundary value problem for the systems of loaded differential equations with parameter:

$$\frac{dx}{dt} = A(t)x + K_0(t)x(\theta_0) + K_1(t)x(\theta_1) + K_2(t)x(\theta_2) + A_0(t)\mu + f(t),$$

$$t \in [0, T],\ x \in R^2,\ \mu \in R^3, \tag{20}$$

$$Bx(0) + Cx(T) + B_0\mu = d,\ d \in R^5, \tag{21}$$

where $A(t) = \begin{pmatrix} t^2 & t+1 \\ 2 & 3t \end{pmatrix}$, $K_0(t) = \begin{pmatrix} t & 2 \\ t^2 & t-4 \end{pmatrix}$, $K_1(t) = \begin{pmatrix} 8 & t^2+1 \\ t & 3t \end{pmatrix}$, $K_2(t) = \begin{pmatrix} t^3 & 0 \\ t & 4 \end{pmatrix}$,

$$A_0(t) = \begin{pmatrix} 1 & t & t^2 \\ t+3 & 2 & t^3 \end{pmatrix},\quad f(t) = \begin{pmatrix} -2t^4 - 2t^3 - \frac{59}{8}t^2 + 7t + \frac{101}{8} \\ -3t^4 - 12t^3 + 2t^2 + \frac{117}{8}t - 5 \end{pmatrix},$$

$$B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 3 & -2 \\ 0 & 4 \\ 7 & 9 \end{pmatrix},\ C = \begin{pmatrix} 5 & -2 \\ 0 & 4 \\ 5 & 3 \\ 8 & 11 \\ 7 & 8 \end{pmatrix},\ B_0 = \begin{pmatrix} 1 & 3 & -5 \\ 0 & 2 & 4 \\ 4 & -2 & 3 \\ -1 & 3 & 8 \\ 4 & -2 & 0 \end{pmatrix},\ d = \begin{pmatrix} -51 \\ 28 \\ 36 \\ 57 \\ -17 \end{pmatrix}.$$

In this example, the matrix of differential part is variable and the construction of fundamental matrix fails. We use a numerical implementation of algorithm of the parametrization method. Below we provide the results of numerical implementation of the algorithm by partitioning the subintervals [0,0.5], [0.5,1] with step $h_1 = h_2 = 0.05$.

Solving the system of equations (19) we obtain the numerical values of the parameters:

$$\lambda_1^{\tilde{h}} = \begin{pmatrix} -0.99999562 \\ -2.00000517 \end{pmatrix},\ \lambda_2^{\tilde{h}} = \begin{pmatrix} -0.74999836 \\ -1.62499829 \end{pmatrix},\ \lambda_3^{\tilde{h}} = \begin{pmatrix} 0.00000017 \\ 0.00000511 \end{pmatrix}.$$

We find the numerical solutions at the other points of the subintervals applying the Runge-Kutta method of the 4th order to the following Cauchy problems:

$$\frac{dx_1}{dt} = A(t)x_1 + K_0(t)\lambda_1^{\tilde h} + K_1(t)\lambda_2^{\tilde h} + K_2(t)\lambda_3^{\tilde h} + B(t)\lambda_4^{\tilde h} + f(t), \quad t \in \left[0, \frac{1}{2}\right), \quad x_1(0) = \lambda_1^{\tilde h},$$

$$\frac{dx_2}{dt} = A(t)x_2 + K_0(t)\lambda_1^{\tilde h} + K_1(t)\lambda_2^{\tilde h} + K_2(t)\lambda_3^{\tilde h} + B(t)\lambda_4^{\tilde h} + f(t), \quad t \in \left[\frac{1}{2}, 1\right), \quad x_2\left(\frac{1}{2}\right) = \lambda_2^{\tilde h}.$$

The exact solution of the problem (20)-(21) is a pair $(\mu^*, x^*(t))$, where $\mu^* = \begin{pmatrix} 1 \\ -2 \\ 9 \end{pmatrix}$,

$x^*(t) = \begin{pmatrix} t^2 - 1 \\ t^3 + t^2 - 2 \end{pmatrix}, \quad t \in [0,1].$

The results of calculations of numerical and exact solutions at the partition points are presented in the following table:

t	$\tilde x_1(t)$ (numerical solution)	$x_1^*(t)$	$\tilde x_2(t)$ (numerical solution)	$x_2^*(t)$
0	-0.99999562	-1	-2.00000517	-2
0.05	-0.99749604	-0.9975	-1.99737924	-1.997375
0.1	-0.98999643	-0.99	-1.98900339	-1.989
0.15	-0.97749678	-0.9775	-1.97412762	-1.974125
0.2	-0.9599971	-0.96	-1.9520019	-1.952
0.25	-0.93749739	-0.9375	-1.92187622	-1.921875
0.3	-0.90999764	-0.91	-1.88300059	-1.883
0.35	-0.87749786	-0.8775	-1.83462498	-1.834625
0.4	-0.83999805	-0.84	-1.7759994	-1.776
0.45	-0.79749822	-0.7975	-1.70637383	-1.706375
0.5	-0.74999836	-0.75	-1.62499829	-1.625
0.55	-0.69749848	-0.6975	-1.53112275	-1.531125
0.6	-0.63999858	-0.64	-1.42399724	-1.424
0.65	-0.57749867	-0.5775	-1.30287174	-1.302875
0.7	-0.50999875	-0.51	-1.16699627	-1.167
0.75	-0.43749884	-0.4375	-1.01562083	-1.015625
0.8	-0.35999893	-0.36	-0.84799544	-0.848
0.85	-0.27749906	-0.2775	-0.66337012	-0.663375
0.9	-0.18999923	-0.19	-0.46099489	-0.461
0.95	-0.09749948	-0.0975	-0.2401198	-0.240125
1	0.00000017	0	0.00000511	0

$\tilde\mu_1 = \lambda_{41}^{\tilde h}$ (numerical solution)	μ_1^*	$\tilde\mu_2 = \lambda_{42}^{\tilde h}$ (numerical solution)	μ_2^*	$\tilde\mu_3 = \lambda_{43}^{\tilde h}$ (numerical solution)	μ_3^*
0.9999919	1	-2.00000308	-2	8.99999553	9

For the difference of the corresponding values of the exact and constructed solutions of the problem the following estimate is true:

$$\max\left\|\mu^* - \tilde\mu\right\| < 0.0000081,$$

$$\max_{j=0,20}\left\|x^*(t_j) - \tilde x(t_j)\right\| < \varepsilon, \quad \varepsilon = 0.000005.$$

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ПАРАМЕТРІ БАР ЖҮКТЕЛГЕН ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕР ЖҮЙЕСІ ҮШІН ШЕТТІК ЕСЕПТІ ШЕШУДІҢ САНДЫҚ ЖҮЗЕГЕ АСЫРЫЛУЫ

Аннотация. Параметрі бар жүктелген дифференциалдық теңдеулер үшін сызықты екі нүктелі шеттік есеп қарастырылады. Аталған есеп параметрлеу әдісі арқылы зерттеледі. Параметрі бар жүктелген дифференциалдық теңдеулер жүйесі үшін шеттік есептің шешімін табудың алгоритмі ұсынылады. Алдымен бастапқы есеп ішкіаралықтардағы параметрлері бар жәй дифференциалдық теңдеулер жүйесі үшін Коши есебін және енгізілген параметрлерге қатысты функционалдық қатынастарды қамтитын пара-пар есепке келтіріледі. Параметрлердің бекітілген мәнінде ішкіаралықтағы жәй дифференциалдық теңдеулер жүйесі үшін Коши есебінің жалғыз шешімі бар болады. Бұл шешім жүйенің фундаменталдық матрицасы арқылы кейіптеледі. Осы кейіптемелерді пайдалана отырып параметрлерге қатысты сызықты алгебралық теңдеулер жүйесін құрамыз. Пара-пар есептің сандық шешімін табуға арналған алгоритм ұсынылады. Бұл алгоритм жәй дифференциалдық теңдеулер жүйесі үшін Коши есептерін сандық шешуді және алгебралық теңдеулер жүйесін шешуді қамтиды. Коши есептерін сандық түрде шешу үшін төртінші ретті Рунге-Кутганың әдісі қолданылады. Ұсынылып отырған сандық жүзеге асырылу мысалмен көрнектеледі.

Кілттік сөздер: параметрі бар шеттік есеп, жүктелген дифференциалдық теңдеу, сандық әдіс, алгоритм.

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ЧИСЛЕННАЯ РЕАЛИЗАЦИЯ РЕШЕНИЯ КРАЕВОЙ ЗАДАЧИ ДЛЯ СИСТЕМЫ НАГРУЖЕННЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С ПАРАМЕТРОМ

Аннотация. Рассматривается линейная двухточечная краевая задача для системы нагруженных дифференциальных уравнений с параметром. Данная задача исследуется методом параметризации. Предлагается алгоритм нахождения решения краевой задачи для системы нагруженных дифференциальных уравнений с параметром. Вначале исходная задача сводится к эквивалентной задаче, состоящей из задач Коши для системы обыкновенных дифференциальных уравнений с параметрами на подинтеравалах и функциональных соотношений относительно введенных дополнительных параметров. При фиксированных значениях параметров задача Коши для системы обыкновенных дифференциальных уравнений на подинтервале имеет единственное решение. Это решение представляется через фундаментальную матрицу системы. Используя эти представления составляется система линейных алгебраических уравнений относительно параметров. Предлагается алгоритм нахождения численного решения эквивалентной задачи. Данный алгоритм включает численное решение задач Коши для системы обыкновенных дифференциальных уравнений и решение линейной системы алгебраических уравнений. Для численного решения задачи Коши применяется метод Рунге-Кутта четвертого порядка. Предлагаемая численная реализация иллюстрируется примером.

Ключевые слова: краевая задача с параметром, нагруженное дифференциальное уравнение, численный метод, алгоритм.

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