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THE COORDINATED SOLUTION OF TWO DIFFERENTIAL EQUATIONS IN PRIVATE DERIVATIVES OF THE THIRD ORDER

Abstract. In the study the possibility of constructing a solution near a variety of special features of the system consisting of two differential equations in partial derivatives of the third order was investigated. There definitive aspects of such systems in comparison with systems consisting of two differential equations in partial derivatives of second order were established. The classification of regular and irregular singularities was carried out, the relevant solutions are constructed. A specific example shows that the application of the Frobenius - Latysheva method to the construction of a solution of a degenerate hypergeometric system consisting of two third-order partial differential equations. A variety of properties of system solution were considered, which shows that these solutions have the properties of generalized hypergeometric functions.

Key words: solution, constructed, system, features, regular, irregular, generalized, third order.

Introduction. The solutions of particular cases of systems of differential equations of second order type

$$\left.\begin{aligned} G^{(0)}\cdot Z_{xx}+G^{(1)}\cdot Z_{xy}+G^{(2)}\cdot Z_x+G^{(3)}\cdot Z_y+G^{(4)}\cdot Z=0\\ Q^{(0)}\cdot Z_{yy}+Q^{(1)}\cdot Z_{xy}+Q^{(2)}\cdot Z_x+Q^{(3)}\cdot Z_y+Q^{(4)}\cdot Z=0\end{aligned}\right\} \tag{1}$$

where $G^{(i)}=G^{(i)}(x,y), Q^{(i)}=Q^{(i)}(x,y), (i=\overline{0,4})$ analytic functions or polynomials of two variables, $Z=Z(x,y)$ – total unknown, are the generalized hypergeometric functions of two variables. The general theory of systems type (1) was established by American mathematician E. Wilczynski [1] - [2] using it to support special cases of projective differential geometry. He proved compatibility and integrability conditions

$$1-\frac{G^{(1)}}{G^{(0)}}\cdot\frac{Q^{(1)}}{Q^{(0)}}\neq 0 \tag{2}$$

of system type (1) [1].

In carrying out these important conditions the following statement is fair.

Theorem 1. Suppose that the conditions of compatibility and integrability are fulfilled (2). Then, system (1) has four linearly independent particular solutions of the form

$$Z_j(x,y)=\sum_{m,n=0}^{\infty}A_{m,n}\cdot x^m\cdot y^n,\ (A_{0,0}\neq 0),\ \ (j=1,2,3,4) \tag{3}$$

The overall solution of system (1) is represented as the sum

$$Z_k(x,y)=\sum_{k=1}^{4} G_j \cdot Z_k(x,y),\ \ k=1,2,3,4 \tag{4}$$

where G_j- is arbitrary constant, that is, the general solution of a system of partial differential equations depends on arbitrary constants, and not on an arbitrary constant.

Theorem 2. If the compatibility conditions are satisfied, and the integrability condition is not satisfied, that is,

$$1-\frac{G^{(1)}}{G^{(0)}}\cdot\frac{Q^{(1)}}{Q^{(0)}}=0 \tag{5}$$

then system (1) has no more than three linearly independent solutions, or the number of them will be countless.

Further development of the study was obtained in the works of J. Horn [3] - [4], P. Appell, J. Kampe de Feriet [5], W. Sternberg [6], A. Erdelyi [7] - [8], P. Humbert [9], E. Ince [10], L. Borngesser [11] and others, in connection with the study of the theory of generalized hypergeometric series of two variables [12] - [13] - [14]. Using the concepts of rank $p=1+k$ ($k-$subrank) and antiranga $m=-1-\lambda$ ($\lambda-$ antipodrang). Zh.N. Tasmambetov developed [15] efficient algorithms for constructing normal, subnormal series, and normal-regular and final solutions of systems of the form (1) consisting of two second-order partial differential equations.

The aim of this work is to study a special system consisting of two differential equations in partial derivatives of the third order; to establish the distinctive features of such systems; to classify regular and irregular features and construct the corresponding solutions; to show the application of the Frobenius-Latysheva method to constructing a solution of a degenerate hypergeometric system consisting of two third-order partial differential equations by using a specific example.

THE COORDINATED SOLUTION OF TWO DIFFERENTIAL EQUATIONS IN PRIVATE DERIVATIVES OF THE THIRD ORDER

Statement. Studying the possibility of coordinated solving a homogeneous system of third-order partial differential equations consisting of two equations of the form

$$\begin{aligned} &x^3g^{(0)}p_{30}+x^2yg^{(1)}p_{21}+x^2g^{(2)}p_{20}+xyg^{(3)}p_{11}+xg^{(4)}p_{10}+yg^{(5)}p_{01}+g^{(6)}p_{0,0}=0,\\ &y^3g^{(0)}p_{03}+xy^2g^{(1)}p_{12}+y^2g^{(2)}p_{02}+xyg^{(3)}p_{11}+xg^{(4)}p_{10}+yg^{(5)}p_{01}+g^{(6)}p_{0,0}=0,\end{aligned} \tag{6}$$

where

$$\begin{aligned} &p_{30}=Z_{xxx},p_{03}=Z_{yyy},p_{21}=Z_{xxy},p_{12}=Z_{xyy},p_{20}=Z_{xx},p_{02}=Z_{yy},\\ &p_{11}=Z_{xy},p_{10}=Z_x,p_{01}=Z_y,p_{0,0}=Z(x,y)\end{aligned} \tag{7}$$

– total unknown, and the coefficients

$$\begin{aligned} &g^{(i)}=g^{(i)}(x,y)=a_{00}^{(i)}+a_{10}^{(i)}\cdot x,\\ &q^{(i)}=q^{(i)}(x,y)=b_{00}^{(i)}+b_{01}^{(i)}\cdot y,(i=\overline{0,6}).\end{aligned} \tag{8}$$

It is required to classify regular and irregular singularities and establish the type of solution near each singularity; to determine the number of linearly independent solutions near these features; to examine the relationship of the original system with systems solutions that are generalized hypergeometric functionы of two variables and properties of such functions.

Solution of a system of differential equations.

To construct solutions of the system of equations (6), it is required to carry out a number of conditions, let us discuss the major of them

1. It must be carried out the compatibility conditions. However, the general compatibility condition, as for system (1), is difficult to control. Usually, such checking is carried out for specific systems solutions which are generalized hypergeometric functions of two variables. The Kampe de Feriet method can be applied [5, p.155], [16, p.21] - [17].
2. Integrability condition

$$1-\frac{g^{(1)}}{g^{(0)}}\cdot\frac{q^{(1)}}{q^{(0)}}\neq 0. \quad (9)$$

3. Unknown function $p_{0,0}=Z(x,y)$ depends on two variables, therefore, as before, depending on the regularity and irregularity of the singular curves, solutions should be sought in the form of generalized, normal and normal-regular series of two variables [15, p.159].

4. If in (6) the coefficients at higher derivatives p_{30} and p_{03}: $x^3\cdot g^{(0)}(x,y)\equiv 1$ and $y^3\cdot g^{(0)}(x,y)\equiv 1$ then solution can be sought as a simple series of two variables (3) since there are no singularities. Then, in order to formulate Theorems 1 and 2 for this case, it is necessary to establish the number of linearly independent particular solutions of system (6), which we will do next.

5. The special curves of the system (6) are established by equating to zero the coefficients of the highest derivatives. $p_{30}=Z_{xxx}$ и $p_{03}=Z_{yyy}$:

$$x^3\cdot(a_{00}^{(0)}+a_{10}^{(0)}\cdot x)=0,\ \ x_1=0,\ \ x_2=-\frac{a_{00}^{(0)}}{a_{10}^{(0)}},$$

$$y^3\cdot(b_{00}^{(0)}+b_{01}^{(0)}\cdot y)=0,\ \ y_1=0,\ \ y_2=-\frac{b_{00}^{(0)}}{b_{01}^{(0)}}.$$

Composed from them pairs $(0,0),(0,-\frac{b_{00}^{(0)}}{b_{01}^{(0)}}),(-\frac{a_{00}^{(0)}}{a_{01}^{(0)}},0),(-\frac{a_{00}^{(0)}}{a_{01}^{(0)}},-\frac{b_{00}^{(0)}}{b_{01}^{(0)}})$ - represent the final features, $(0,\infty),(\infty,0),(\infty,-\frac{b_{00}^{(0)}}{b_{01}^{(0)}}),(-\frac{a_{00}^{(0)}}{a_{01}^{(0)}},\infty)$ and (∞,∞) - features at infinity. Normally, when building solutions two pairs of features identified (0,0) and (∞,∞). Classification for regularity and irregularity will be carried out using simple rules [15].

Rule I. If in (6) coefficient $a_{00}^{(0)}\neq 0, b_{00}^{(0)}\neq 0,$ then the feature (0,0) is special regular. When $a_{00}^{(0)}=0, b_{00}^{(0)}=0,$ then (0,0) is a special irregular.

Rule II. If in (6) coefficient $a_{10}^{(0)}\neq 0$ and $b_{01}^{(0)}\neq 0,$ then the feature (∞,∞) – special regular, and when $a_{10}^{(0)}=0, b_{01}^{(0)}=0$ – special irregular.

Features of constructing a solution by the Frobenius-Latyshev method.

The Frobenius-Latyshev method allows determining directly the structure and construct solutions of the system near particular curves. So, when the feature (0,0) is regular, the desired solution is represented as a generalized power series of two variables

$$Z(x,y)=x^{\rho}\cdot y^{\sigma}\cdot\sum_{m,n=0}^{\infty}A_{m,n}\cdot x^m\cdot y^n, \quad (10)$$

where ρ,σ and $A_{m,n}$ $(m,n=0,1,2,3,...)$ – unknown constants to be determined. If the feature (0,0) is irregular, then the desired solution is represented as

$$Z(x,y)=\exp Q(x,y)\cdot x^{\rho}\cdot y^{\sigma}\cdot\sum_{m,n=0}^{\infty}A_{m,n}\cdot x^m\cdot y^n,\quad A_{0,0}\neq 0 \quad (11)$$

$Q(x,y)$ – polynomial of two variables

$$Q(x,y)=\frac{\alpha_{p1}\cdot x^p}{p}+\frac{y^p\cdot\alpha_{0p}}{p}+...+\alpha_{11}\cdot x\cdot y+\alpha_{10}\cdot x+\alpha_{01}\cdot y, \quad (12)$$

with uncertain parameters $\alpha_{p,0},\alpha_{0,p},...,\alpha_{11},\alpha_{10},\alpha_{01.}$

Similarly, near the regular singularity (∞,∞) the solution is constructed in the form

$$Z(x,y) = x^{\rho} \cdot y^{\sigma} \cdot \sum_{m,n=0}^{\infty} B_{m,n} \cdot x^{-m} \cdot y^{-n}, \quad B_{0,0} \neq 0 \tag{13}$$

and near irregular features in the form

$$Z(x,y) = \exp(x,y) \cdot x^{\rho} \cdot y^{\sigma} \cdot \sum_{m,n=0}^{\infty} B_{m,n} \cdot x^{-m} \cdot y^{-n}, \ B_{0,0} \neq 0 \tag{14}$$

where ρ,σ and $B_{m,n}$ $(m,n = 0,1,2,3,...)$ – unknown constant.

Polynomial $Q(x,y)$ is common to the type (11) and (14). Its degree is determined by the rank value

$$p = 1 + k, \ k = \max_{(1\leq s\leq n)} \frac{\beta_s - \beta_0}{s} \tag{15}$$

introduced by A. Poincare to study ordinary differential equations, and β_0, β_s - the greatest degrees of the coefficients of the equation [18].

MAIN RESULTS

We will start the application of the Frobenius-Latysheva method with drawing up a system of Frobenius characteristic functions [15].

Definition 1. The system of characteristic functions of Frobenius is called the system

$$\begin{aligned} L_1\left[x^{\rho} \cdot y^{\sigma}\right] &\equiv x^{\rho} \cdot y^{\sigma} \cdot \left[f_{00}^{(1)}(\rho,\sigma) + f_{11}^{(1)}(\rho,\sigma) \cdot x\right], \\ L_2\left[x^{\rho} \cdot y^{\sigma}\right] &\equiv x^{\rho} \cdot y^{\sigma} \cdot \left[f_{00}^{(2)}(\rho,\sigma) + f_{11}^{(1)}(\rho,\sigma) \cdot y\right], \end{aligned} \tag{16}$$

where

$$f_{00}^{(1)}(\rho,\sigma) = a_{00}^{(0)}\rho(\rho-1)(\rho-2) + a_{00}^{(1)}\rho(\rho-1)\sigma + a_{00}^{(2)}\rho(\rho-1) + a_{00}^{(3)}\rho\sigma + a_{00}^{(4)}\rho + a_{00}^{(5)}\sigma + a_{00}^{(6)} \tag{16.1}$$

$$f_{00}^{(2)}(\rho,\sigma) = b_{00}^{(0)}\sigma(\sigma-1)(\sigma-2) + b_{00}^{(1)}\rho\sigma(\sigma-1) + b_{00}^{(2)}\sigma(\sigma-1) + b_{00}^{(3)}\rho\sigma + b_{00}^{(4)}\rho + b_{00}^{(5)}\sigma + b_{00}^{(6)} \tag{16.2}$$

$$f_{10}^{(1)}(\rho,\sigma) = a_{10}^{(0)}\rho(\rho-1)(\rho-2) + a_{10}^{(1)}\rho(\rho-1)\sigma + a_{10}^{(2)}\rho(\rho-1) + a_{10}^{(3)}\rho\sigma + a_{10}^{(4)}\rho + a_{10}^{(5)}\sigma + a_{10}^{(6)} \tag{16.3}$$

$$f_{01}^{(2)}(\rho,\sigma) = b_{01}^{(0)}\sigma(\sigma-1)(\sigma-2) + b_{01}^{(1)}\rho\sigma(\sigma-1) + b_{01}^{(2)}\sigma(\sigma-1) + b_{01}^{(3)}\rho\sigma + b_{01}^{(4)}\rho + b_{01}^{(5)}\sigma + b_{01}^{(6)} \tag{16.4}$$

obtained by substituting into system (6) with coefficients of the form (8) instead of $Z = x^{\rho} \cdot y^{\sigma}$. Define the system of determining equations for features (0,0) and (∞,∞) from (16).

Definition 2. The system of defining equations for the feature (0,0) is called the system

$$f_{00}^{j}(\rho,\sigma) = 0 \quad (j = 1,2) \tag{17}$$

from which the indicators of series (10) and (11) are determined in the form of pairs (ρ_t,σ_t).

Definition 3. A system of defining equations for a feature (∞,∞) is called a system

$$\begin{aligned} f_{10}(\rho,\sigma) &= 0, \\ f_{01}(\rho,\sigma) &= 0 \end{aligned} \tag{18}$$

from which the indicators of series (13) and (14) are determined in the form of pairs (ρ_t,σ_t).

In this pairs (ρ_t,σ_t) it is important to determine the index t, since the number of such pairs allows determining the number of linearly independent solutions of the original system (6) near the specified features.

Theorem 3. For the existence the solutions of the form (10) in the system (6) with coefficients (8) near the singularity (0.0) requires the equality (17).**Theorem 4.** For the existence of solutions of the form (13) in the system (6) with coefficients (8) near the singularity (∞,∞) equality is necessary (18).

Let's determine how many roots have the systems of constitutive equations (17) and (18). In view of this, the system (17) can be written in expanded form, using (16.1) and (16.2). From $f_{0,0}^{(1)}(\rho,\sigma)=0$ we will find out

$$\sigma=\frac{a_{00}^{(0)}\rho(\rho-1)(\rho-2)+a_{00}^{(2)}\rho(\rho-1)+a_{00}^{4}\rho+a_{10}^{(6)}}{a_{00}^{(1)}\rho(\rho-1)+a_{00}^{(3)}\rho+a_{00}^{(5)}}$$

and substituting in the second equation $f_{00}^{(2)}(\rho,\sigma)=0$ of the system (17), after exclusion ρ, we obtain the ninth degree equation for σ. If there are only simple roots, then from here we define nine roots σ_t $(t=\overline{1,9})$. Similarly, you can define nine simple roots ρ_t $(t=\overline{1,9})$. Of these, make nine pairs of roots (ρ_t,σ_t) $(t=\overline{1,9})$ of system (17), which allows us to construct nine linearly independent particular solutions of system (17) near the singularity (0,0).

Theorem 5. Suppose the system (6) with coefficients of the form (8), where $a_{00}^{(0)}\neq 0, b_{00}^{(0)}\neq 0$ conditions of compatibility and integrability are satisfied (9). Then system (6) has nine linearly independent regular particular solutions of the form

$$Z_t(x,y)=x^{\rho_t}\cdot y^{\sigma_t}\cdot\sum_{m,n=0}^{\infty}A_{m,n}^{(t)}\cdot x^m\cdot y^n,\quad A_{0,0}^{(t)}\neq 0,\ (t=\overline{1,9}), \tag{19}$$

near feature (0,0), where ρ_t,σ_t $(t=\overline{1,9})$, $A_{m,n}^{(t)}$ $(m,n=0,1,2,...)$.

A similar argument can be sure that the system (6) also has nine linear - independent particular solutions near the singularity (∞,∞).

Theorem 6. Suppose the system (6) with coefficients of the form (8), where $a_{10}^{(0)}\neq 0, b_{01}^{(0)}\neq 0$ conditions of compatibility and integrability are satisfied (9). Then the system (6) has nine linearly independent regular particular solutions of the form

$$Z_t(\rho,\sigma)=x^{\rho_t}\cdot y^{\sigma_t}\cdot\sum_{m,n=0}^{\infty}B_{m,n}^{(t)}\cdot x^m\cdot y^n,\quad B_{0,0}^{(t)}\neq 0,\ (t=\overline{1,9}), \tag{20}$$

near feature (∞,∞), where $\rho_t,\sigma_t,(t=\overline{1,9}),B_{m,n}^{(t)}$ $(m,n=0,1,2,...)$ - unknown constant.

In the theorem 5 and 6, conditions $a_{00}^{(0)}\neq 0, b_{00}^{(0)}\neq 0$ and $a_{10}^{(0)}\neq 0, b_{01}^{(0)}\neq 0$ essential since, ninth degree equations are relatively ρ and σ it turns out only when they are non-zero. Therefore, near regular singular curves (0,0) and (∞,∞) there are nine regular linearly independent particular solutions. Then, the overall solution is represented as the following sum

$$Z(x,y)=\sum_{t=1}^{9}C_t\cdot Z_t(x,y),\ (t=\overline{1,9}). \tag{21}$$

Unknown row coefficients (19) $A_{m,n}^{(t)}$ $(m,n=0,1,2,...)$ are determined from systems of recurrent sequences

$$\sum_{m,n=0}^{\infty}A_{\mu-m,\nu-n}^{(t)}\cdot f_{m,n}^{(j)}(\rho+\mu-m,\sigma+\nu-n)=0 \tag{22}$$

$(\mu,\nu=0,1,2,...,;j=1,2;t=\overline{1,9})($ obtained by substituting series (19) into system (6) with coefficients (8).

Application to the construction of a specific system solutions.

All hypergeometric functions of two variables satisfy a system consisting of two partial differential equations of second order. The coefficient of such systems are polynomials x and y. These coefficients can be calculated if in the hypergeometric functions defined by double series

$$F(x,y) = \sum_{m,n} a_{m,n} \cdot x^m \cdot y^n, \tag{23}$$

the coefficients satisfy the following relations

$$\frac{a_{m+1,n}}{a_{m,n}} = \frac{P(m,n)}{R(m,n)}, \ \frac{a_{m,n+1}}{am,n} = \frac{Q(m,n)}{S(m,n)}, \tag{24}$$

where P, R, Q, S are known polynomials.

The use of such an approach enabled the intensification of the study of relations between systems consisting of two second-order equations and generalized hypergeometric series of two variables.

However, the relationship between systems consisting of two equations of the third and fourth orders and generalized hypergeometric series has not been studied at the proper level. Here, it should be noted the work of Kampe de Feriet [5], [16] - [17], which provides a method for constructing a series of third and fourth order systems, while ensuring the compatibility of such systems. In this paper we will also proceed from the studies of J. Kampe de Feriet [5, p.155-169].

Theorem 7. The system of partial differential equations consisting of two equations of third order

$$\begin{aligned} &x^2 \cdot p_{30} + (\delta_1 + \delta_2 + 1) \cdot x \cdot p_{20} + (\delta_1 \cdot \delta_2 - x) \cdot p_{10} - y \cdot p_{01} - \alpha \cdot p_{0,0} = 0, \\ &y^2 \cdot p_{30} + (\delta_1^{'} + \delta_2^{'} + 1) \cdot y \cdot p_{02} + (\delta_1 \cdot \delta_2 - y) \cdot p_{01} - x \cdot p_{10} - \alpha \cdot p_{0,0} = 0, \end{aligned} \tag{25}$$

where $p_{30} = Z_{xxx}, p_{03} = Z_{yyy}, p_{20} = Z_{xx}, p_{02} = Z_{yy}, p_{10} = Z_x, p_{01} = Z_y, p_{0,0} = Z(x,y)$ – common unknown, has nine linearly independent regular particular solutions, and the general solution is represented as the sum (21).

Argument. We will carry out the proof by the Frobenius-Latyshev method, while revealing the additional properties of system (25) can be obtained from the original system (6) with coefficients of the form (8) using the Kampe de Feriet method [5, p.155-164]. As we noted above, this ensures the compatibility of the two equations of the system (25). The integrability condition (9) is also satisfied, since $g^{(1)} = q^{(1)} \equiv 0$.

Coefficients $a_{00}^{(0)} \neq 0$ и $b_{00}^{(0)} \neq 0$, so the feature (0,0) is regular based on the Rule I, a $a_{10}^{(0)} = 0, b_{01}^{(0)} = 0$, therefore, on the basis of the Rule II feature (∞,∞) – irregular. Let us build regular solutions in the form (10) near feature (0,0).

To this end, we compose a system of Frobenius characteristic functions (15) - (16) and define systems of defining equations (17) with respect to the singularity (0,0), in a transformed expanded form

$$\left.\begin{aligned} f_{00}^{(1)}(\rho,\sigma) = \rho \cdot (\rho - 1 + \delta_1) \cdot (\rho - 1 + \delta_2) = 0, \\ f_{00}^{(2)}(\rho,\sigma) = \sigma \cdot (\sigma - 1 + \delta_1) \cdot (\sigma - 1 + \delta_2) = 0. \end{aligned}\right\} \tag{26}$$

It has nine pairs of roots $(\rho_t, \sigma_t, t = \overline{1,9})$:

1.$(\rho_1 = 0, \sigma_1 = 0)$, 2.$(\rho_2 = 1 - \delta_{,1}\, \sigma_1 = 0)$, 3.$(\rho_1 = 0, \sigma_2 = 1 - \delta_1^{'})$, 4.$(\rho_3 = 1 - \delta_2, \sigma_1 = 0)$,
5.$(\rho_1 = 0, \sigma_3 = 1 - \delta_2^{'})$, 6.$(\rho_2 = 1 - \delta_1, \sigma_2 = 1 - \delta_1^{'})$, 7.$(\rho_2 = 1 - \delta_1, \sigma_3 = 1 - \delta_2^{'})$,
8.$(\rho_3 = 1 - \delta_2, \sigma_2 = 1 - \delta_1^{'})$, 9. $(\rho_3 = 1 - \delta_2, \sigma_3 = 1 - \delta_2^{'})$.

Then, on the basis of Theorem 5, the system (25) near the singularity (0,0) has nine linearly - independent regular partial solutions, corresponding to these indicators.:

$$
\begin{aligned}
&Z_1 = \Phi(\alpha,\delta_{1,}\delta_1^{'},\delta_2,\delta_2^{'};x,y),\\
&Z_2 = x^{1-\delta_1}\cdot\Phi(\alpha+1-\delta_{1,};2-\delta_1^{'},\delta_1^{'},\delta_2+1-\delta_1,\delta_2^{'};x,y),\\
&Z_3 = y^{1-\delta_1^{'}}\cdot\Phi(\alpha+1-\delta^{'}_{1,},\delta_1;2-\delta_1^{'},\delta_2,\delta_2^{'}+1-\delta^{'}_{1};x,y),\\
&Z_4 = x^{1-\delta_2}\cdot\Phi(\alpha+1-\delta_2;\delta_1+1-\delta\ ,\delta_1^{'},2-\delta_2^{'};x,y),\\
&Z_5 = y^{1-\delta_2^{'}}\cdot\Phi(\alpha+1-\delta_2^{'},\delta_1;\delta_1^{'}+1-\delta_2^{'},2-\delta_2^{'};x,y),\\
&Z_6 = x^{1-\delta_1}\cdot y^{1-\delta_1^{'}}\cdot\Phi(\alpha+2-\delta_1,\delta_1^{'};2-\delta_1,2-\delta_1^{'},\delta_1^{'},\delta_2+1-\delta_1,\delta_2^{'}+1-\delta_1^{'};x,y),\\
&Z_7 = x^{1-\delta_1}\cdot y^{1-\delta_2^{'}}\cdot\Phi(\alpha+2-\delta_1-\delta_2^{'};2-\delta_1,2-\delta_1\cdot\delta_1^{'},\delta_1^{'}+1-\delta_2^{'},\delta_2+1-\delta_1,2-\delta_1^{'};x,y),\\
&Z_8 = x^{1-\delta_2}\cdot y^{1-\delta_2^{'}}\cdot\Phi(\alpha+2-\delta_2-\delta_1,\delta_1+1-\delta_2,2-\delta_1^{'}2-\delta_2,\delta_2^{'}+1-\delta_1^{'};x,y),\\
&Z_9 = x^{1-\delta_2}\cdot y^{1-\delta_2^{'}}\cdot\Phi(\alpha+2-\delta_2-\delta_2^{'},\delta_1+1-\delta_2,\delta_1+1-\delta_2.\delta_1^{'}+1-\delta_2^{'},2-\delta_2,2-\delta_2^{'};x,y).
\end{aligned}
\tag{27}
$$

All the solutions obtained coincide exactly with the solutions in the monograph. [5, p.166].

Conclusions: We note some properties of the studied system and its solution.

Properties 1. The monograph [5] does not classify the regular and irregular features of the system (25). In system (25) $a_{10}^{(0)}=0$ and $b_{01}^{(0)}=0$. Therefore, on the basis of Rule II, we conclude that the peculiarity (∞,∞) irregular and system (25) must have a solution in the form of a normal Tome series (14). However, in this case, the system of defining equations will be written as

$$f_{10}(\rho,\sigma)=-(\rho+\sigma+2)=0,$$
$$f_{01}(\rho,\sigma)=-(\rho+\sigma+2)=0$$

that is, it boils down to one equation and defines an infinite number of solutions. Then, due to the uncertainty of the indicators (ρ_t,σ_t) there is no solution in the form of a normal Tome series (14)

Properties 2. It is also not difficult to make sure that the solutions of system (25) presented in the form of (27) possess many properties of generalized hypergeometric series [5],[14]. Thus, the first derivatives (27) are represented as

$$
\begin{aligned}
&\frac{\partial Z_1}{\partial x}=\frac{\partial\Phi\left[\alpha,\delta_1,\delta_1^{'},\delta_2,\delta_2^{'};x,y\right]}{\partial x}=\frac{\alpha}{\delta_1\cdot\delta_2}\cdot\Phi\left[\alpha+1,\delta_1+1,\delta_1^{'};1+\delta_2,\delta_2^{'};x,y\right]\\
&\frac{\partial Z_1}{\partial y}=\frac{\partial\Phi\left[\alpha,\delta_1,\delta_1^{'},\delta_2,\delta_2^{'};x,y\right]}{\partial y}=\frac{\alpha}{\delta_1^{'}\cdot\delta_2^{'}}\cdot\Phi\left[\alpha+1,\delta_1,\delta_1^{'}+1,\delta_2,\delta_2^{'}+1;x,y\right]\\
&\frac{\partial Z_1}{\partial x\partial y}=\frac{\partial\Phi\left[\alpha,\delta_1,\delta_1^{'},\delta_2,\delta_2^{'};x,y\right]}{\partial x\partial y}=\frac{\alpha(\alpha+1)}{\delta_1\cdot\delta_2\cdot\delta_1^{'}\cdot\delta_2^{'}}\cdot\Phi\left[\alpha+2,\delta_1+1,\delta_1^{'}+1,\delta_2+1,\delta_2^{'}+1;x,y\right]
\end{aligned}
\tag{28}
$$

Properties 3. The first solution $Z_1(x,y)$ represents a double row depending on five parameters $\alpha,\delta_1,\delta_2,\delta_1^{'},\delta_2^{'}$ [14]:

$$Z_1(x,y)={}_1\Phi_4\begin{bmatrix}\alpha\\ \delta_1,\delta_1^{'},\delta_2,\delta_2^{'}\end{bmatrix};x,y=\sum_{m,n=0}^{\infty}\frac{(\alpha)_{m+n}}{(\delta_1)_m,(\delta_1^{'})_n(\delta_2)_m(\delta_2^{'})_n}\cdot\frac{x^m}{m!}\cdot\frac{x^n}{n!} \tag{29}$$

In the same way, it can be verified that the remaining solutions (27) have similar properties, that is, all the functions listed in (27) are generalized hypergeometric functions [14], [19]. The question of the computational application of special functions of several variables, as in monographs, remains topical [20]. The development [21] of a numerical method for calculating the values of the degenerate hypergeometric Humbert functions is also required $\Psi_2(\alpha,\gamma,\gamma';x,y)$ through the products of Laguerre polynomials [22].

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ҮШІНШІ РЕТТІ ДЕРБЕС ТУЫНДЫЛЫ ЕКІ ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕРДІҢ БІРЛЕСКЕН ШЕШІМІ

Аннотация. Мақалада үшінші ретті дербес туындылы екі дифференциалдық теңдеулерден тұратын арнайы жүйенің ерекше нүктелерінің маңайындағы шешімдерін тұрғызу мүмкіндіктері зерттелді. Мұндай жүйелердің айрықша ерекше нүктелері екінші ретті дербес туындылы екі дифференциалдық теңдеулерден тұратын жүйелермен салыстыру арқылы алынды. Регулярлы және иррегулярлы ерекшеліктеріне жіктеу жүргізіліп, олардың сәйкес шешімдері тұрғызылды. Нақты мысал арқылы үшінші ретті дербес туындылы екі дифференциалдық теңдеулерден тұратын туындалған гипергеометриялық жүйенің шешімін табуда Фробениус-Латышева әдісінің қолдануы келтірілген. Жүйе шешімдерінің бірқатар қасиеттері келтіріліп, бұл шешімдердің өзі жалпыланған гипергеометриялық функциялардың да қасиеттеріне ие екені көрсетілген.

Түйін сөздер: шешім, тұрғызылған, жүйе, ерекше нүктелер, регулярлы, иррегулярлы, жалпыланған, үшінші ретті.

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СОВМЕСТНОЕ РЕШЕНИЕ ДВУХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ В ЧАСТНЫХ ПРОИЗВОДНЫХ ТРЕТЬЕГО ПОРЯДКА

Аннотация. В работе изучена возможности построения решения вблизи различных особенностей специальной системы состоящим из двух дифференциальных уравнений в частных производных третьего порядка. Установлены отличительные особенности таких систем по сравнению с системами состоящих из двух дифференциальных уравнений в частных производных второго порядка. Проведена классификация регулярных и иррегулярных особенностей, построены соответствующие им решения. На конкретном примере показано применение метода Фробениуса-Латышевой к построению решения вырожденной гипергеометрической системы состоящих из двух дифференциальных уравнении в частных производных третьего порядка. Рассмотрены ряд свойств решения системы, которое показывает, что эти решения обладают свойствами обобщенных гипергеометрических функций.

Ключевые слова: решение, построены, система, особенности, регулярные, иррегулярные, обобщенная, третьего порядка.

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