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# FERMION THEORY OF COLLECTIVE STATES OF NUCLEI, ITS APPLICATION TO THE STRUCTURE OF REAL SYSTEMS

**Abstract.** Based on the nucleon-pair shell model, in which the fermionic space is cut by the "realistic" SD-operators by the generalized senority method, the microscopic structure of the collective states of the nuclei of the average atomic weight is studied. In this case, the effects of splitting of single-particle levels on the collective-pair structure of the system are taken into account. To solve such a multiparticle problem, we use the generalized quasispin method and double tensors, which facilitate the calculation of the matrix elements of pair interactions of nucleons. The total Hamiltonian is diagonalized exactly in fermionic space without applying the procedure for mapping fermion operators into bosonic operators. The parameters of the interacting boson model are calculated on the basis of the permuted fermion approach. The theory is applied to the study of the properties of the collective states of even isotopes of ruthenium with N = 58-66. The spectrum of low-energy states is also calculated for the probabilities of E2 transitions in them and they are compared with the available experimental data.

## I. Introduction

The low-energy collective states of the nuclei of medium and heavy atomic nuclei are well described by the interacting boson model (IBM) [1-4]. The parameters of such a phenomenological theory are usually chosen from comparison with experimental data, and they smoothly change with an increase in the number of nucleons in the isotopes of systems.

On the other hand, the observed changes in the parameters of the IBM model as a function of N and Z are in good agreement with the first approximation of the approach that takes into account the severity schemes in interacting fermion systems [5-7].Attempts were also made to substantiate the IBM by calculating model parameters from detailed microscopic approaches. The description of collective states in terms of fermion degrees of freedom is an interesting and important problem in the theory of nuclear structure.But because of the difficulty of carrying out numerical calculations in a huge shell-model space for nuclei with a sufficiently large number of particles, we have to use some types of truncating schemes for the fermion space.In many cases, the so-called SD –pair circumcision of the enormous fermion-shell space is used, however, difficulties remain regarding the application of “realistic” SD –nucleated pairs as building blocks in the model [8-11].To overcome such obstacles in the microscopic calculations, the parameters of the IBMare determined by the method of mapping the collective shell space into the bosonic one, and then systematic calculations are carried out in the bosonic space. The Ohtsuka-Arima-Yakello (OAY) map [12,13] based on the generalized seniority scheme was used most of all [14].

In this work, we use the nucleon – pair shell model in which the fermion space is cut off by ″ realistic ″ SD – pair operators taking into account the effect of splitting of single-particle levels into a collective – pair structure of the system.Also here, the full Hamiltonian is diagonalized exactly in the fermion space with generalized senority, without using the mapping procedure, which gives a fermion pattern of collective excitations of nuclear states. The used approach to the study of collective states goes into the so-called fermion-dynamic symmetric model (FDSM) [15-24], in the case of neglecting splittings of one-particle states that affect the collectivity of levels.

In this work, systematic calculations of the parameters of the model of interacting bosons are carried out, as applied to the study of the properties of low-energy states of even Ru isotopes with neutron numbers N = 58-66. Selection of the nuclei due to the fact that, firstly, they generalized quantum number seniority (quasi- spin generalized) is most purely secondly due to prostate configurations of excited nuclear states application of this method is simple and clear. In addition to the spectra of nuclei, the behavior of the wave functions of states is also discussed by calculating the ratios B (E2) and δ (E2 / M1) for lower states. The obtained values are compared with known experimental data.

**II.Generalized quasispin space and pair interaction of fermions**

The formalism of the generalized quasi-spin (generalized senority) [3, 4] is used, which allows one to exactly solve a multi-pair fermion problem with a fixed number of particles with given internuclone forces.It introduces a double tensor in the usual and generalized seniority spaces, by means of which the eigenfunctions and eigenvalues of numerous pairwise operators are easily found.Generalized quasispin fermion space we introduce via operators:

$$S_{+=}\sum_j \alpha_j S_+^j, S_{-=}\sum_j \frac{1}{\alpha_j} S_-^j, \quad S_{0=}\sum_j S_0^j = {}^{(N-\Omega)}/_2, \tag{2.1}$$

in which $\alpha_j$ – are coefficients reflecting the amplitudes of the probability of population of the orbit j and they are normalized:

$$\Omega = \sum_j \alpha_j \Omega_j = \sum \alpha_j (j + {}^1/_2).$$

The quasi-spin operators of the shell configuration $j^n$, satisfy the usual commutation relations, which the angular momentum operators obey:

$$S_+^j = \sqrt{\Omega} A^+(jj00) = \sqrt{\Omega}\left(jm_1 jm_2 \middle| 00\right\rangle a^+_{jm_1} a^+_{jm_2}, S_-^j = \sqrt{\Omega}\tilde{A}(jj00),$$
$$S_0^j = {}^1/_2\left(N^j - \Omega^j\right) = \frac{1}{2}(\sum a^+_{jm}\tilde{a}_{j-m}(-)^{j-m} - \Omega^j). \tag{2.2}$$

In addition, double tensors are introduced in the spaces of both quasispins with the moment λ, as well as in the usual with tensors of rank k and their corresponding Z-projections s and q.For the case when k is even, they are written in the form:

$$T^{(1,k)}_{1,q}(jj) = A^+(jjkq), T^{(1,k)}_{1,q}(jj) = \tilde{A}(jjkq),$$
$$T^{(1,k)}_{0,q}(jj) = U(jjkq) + \sqrt{{}^{\Omega}/_2}\,\delta(k,0). \tag{2.3}$$

Here they are double tensors; in the usual space of rank k, and simultaneously a tensor of rank 1 in a quasispin space. Any single-particle operator is proportional to the double tensor of the first rank in a quasispin space, and in the ordinary one it is proportional to the k-rank tensor $T^{(1,k)}(jj)$.

The reduced matrix element of the single-particle operator of n particles is written through a matrix of single particles with senority ϑ.

$$<j^n\vartheta\alpha J||\sum_j f_i^n||j^n\vartheta\alpha' J'>=\frac{\Omega-n}{\Omega-\vartheta}\sqrt{\frac{(n-\vartheta+2)(2\Omega-n-\vartheta+2)}{4(\Omega-\vartheta+1)}}<j^\vartheta\vartheta\alpha J\left|\left|\sum_j f_i^k\right|\right|j^\vartheta\vartheta-2,\alpha' J'>. \tag{2.4}$$

A completely similar method can be used to simplify the calculation of two-particle matrix elements using similar reduction formulas. Usingthedoubletensors (2.3), thepairinginteraction operator is written:

$$V = \sum_{j_1 j_2 j_3}\sqrt{2J+1}\,G_1(j_1 j_2 j_3 j_4)[A^+(j_1 j_2)\times\tilde{A}(j_3 j_4)]^{(0)}_0 \tag{2.5}$$

$$G_J = {}^{(1+\delta j_1 j_2)(1+\delta j_3 j_4)}/_4 \cdot < j_1 j_2 J|V|j_3 j_4 J >,$$

can also be expressed in terms of double tensor

$$V = -\sum_J \sqrt{2J+1}\, G_J \left[T_{jj}^{(1,J)} \times T_{jj}^{(1,J)}\right]_{00}^{(\lambda,0)} (111-1|\lambda 0) = T^{(0)} + T^{(1)} + T^{(2)} \quad (2.6)$$

For example, $$T^{(\lambda)} = -\frac{1}{2}\sum_{JM}(-)^{J-M} G_J \left[T_{1M}^{(1,J)} - T_{-1-M}^{(1,J)}\right] = (N-\Omega)F_0 \quad (2.7)$$

Here, for example, for $= 0: F_0 = -\frac{1}{2\Omega}\sum_J (2J+1)G_J$.

In additiononly$T^{(2)}$) can change senority $\vartheta$ by $\vartheta' = \vartheta + 2, \vartheta + 4$, then according to the Wigner-Eckart theorem the reduction formula follows:

$$< j^n \vartheta \alpha J|V|j^n \vartheta' \alpha' J' > = \frac{f_2(n)}{f_2(\vartheta)} < j^2 \vartheta \alpha J\left|T^{(2)}\right|j^2 \vartheta' \alpha' J' > \quad (2.8)$$

where $f_2(n)$=($\frac{1}{2}(\Omega-\vartheta')2\frac{1}{2}(n-\Omega)0|\frac{1}{2}(\Omega-\vartheta')\frac{1}{2}(n-\Omega)$)are the Clebsch-Gordan coefficients.

Matrix elements that are diagonal in senority include the contributions of all three tensors. $T^{(0),(1),(2)}$, which are discussed in detail in the works[4,9].

Operators of generalized quasi-spin(2.1) obey also the usual commutation ratios, however, remain non-Hermitian:$(S_-)^+ \neq S_+$.

$$\left[S_+, S_-\right] = 2S_0\,,\ \left[S_0, S_\pm\right] = \pm 2S_\mp \quad (2.9)$$

All Lie groups (2.1) for all values of $\alpha_j$are isomorphic to each other. Therefore, for any set of operators (2.1), we can introduce the complete generalized quasispin operator: $S^2 = S_+S_- + S_0^2 - S_0$.

The state vectors of the quasi-spin operators S and $S_0$are determined by the quantum numbers s and $s_0$, which are associated with the quantum number seniority and the total number of nucleons N in the form:

$$S = \frac{1}{2}(\Omega - \vartheta) and S_0 = \frac{1}{2}(N-\Omega). \quad (2.10)$$

Then using commutation relations between $S_\pm$, $S_0$we have:

$$\begin{aligned} S_-\left|s\ \ , s_0, q\right> &= const\left|s, s_0 - 1, q\right>, \\ S_+\left|s, s_0, q\right> &= const\left|s, s_0 + 1, q\right>, \\ S_-|s, s_0, &= -s, q> = 0. \end{aligned} \quad (2.11)$$

Thus, according to the quantum number of the generalized quasi-spin s, it is possible to classify the states of the system by the ratio of the rotation of the system in the quasispin space.Therefore, this method is one of the easiest ways to solve many-particle problems.Many-particle matrix elements are expressed in terms of two-particle with the help of reduction formulas and commutation relations between tensors.For example, Hamiltonian pairing interaction

$$H_S = \varepsilon N - GS_+S_- \quad (2.11)$$

is diagonal in the representation of a generalized seniority.

Wave functions of system states with quantum numbers$\left|s, JM\right\rangle$, therefore, are expressed in form:

$$|s, s_0 JM> = \left\{\frac{(\Omega-\vartheta-n)!}{n!(\Omega-\vartheta)!}\right\}^{\frac{1}{2}} (S_+^1)^n |s, -s, JM> \quad (2.12)$$

where$n = (\mathrm{N} - \vartheta)/2$ -the number of paired particles.

Now we consider a many-particle problem in the space of a generalized quasispin with an arbitrary pair interaction operator. The full Hamiltonian, in this case, is conveniently divided into two parts, selecting from it the pairing interaction Hs in the generalized quasispin representation: $H = H_s + W$ ,where W-operator, expressing the rest of the interaction of particles, but diagonal in the representation of generalized quasispin s

$$W = \sum_{j_1 j_2 j_3 j_4} \left\langle j_1 j_2 \left| V' \right| j_3 j_4 \right\rangle A_+ \left( j_1 j_2 JM \right) A_- \left( j_3 j_4 JM \right). \tag{2.13}$$

Then the eigenvalue problem for the complete Hamiltonian H, which is diagonal in the s-representation, reduces to solving the equation:

$$H \left| s, S_0 \right\rangle = E\left( n = s + s_0, v = \Omega - 2s, q \right) \left| s, s_0 q \right\rangle_0 . \tag{2.14}$$

The total energy of the system is also divided into two parts.

$$E\left( n, v, q \right) = E\left( n = 2n + v, v \right) + E'\left( n, v, q \right), \tag{2.15}$$

where, Es is the eigenvalues of the pairing part of the Hamiltonian Hs.

Let us find the conditions under which the full Hamiltonian H is diagonal in the representation of a generalized quasispin. For this, it is necessary that functions (2.14) be eigenfunctions of the operator W:

$$W \left| s, s_0, q \right\rangle = E'\left( n, v, q \right) \left| s, s_0, q \right\rangle . \tag{2.16}$$

This equation can be reduced to several easily solvable, independent of n equations. Forthispurpose, considerthecommutator:

$$\left[ W, S_+ \right] = 2 \sum \left\langle j_1 j_2 \left| V' \right| j_3 j_4 \right\rangle A_+ \left( j_1 j_2 JM \right) \left\{ \sqrt{\Omega^{j_3}} \alpha j_3 \delta_{JM}^{00} / \sqrt{2} - (-)^{J-M} \alpha j_3 T_{-M}^J \left( j_3 j_4 \right) + (-)^{j_3 + j_4 - M} \alpha j_4 T_{-M}^J \left( j_3 j_4 \right) \right\}$$

$$T_M^J \left( j j' \right) = \left( 2\sqrt{1 + \delta_{jj'}} \right)^2 \sum_{mm'} \left( j j m - m' \left| JM \right. \right) a_{jm}^+ \tilde{a}_{j'm'} , \tag{2.17}$$

where, single-particle operator satisfying the relations:

$$\left[ T_M^J \left( j j' \right), S_+'' \right] = 2 \delta_{jj'} A_+ \left( j j' JM \right). \tag{2.18}$$

This operator breaks a pair of particles in the $S_+ \left| 0 \right\rangle$ state and puts them into the excited state $A_+ \left( j j' JM \right) \left| 0 \right\rangle$ .In addition, we introduce the operator of the creation of unpaired particles v with a common angular momentum J:

$$Q^+ \left( v, JM \right) \left| 0 \right\rangle = \sum_j \gamma_j^{v,J} Q^+ \left( j^v, JM \right) \left| 0 \right\rangle . \tag{2.19}$$

From the normalization condition of the wave functions, we have: $\sum_j \left( \gamma_j^{v,J} \right)^2 = 1$ .

Then equation (2.16) can be rewritten in the form:

$$W \left( S_+ \right)^n Q^+ \left( v, JM \right) \left| 0 \right\rangle = E'\left( n, v, q \right) Q^+ \left( v, JM \right) \left| 0 \right\rangle . \tag{2.20}$$

Expression (2.19) will be satisfied, if only the equalities hold:

$$\left[ \left[ W, S_+ \right] Q^+ \left( v, JM \right) \right] = \lambda \left( v, J \right) S_+ Q^+ \left( v, JM \right),$$

$$W Q^+ \left( v, JM \right) \left| 0 \right\rangle = E'\left( n, v, q \right) Q^+ \left( v, JM \right) \left| 0 \right\rangle ,$$

$$E'\left( n, v, q \right) = E'\left( n, v, q \right) + n\lambda \left( v, J \right).$$

As a result, the condition of diagonalization of the complete Hamiltonian $H = H_s + W$ in the s-representation reduces to solving the system of equations

$$\left.\begin{aligned} &HS_+^+|0\rangle = E_0 S_+^+|0\rangle, \\ &[[H,S_+^+]S_+^+] = 2G(S_+^+)^2, \\ &HQ^+(\nu,JM)|0\rangle = E(\nu,J)Q^+(\nu,JM)|0\rangle, \\ &[[H,S_+^+],Q^+(\nu,JM)] = (\nu G + \lambda(\nu,J))S_+^+Q^+(\nu,JM), \end{aligned}\right\} \quad (2.22)$$

where, $E_+ = S_s\left(N=2,\nu=0=2\varepsilon-G\Omega.\right)$.

The hollow energy of the system is determined by equality (2.15). In the case of the invariance of W with respect to its rotation in the s-representation, the equality $\lambda(\nu,J)=0$ must be fulfilled.

Thus, the solution of the problem with the full Hamiltonian H leads to the lifting of the degeneracy of levels by the angular momentum J in multiplets characterized by a quantum number of generalized senority v whose positions linearly depend on the number of pairs in the system.

**III. Fermion structure of collective states of even ruthenium isotopes**

The stated microscopic method for calculating the collective state of nuclei is applied to the study of low-energy states of even ruthenium isotopes with atomic weights A = 100-106.

The single-particle functions of a symmetric harmonic oscillator are taken as the basis of the calculations. The potential of the nucleon-nucleon interaction is selected in the form:

$$V = (U_w + U_S\pi_S + U_T S_{12})f(r,r_0) + U_C \quad (3.1)$$

where,$U_w, U_S, U_T$–parameters ofwigner, singlet and tensor forces,$\pi_S$and $S_{12}$- operators singlet and tensor projection n.Radial force dependence$f(r,r_0)$selected as Gauss potential,$U_C$ -Coulomb potential. The full potential of the pair interaction of nucleons is equal to:

$$V = V_{nn} + V_{pp} + V_{np}$$

As the core of this nuclei in protons and neutrons states is taken the low energy states of the strontium nucleus with Z = 38 and N = 50 [10].The proton shell of the strontium nucleus is completed by filling the $2P_{3/2}$level.

The distance between this overhead and free shell $2P_{1/2}$ of order 3 MeV. Then the shell $2P_{3/2}$can be consideredsemimagic. As a single-particle proton states can be taken, the lower state $^{89}$Y: the binding energy of which is:$\varepsilon_{p_{1/2}} = -7{,}07$ MeV, $\varepsilon_{g_{9/2}} = -6{,}16$MeV.

As single-particle neutron states, taken thehole states of $^{131}_{50}Sn$ in MeV:

$$\varepsilon_n = 0\left(d_{3/2}\right), 0{,}24(h_{11/2}), 0{,}33(s_{1/2}), 1{,}66\left(d_{3/2}\right), 2{,}34\left(g_{7/2}\right). \quad [11]$$

With the above experimental data on single-particle states of near magic nuclei Z = 38, N = 50realizethe corresponding choices of the parameters of paired interactions of valence nucleons.The parameters of pair interactions of nucleons are determined from the description of experimental spectra of even Ru isotopes.

The depth of the proton-proton interaction Vpp should vary slowly in nuclear isotopes, but differ from each other by a small amount. In this paper, they were chosen the same for all isotopes and equal tothe amplitude of the tensor interaction was considered negligible. The selected parameters of the nn and np interactions are shown in Table 1.These values turned out to be close to the values obtained in [12], for heavy nuclei.They vary with the number of neutrons monotonously and slowly. In addition, $V_{pp} > V_{nn}$for all isotopes.This is due to the fact that the single-particle energies splinter for protons slightly more than for neutron holes. The table shows that the depth of the neutron-proton interaction is also somewhat greater for all isotopes and it slowly decreases with a decrease in the number of neutrons.

| | $U_W^n$ | $U_S^n$ | $U_W^{np}$ | $U_S^{np}$ |
|---|---|---|---|---|
| $^{100}Ru$ | -22 | -16 | -30 | -24 |
| $^{102}Ru$ | -19 | -14 | -27 | -22 |
| $^{104}Ru$ | -17 | -11 | -24 | -19 |
| $^{106}Ru$ | -15 | -10 | -22 | -17 |
| $^{108}Ru$ | -12 | -8 | -20 | -14 |

They computed the entire low-lying spectrum of even ruthenium isotopes with N = 58, 60, 62, 64, 66. The obtained energy values of the levels of these nuclei are compared with their experimental data, which are listed in Table 2.

| Ядра | $^{100}Ru$ | | $^{102}Ru$ | | $^{104}Ru$ | | $^{106}Ru$ | | $^{108}Ru$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Exp. | Theor. | Exp. | Theor. | Exp. | Theor. | Exp. | Theor. | Exp. | Theor. |
| $0_1^+$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $2_1^+$ | 0,54 | 0,52 | 0,48 | 0,46 | 0,36 | 0,35 | 0,27 | 0,25 | 0,24 | 0,23 |
| $4_1^+$ | 1,23 | 1,18 | 1,11 | 1,09 | 0,89 | 0,81 | 0,71 | 0,66 | 0,67 | 0,61 |
| $6_1^+$ | 2,08 | 1,97 | 1,87 | 1,72 | 1,56 | 1,45 | 1,30 | 1,19 | 1,22 | 1,11 |
| $8_1^+$ | 3,06 | 2,80 | 2,70 | 2,51 | 2,32 | 2,13 | 1,97 | 1,76 | - | 1,61 |
| $10_1^+$ | 4,09 | 3,56 | 3,43 | 3,15 | 3,11 | 2,91 | - | 2,46 | - | 2,13 |
| $0_2^+$ | 1,13 | 1,05 | 0,94 | 1,01 | 0,99 | 0,90 | 0,99 | 0,91 | 1,09 | 1,05 |
| $2_3^+$ | 1,87 | 1,69 | 1,58 | 1,45 | 1,52 | 1,37 | 1,39 | 1,29 | 1,25 | 1,12 |
| $2_2$ | 1,36 | 1,22 | 1,10 | 1,02 | 0,89 | 0,75 | 0,79 | 0,81 | 0,71 | 0,80 |
| $3_1$ | 1,88 | 1,65 | 1,52 | 1,41 | 1,24 | 1,04 | 1,09 | 0,94 | 0,97 | 0,88 |
| $4_2$ | - | 1,24 | 1,80 | 1,34 | 1,50 | 1,35 | - | 1,29 | 1,18 | 1,01 |
| $5_1$ | - | 1,89 | 2,22 | 1,96 | 1,87 | 1,59 | - | 1,32 | 1,49 | 1,18 |

The choice of these nuclei for research is primarily due to the fact that their low-energy states manifest themselves most purely in the presentation of generalized seniority, as noted in [4, 12]. It should be noted that there is a good agreement between the calculated values of E and the experiment for states with small angular valuesJ,

Table 3 - Relationships of E2 transitions between states in Ru nuclei.

| $\frac{J_i \to J_f}{J_i' \to J_f'}$ | $^{100}Ru$ | | $^{102}Ru$ | | $^{104}Ru$ | | $^{106}Ru$ | | $^{108}Ru$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Exp. | Theor. | Exp. | Theor. | Exp. | Theor. | Exp. | Theor. | Exp. | Theor. |
| $\frac{4_1 \to 2_1}{2_1 \to 0_1}$ | 1,50 ±0,21 | 1,47 | 1,6 ±0,3 | 1,5 | 1,43 ±0,20 | 1,44 | - | 1,75 | - | 1,83 |
| $\frac{0_2 \to 2_1}{2_1 \to 0_1}$ | 0,99 ±0,20 | 0,95 | 0,76 ±0,15 | 0,71 | 0,46 ±0,06 | 0,41 | - | | - | |
| $\frac{2_2{}^+ \to 2_1}{2_1 \to 0_1}$ | 0,91 ±0,18 | 0,83 | 0,90 ±0,15 | 0,84 | 1,0 ±0,02 | 0,82 | 11,6 ±1,2 | 5,2 | 10,2 ±1,0 | 6,1 |
| $\frac{2_2 \to 2_1}{2_2 \to 0_1}$ | 15,4 ±0,5 | 12,0 | - | 11,4 | 25 ±4,0 | 14,5 | - | 16,0 | - | 22,4 |
| $\frac{2_3 \to 4_1}{2_3 \to 2_1}$ | 16,2 ±2,5 | 17,4 | 8,0 ±2,0 | 12,6 | - | 15,2 | | 18,3 | | 18,9 |
| $\frac{3_1 \to 2_2}{3_1 \to 2_1}$ | 11 ±5,6 | 13,2 | 27 ±3,0 | 29,3 | 26 ±3,0 | 10,4 | 26 ±10 | 22,4 | 17 ±3 | 25,6 |

for which the main role is played by the interaction potentials of like nucleons$V_{pp}$,$V_{nn}$.The usefulness of these potentials in these cases is determined by their properties, which well preserve the scheme of generalized seniority.At the same time, such a purity of the quantum number of the generalized seniority$\vartheta$, greatly simplifies the calculation procedure and leads to close real energy values for small values of the spins J, as can be seen from Table 2,3,4.

Table 4 - Relationships of $\delta(E2/M1)$ transitions between states in Ru nuclei

| $J_i - J_f$ | $^{100}Ru$ | | $^{102}Ru$ | | $^{104}Ru$ | | $^{106}Ru$ | | $^{108}Ru$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Exp. | Theor. | Exp. | Theor. | Exp. | Theor. | Exp. | Theor. | Exp. | Theor. |
| $2_2 \to 2_1$ | 3,4 ±0,8 | 2,9 | -6,0 ±0,2 | -2,7 | -9,0 | -4,4 | 7,1 ±1,4 | 5,6 | 4,3 ±0,8 | 2,9 |
| $2_3 \to 2_1$ | | 3,6 | 0,25 ±0,03 | 1,2 | 0,43 ±0,11 | 0,25 | 0,24 ±0,12 | 0,43 | 0,87 ±0,56 | 0,56 |
| $3_1 \to 2_1$ | 0,45 ±0,1 | 0,9 | 0,90 ±0,15 | 0,6 | -3,2 ±0,4 | -1,7 | -3,8 ±1,4 | -5,4 | -3,0 ±0,95 | -1,4 |
| $4_2 \to 4_1$ | - | 0,17 | - | 0,26 | 0,11 ±0,11 | 0,18 | - | 0,22 | - | 22,4 |
| $3_1 \to 2_1$ | - | -4,3 | -7,2 (10) | -3,2 | - | - | - | -2,3 | - | 0,27 |
| $4_1 \to 2_1$ | - | - | 0,01 (5) | - | - | - | - | - | - | 3,1 |
| $5_1 \to 4_1$ | - | - | -1,05 ±0,05 | - | - | - | - | - | - | |

However, in neutron-protoninteractions$V_{np}$-begin to dominate the quadrupole-quadrupole interaction, which strongly mixes states with different senorities.If we restrict ourselves to several protons or neutrons in the valence shell, then the mixing of components in states with high senority, mainly the lower excited states, can be included in the calculations of the lower approximation of matrix diagonalization.Due to the quadrupole nature of the n-p forces, the components of the wave functions of such as$S_n^N S_p^N (D_n D_p)^0$fractions will mix states of different senoriths, which strongly wag the collectivity of D-pair-fermion states. Because of this, the discrepancy between the calculated and experimental values of the energy of states with large J becomes stronger, as can be seen from Table 2,3,4.Apparently, the hexadecapole component in pn-forces will play a certain role here.In addition, in such calculations, the single-particle energies were taken constant for all isotopes of the nucleus, whereas they can be varied by changing the number of nucleons in the shells.However, as shown by calculations for deformed isotopes$^{106}Ru$the main properties of the states in terms of energy are reproduced quite satisfactorily, since they change smoothly.

Table 3 and 4 shows the probability ratios of E2 transitions between different states of Ru isotopes. They also show satisfactory agreement of their calculated values with experimentally measured functions, especially for low-lying states.For levels with higher energies, these values differ significantly more. This is also mainly due to the strong mixing of states with different senorities due to quadrupole n-p interaction, as well as the neglect of the contributions of G-pair configurations to computational procedures.In many microscopic models with the method of mapping the fermion space into the bosonic one, the mapping methods are carried out not through the operators of fermion-boson transformations, but through the equalities of the matrix elements of the states in two spaces.As was noted [5, 6], these two spaces are interconnected through the bosonic and fermionic seniority. Such a connection is especially important for states with high senority, in which np-forces between particles play the main role. And they can give a fairly large contribution and bosons with large orbital moments.This leads to a change in the energies of the d-boson states. Therefore, in such calculations with space, it was necessary to renormalize the parameters of bosons included in the theory or parameters of quadrupole interactions in microscopic calculations. In our calculations, it was not necessary to lead such processes of renormalization of the parameters of theories. Despite this, the fermion theory with generalized senority, in general, gives a satisfactory smooth description of the properties of nuclei with average atomic weights.

**CONCLUSION**

On the basis of the generalized quasispin approach, the microscopic structure of the collective states of even ruthenium isotopes in the low-energy region was studied. To solve the many-particle problem in the space of generalized quasispins of, the potentials of nn, pp, npinteractions are taken in the most general form, the parameters of which are chosen from a comparison of the calculated values with their experimental values.

The description of microstates of nuclei with the help of generalized senority and double tensors, which express pairwise interaction of nucleons, greatly simplifies the procedure for calculating matrix elements, which give a good confirmation of the experimental facts on energy of states and on probabilities of electromagnetic transitions between them, especially for states with small quantum numbers J. However, the quadrupole interaction operator between different nucleons $V_{pn}$-strongly mixes states with different senorities.

This fact strongly influences the formation of the collectivity of the D-fermion states in the systems, which weakens the consistency of the calculated and experimental values of the energy levels, as well as the relative probabilities of E2 transitions between them, with large spins J.Nevertheless, the main properties of deformed nuclei are transferred quite satisfactorily. In some microscopic calculations using the method of mapping the fermion space into the bosonic space due to the connection of these representations through fermionic and bosonicsenority, the contributions of pairwise states with large orbital moments, for example, G-states, increase.Apparently, the divergence of our calculations for a state with large angular moments and with higher excitation energies also requires taking into account the contributions to them of G-pair formations. In addition, in the composition of high-energy levels, the role of that part of the complete Hamiltonian that was not included in the generalized seignity scheme is important.

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**ЯДРОЛАР КОЛЛЕКТИВТІК КҮЙЛЕРІНІҢ ФЕРМИОНДЫҚ ТЕОРИЯЛАРЫН ЖӘНЕ ОНЫ НАҚТЫ ЖҮЙЕГЕ ҚОЛДАНУ**

**Аннотация.** Атомдық салмағы орташа ядролар коллективтік күйлерінің микроскоптық құрылымы қосарланған нуклондық қабықша моделінде зерттеледі. Теорияда үлкен фермиондық кеңістік жалпыланған сеньорити әдісі көмегімен реалды SD-операторлар шеңберіне дейін қысқара кесілді және бірбөлшектік деңгейлер жіктелуінің жүйедегі күйлерін коллективтік құрылысына әсері де есепке алынды. Осындай көпбөлшектік мәселені шешу үшін жалпыланған квазиспин әдісі мен қосарлы тензорлар ұғымы қолданылды. Олар қосарлы потенциалдардың матрицаларын есептеуге өте қолайлы. Толық гамильтониан таза фермиондық кеңістікте дәл диагональдауы бірге енгізілді. Мұнда фермиондық операторларды бозондық түрге айналдырудың қажеті болмады. Әсерлесуші бозондар моделінің параметрлері фермиондық жолмен есептелді. Құрылған теория рутений ядросының N=58-66 жұпты изотоптары құрылысына қолданылды. Олардағы төмегі энергиялы деңгей спектрі және электромагниттік E2-ауысу ықтималдығы есептеліп, алнған шамалар экспериментте табылған мәндерімен салыстырылды.

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**ФЕРМИОННАЯ ТЕОРИЯ КОЛЛЕКТИВНЫХ СОСТОЯНИЙ ЯДЕР ЕЕ ПРИЛОЖЕНИЕ К СТРУКТУРЕ РЕАЛЬНЫХ СИСТЕМ**

**Аннотация.** На основе нуклонно-парной оболочечной модели, в которой методом обобщенной сеньорити обрезается фермионное пространство ″реалистическими″ SD-операторами, изучены микроскопическая структура коллективных состояний ядер среднего атомного веса. При этом учтены влияния расщепления одночастичных уровней на коллективно-парную структуру системы. Для решения такой многочастичной задачи используется метод обобщенного квазиспина и двойные тензоры, облегчающие вычисления матричных элементов парных взаимодействий нуклонов. Полный гамильтониан диагонализуется точно в фермионном пространстве без применения процедуры отображения фермионных

операторов в бозонные. Вычислены параметры модели взаимодействующих бозонов на основе перелагаемого фермионного подхода. Теория приложена к изучению свойств коллективных состояний четных изотопов рутения с N=58-66. Вычислены спектр состояний низких энергии также вероятности E2-переходов в них и они сравнены с имеющимися экспериментальными данными.

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