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ON SOME EFFECTS IN THE STRUCTURE OF WHITE DWARFS

Abstract. We review the effects of general relativity, finite temperatures, nuclear composition and rotation which make a substantial contributionto the structure of white dwarfs.First, the mass-radius, mass-central density relations and mass, density profiles of a white dwarf with total mass1.415 $M_{\odot}$ are constructed both in Newtonian gravity and general relativity, which clearly show that the general relativistic effects are significant for massive white dwarfs close to the Chandrasekhar mass limit, consequently, in strong gravitational fields.Second, hot white dwarfs are studied in the framework of general relativity. Basic parameters of white dwarfs such as the central density, pressure, mass, radius and etc. are calculated. It is shown that the effects of finite temperatures play a key role in low mass white dwarfs.Third, cold white dwarfs are investigated within general relativity employing the Salpeter equation of state. Finally, we investigate the equilibrium configurations of uniformly rotating white dwarfs, using Chandrasekhar and Salpeter equations of state in Newtonian gravity and plot mass-radius, mass-central density relations. It is demonstrated that the effects of rotation are essential in the structure of white dwarfs in allmass range.

Key words: white dwarfs, general relativity, finite temperature, nuclear composition, rotation.

1. Introduction

A white dwarf or degenerate dwarf is the final stage in the evolution of normal (main sequence) stars with masses from 0.08 $M_{\odot}$ to 8 $M_{\odot}$[1, 2, 3] (even to 12 $M_{\odot}$according to some studies [4]), on the other hand, it is one of the classes of compact objects. The lower limit of a main sequence star mass is associated with the impossibility of the occurrence of a thermonuclear helium synthesis reaction.There are two forces which are counterbalanced with each other in the hydrostatic equilibrium configuration of a non-rotating white dwarf: the outward force of interior pressure gradient and the inward force of gravity. In the case of a rotating white dwarf, the centrifugal force is included.

There is no nuclear fusion in the interior of the white dwarf like a normal star. Consequently, it is not the thermal pressure force keeping the white dwarf in hydrostatic equilibrium.The pressure support in the white dwarf is provided by a degenerate electron gas, whereas most of the mass density is due to a nondegenerate gas of ions[3].

The maximum mass of a non-rotating white dwarf cannot exceed the Chandrasekhar mass limit of 1.44$M_{\odot}$beyond which even the degenerate electron gas cannot prevent the white dwarf from gravitational collapse [2] or type Iasupernova explosion[5], whichtakes place as a result of accretion or merger.In turn, type Ia supernova explosion is used as a standard candle to measure intergalactic distances, understand the past and future expansion of the universe and study the nature of dark energy. From this point of view, it is relevant to study properties of white dwarfs and construct their realistic model.

The paper is organized as follows. We introduce the equations of stellar structure in Sec. 2, namely the equation of hydrostatic equilibrium, mass balance equation for static (non-rotating)white dwarfs in Einstein's relativistic theory of gravity and Newtonian gravity, respectively; with the Hartle's approach that describes the equilibrium configuration of rotating white dwarfs. In Sec. 3, we consider some crucial effects such as the effects of general relativity, finite temperatures, nuclear compositions and rotation.The conclusions are given in Sec. 4. The material and results which were considered here can be used in further studies of astrophysics, cosmology and astronomy.

2. Equations of stellar structure

One can derive the Tolman-Oppenheimer-Volkoff (TOV)equation [6] of hydrostatic equilibrium and mass equation for a non-rotating (spherically symmetric configuration) star within the framework of general relativity in the following form

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\rho(r)}{r^2}\left(1+\frac{P(r)}{\rho(r)c^2}\right)\left(1+\frac{4\pi r^3 P(r)}{m(r)c^2}\right)\left(1-\frac{2Gm(r)}{rc^2}\right)^{-1}, \quad (1)$$

$$\frac{dm(r)}{dr} = 4\pi r^2\rho(r), \rho(r) = \frac{\varepsilon(r)}{c^2}, \quad (2)$$

where$G$ is the gravitational constant, c is the speed of light and$P(r)$,$m(r)$and$\rho(r)$are the pressure, mass and density profiles, respectively, which depend on the radial coordinate of r.Eqs. (1) and (2) reduces to the expressions

$$\frac{dP(r)}{dr} = -\frac{Gm(r)}{r^2}\rho(r), \quad (3)$$

$$\frac{dm(r)}{dr} = 4\pi r^2\rho(r) \quad (4)$$

in their Newtonian limit.

The equations of stellar structure of slowly and uniformly rotating axially symmetric configurations of white dwarfs can be obtained by using the Hartle's approach in Einstein's relativistic theory of gravity and Newtonian gravity [7-9]. In the Hartle's approach, the axially symmetric configuration is given for a uniform angular velocity sufficiently slow so that the changes in pressure($P_{rot} = P_{st} + \delta P$), energy density($\varepsilon_{rot} = \varepsilon_{st} + \delta\varepsilon$), and gravitational field($\Phi_{rot} = \Phi_{st} + \delta\Phi$)are small. These small changes are considered as perturbations of the known non-rotating solution. The field equations are expanded in powers of the angular velocity and the perturbations are calculated by retaining only the first- and second-order terms[7, 8].

The equation of hydrostatic equilibrium and mass balance equation is supplemented by the equation of state and boundary condition to determine the stellar structure. The equation of state is necessary to describe the properties of the interior matter of a white dwarf. It determines the dependence of the total pressure on the total density for the case of a one-parameter equation of state, $P = P(\rho)$, where P is the pressure and ρ is the density of matter. This form of the equation of state is appropriate when the temperature is a known function of the density inside the star [7]. In this work, we have used the Chandrasekhar equation of state at zero and finite temperatures and the Salpeter equation of state, which takes into account the nuclear composition, electrostatic interaction, Thomas-Fermi correction, and inverse beta decay threshold. Details of these equations of state can be found in Refs. [10-13].

2.Results and Discussions

Effects of general relativity.We solve numerically the equations of stellar structure employing the Chandrasekhar EoS ($\mu = A/Z = 2$)with given boundary conditions and obtain the main parameters of white dwarfs, for instance, mass, radius etc. We also construct the dependence of the mass on the radius in Fig. 1 as well as the dependence of the mass on the central density in Fig. 2.The solid curve indicates mass-radius relation in Newtonian gravity, the dashed curve in general relativity. As it can be seen from

Figure 1, the masses of white dwarfs increase by decreasing their radii, which is the main difference of white dwarfs from the main sequence stars. This feature shows the contribution of relativistic and quantum corrections in the EoS.

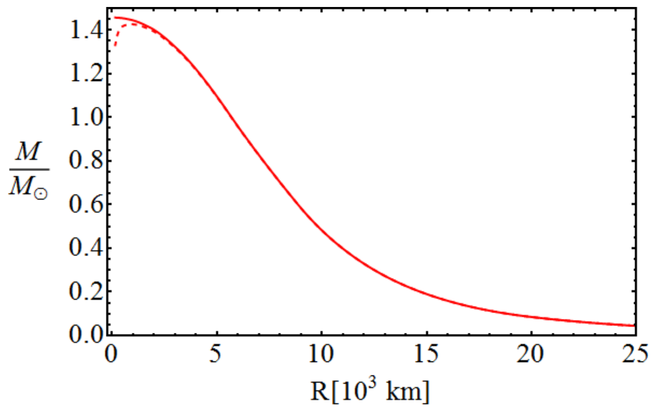


Figure 1 – Mass-radius relations

The mass of a white dwarf also increases by increasing the central density (see Fig. 2). However, it cannot increase infinitely and exceed the Chandrasekhar mass limit $M_{Ch} = 1.44\ M_{\odot}$[5, 6]. The difference between Newtonian and Einstein's gravity is clearly seen for the case of massive white dwarfs. It is related to the presence of general relativistic corrections in the equations of stellar structure.

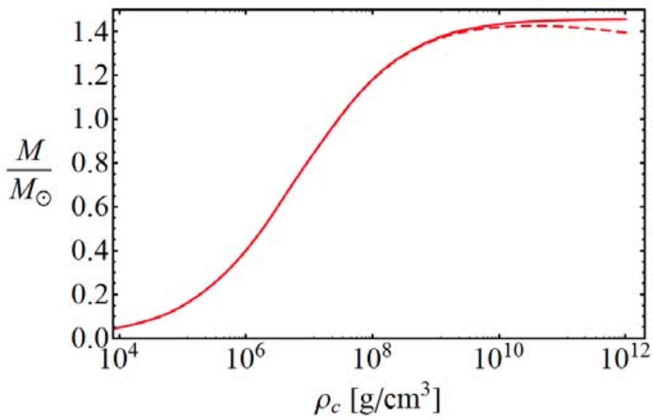


Figure 2 – Mass-central density relations

The growth of mass strengthens gravitational field of white dwarfs. This, in turn, increases pressure, hence in general relativity the maximum mass is less than in Newtonian gravity and it is achieved at finite density. That central density defines stability of white dwarfs in general relativity.

In Newtonian gravity, a white dwarf reaches the maximum mass when the radius tends to zero and the central density and pressure tend to infinity. But, it is impossible, because there is a critical value of central density and central pressure, and consequently, a critical value for the maximum mass. If the value of the central density exceeds this critical value, the white dwarf collapses to a neutron star, or explodes as a type Ia supernova depending on the nuclear composition, temperature etc.The neutronization threshold density is chosen as a critical central density, and the corresponding maximum mass was calculated for white dwarfs composed of ^{12}C in Ref. [14]. The maximum mass M_{max} of a static white dwarf is 1.447 $M_{\odot}$ in Newtonian gravity, and 1.425 $M_{\odot}$ in Einstein's relativistic theory of gravity.

The significance of general relativity for stars can be described by the compactness parameter $z = r_g/R$, where$R$ is the radius of a star, $r_g = 2GM/c^2$ is the gravitational radius (or the Schwarzschild radius), M is the total mass of the star. The compactness parameter of massive white dwarfs close to the

Chandrasekhar mass limit is roughly equal to $z{\sim}0.001$ [14]. One has $z{\sim}0.3$ for neutron stars, $z = 1$ for black holes [15]. That is, the more compactthe object, the more noticeable the role of general relativity [14, 16-18].

We have also reproduced independently the results obtained in the work of Carvalho et al, where they have shown the importance of general relativistic effects for white dwarfs. Following the work [19], in Fig. 3, we show the mass profile of the white dwarf for a fixed total mass $M = 1.415\ \mathrm{M}_{\odot}$, where the importance of general relativistic effects is conspicuous. The total radius of the white dwarf for a fixed total mass$1.415\ \mathrm{M}_{\odot}$ is 938.65 km in general relativity, and 1558.78 km in Newtonian gravity. In Fig. 3 the bluehorizontal dotted line indicates the fixed total mass$M = 1.415\ \mathrm{M}_{\odot}$; the red solid curve indicates the mass profile in Einstein's relativistic theory of gravity; the red dashed curve indicates the mass profile in Newtonian gravity.

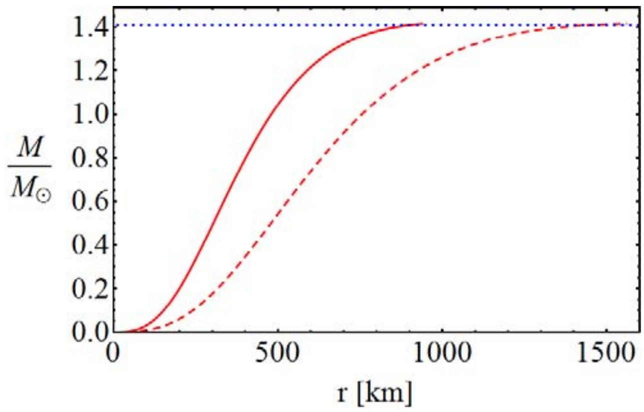


Figure 3 – Mass profiles for a fixed total mass $M = 1.415\ \mathrm{M}_{\odot}$

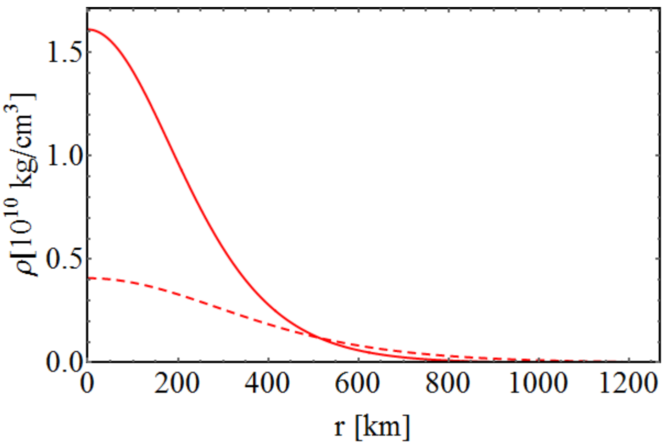


Figure 4 – Density profiles for a fixed mass of$M = 1.415\ \mathrm{M}_{\odot}$

In Fig. 4 we plot the density profile of a white dwarf with fixed mass $M = 1.415\ \mathrm{M}_{\odot}$. The red solid curve denotes the general relativistic density profile; the red dashed curve denotes the Newtonian density profile. From Fig. 4, one can notice that the mass density of the general relativistic white dwarf is larger than the Newtonian one in the central region, where the major part of the white dwarf mass is concentrated. The central density of the white dwarf with a fixed mass $1.415\, M_{\odot}$is $\rho_{cen}^{GR} = 1.61 \times 10^{10} \mathrm{g/cm^3}$ in general relativity, $\rho_{cen}^{NG} = 4.08 \times 10^{9} \mathrm{g/cm^3}$ in Newtonian Gravity [19].

*Effects of finite temperatures.*In Fig. 5, we have constructed the mass-radius relations of general relativistic non-rotating white dwarf cores at finite temperatures$T = (10^4, 10^5, 10^6, 10^7, 4 \times 10^7, 10^8)\ K$ using the Chandrasekhar equation of state $(\mu = 2)$at finite temperatures [20, 21]. From Fig. 5, it can be seen that the effect of finite temperatures increases with decreasing mass and it is significant for low-mass white dwarfs. This indicates that the substance of low-mass white dwarfs cannot be considered completely degenerate.

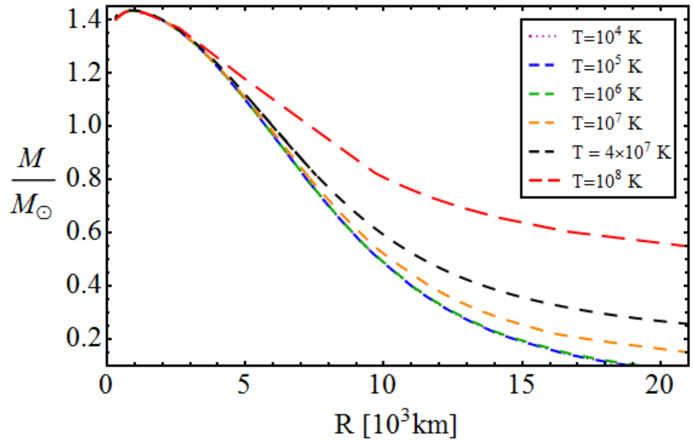
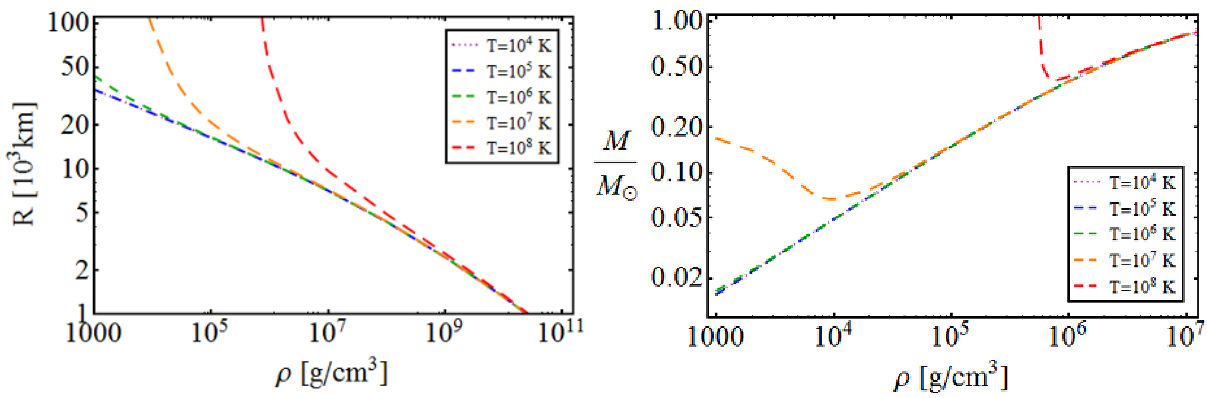


Figure 5 – Mass-radius relations at finite temperatures

Figure 6 shows the radius-central density and mass-central density relations at selected temperatures, where the effects of finite temperatures are more pronounced with a decrease in the central density, and with increasing central density, these effects weaken. That is, the effects of finite temperatures are especially important for white dwarfs with low central densities.



a) b)

Figure 6 – Radius-central density (a) and mass-central density(b) relationsat finite temperatures

*Effects of nuclear composition.*We have considered static white dwarfs by employing the Salpeter equation of state in general relativity and compared them with the results of the Chandrasekhar equation of state.

The Salpeter equation of state allows one to take into account the electrostatic interaction, the Thomas-Fermi correction, and the nuclear composition of white dwarfs. In Figures 7-8, mass-radius, mass-central density and central density-radius ratios were constructed for cold white dwarfs in the general theory of relativity (TOV equation). The plots were constructed for different nuclear compositions of $^{4}_{2}$He, $^{12}_{6}$C, $^{16}_{8}$O, $^{20}_{10}$Ne, $^{24}_{12}$Mg, $^{28}_{14}$Si, $^{56}_{26}$Fe (for the Salpeter EoS) and $\mu = 2$ (for the Chandrasekhar EoS) [22]. The figures show that the heavier the element, the lower the upper limit of the mass of white dwarfs.

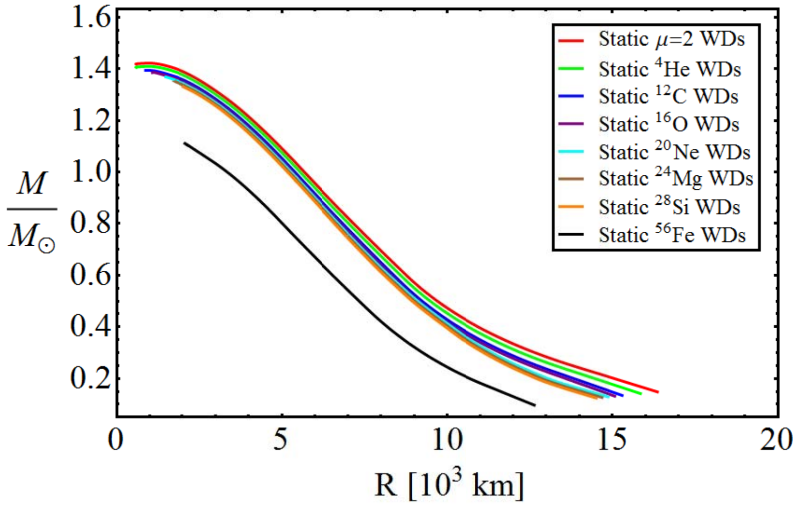
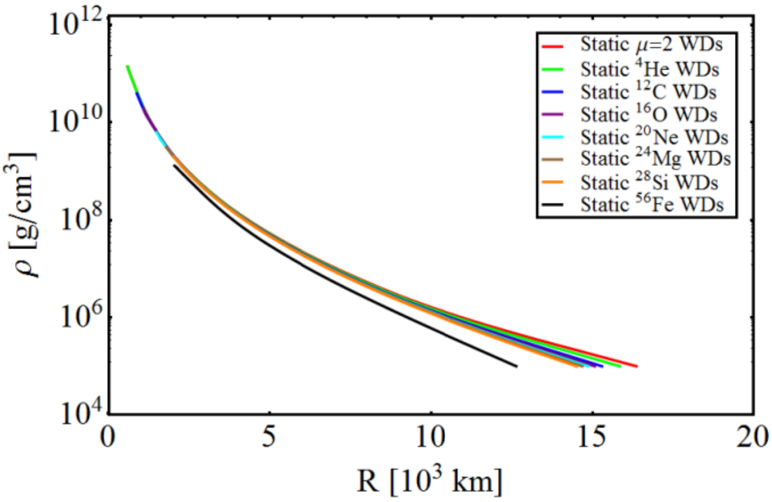
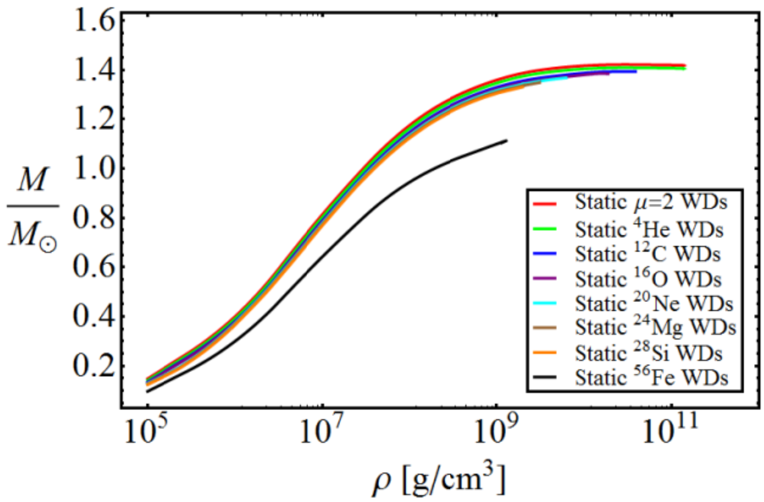


Figure 7 – Mass-radius relations for different nuclear compositions



a)



b)

Figure 8 – Central density-radius (a) and mass-central density (b) relations for different nuclear compositions

In addition, it should be noted that in the Salpeter equation of state, the effect of neutronization (inverse beta decay) for a white dwarf with a uniform nuclear composition was taken into account. This effect sets a limit on the upper limit of the central density(1.37×10^{11}g/cm^3 for ${}^4_2$He, 3.90×10^{10}g/cm^3 for ${}^{12}_6$C, 1.90×10^{10}g/cm^3 for ${}^{16}_8$O, 6.21×10^9g/cm^3 for ${}^{20}_{10}$Ne, 3.16×10^9g/cm^3 for ${}^{24}_{12}$Mg, 1.97×10^9g/cm^3 for ${}^{28}_{14}$Si, 1.14×10^9g/cm^3 for ${}^{56}_{26}$Fe) and, therefore, on the mass of white dwarfs[1].

*Effects of rotation.*There are three forces that act on any rotating star: the outward force pressure, the inward force of gravity and the centrifugal force. We follow the Hartle approach [7] and [9, 23], in order to derive the main equations of stellar structure of a rotating star in the case of uniform and slow rotation. The point of Hartle's approach consists in considering a spherically symmetric non-rotating compact object as starting point to construct a rotating star model. The structure equations are obtained up to the second order in the angular velocity$\Omega = \sqrt{GM_{tot}/r_e}$,G is the gravitational constant, M_{tot} is the total of the star, and r_e is the equatorial radius of the star. Afterwards, we calculate the main parameters of a slow and uniform rotating configuration and plot the relations of the main parameters. We show the significance of the rotation effects for the entire range of mass.

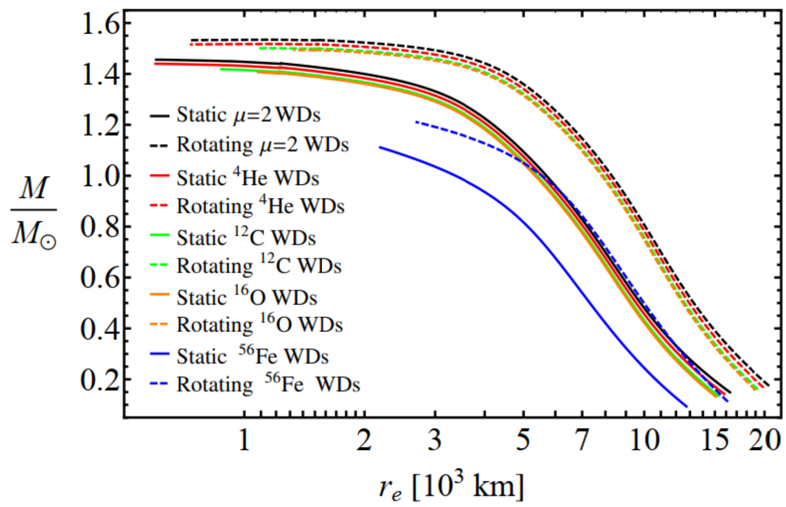


Figure 9 – Mass-equatorial radius relations

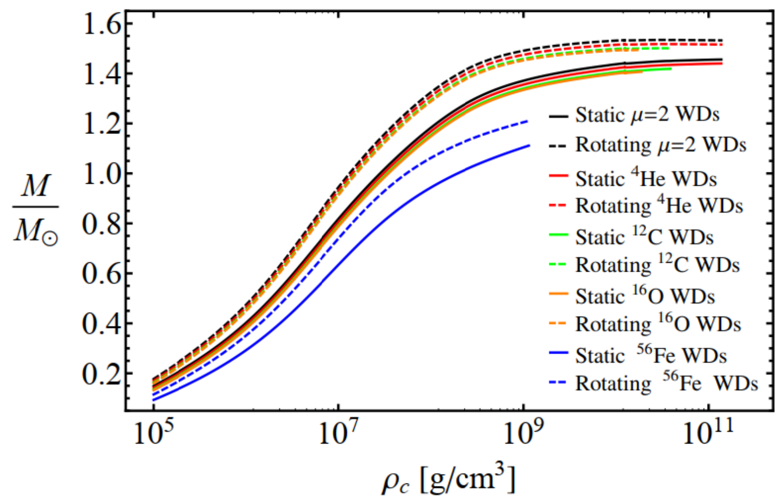


Figure 10 – Mass-central density relations

Fig. 9 shows the mass and equatorial radius relation. We have selected two equations of state: the Chandrasekhar equation of state with average molecular weight μ=2, and the Salpeter equation of state for pure helium ^4He, carbon ^{12}C, oxygen ^{16}O and iron ^{56}Fe white dwarfs, as limiting cases [23]. The equatorial radius for a static case reduces to the static radius. All solid curves indicate non-rotating (static) white dwarfs, whereas all dashed curves indicate rotating white dwarfs at the mass shedding rate. One can see that depending on the equation of state and nuclear composition, white dwarfs display different mass-radius relations.

In Fig. 10, the mass of a white dwarf is shown as a function of the central density. The mass is given in units of one solar mass and the central density is given in $\mathrm{g/cm^3}$. As expected, rotating white dwarfs have larger masses with respect to their static counterparts. In all our computations we restricted the maximum values of the central density to the values of inverse β-decay density to fulfill the stability condition of white dwarfs [17, 24, 25].

5. Conclusions

We considered the structure of non-rotating and slowly rotating, cold and hot, classical and general relativistic white dwarfs in hydrostatic equilibrium. In particular, the mass-radius, mass-central density, radius- central density etc. relations were constructed for white dwarfs using the Chandrasekhar and Salpeter equations of state. We studied the effects of general relativity, finite temperature, nuclear compositions and rotation inthe structure of an equilibrium configuration of white dwarfs. We concluded that:

- the effects of Einstein's general relativity are significant for massive, high density and small radius white dwarfs close to the Chandrasekhar mass limit;
- the finite temperatures considerably affect the structure of white dwarfs at low densities, that is, they play a major role for low-mass white dwarfs;
- the nuclear composition, electrostatic interaction andThomas-Fermi correction are importantfor white dwarfs in all mass range;
- the neutronization threshold is critical near the Chandrasekhar mass limit;
- the uniform rotation are crucial for all white dwarfs in the entire range of mass.

It would be interesting to compare and contrast the observational data for white dwarfs with the theoretical results presented here in analogy to Ref. [26]. In addition, it would be fascinating to investigate the spectral features of white dwarfs in a wide range of X-rays, optical and ultraviolet etc. variabilityfound in symbiotic binary systems in analogy to Ref. [27]. That will be the issue of future studies.

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АҚ ЕРГЕЖЕЙЛІ ЖҰЛДЫЗДАРДЫҢ ҚҰРЫЛЫМЫНДАҒЫ КЕЙБІР ЭФФЕКТТЕР ТУРАЛЫ

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Аннотация. Ақ ергежейлі жұлдыздардың құрылымына үлес қосатын жалпы салыстырмалылық теориясы, шекті температура, ядролық құрам және айналудың әсерлеріқарастырылды. Біріншіден, масса-радиус, масса-орталық тығыздық қатынастары және масса, тығыздық профильдері толық массасы 1.415 $M_\odot$ақ ергежейлі жұлдыз үшін Ньютон гравитациясында, сондай-ақ жалпы салыстырмалылық теориясында тұрғызылды. Олар жалпы салыстырмалылық теориясының әсерлері Чандрасекар массалық шегіне жақын үлкен массадағы ақ ергежейлі жұлдыздар үшін, демек, күшті гравитациялық өрістер үшін маңызды екенін айқын көрсетеді. Екіншіден, ыстық ақ ергежейлі жұлдыздар жалпы салыстырмалылық теориясының шеңберінде зерттеледі. Орталық тығыздық, қысым, масса, радиус және т. б. сияқты ақ ергежейлі жұлдыздардың негізгі параметрлері есептелінді. Массасы аз ақ ергежейлі жұлдыздарда шекті температураның әсерін ескеру қажеттілігі көрсетілді. Үшіншіден, суық ақ ергежейлі жұлдыздар Салпитердің күй теңдеуін қолдана отырып, жалпы салыстырмалылық теориясының шеңберінде зерттеледі. Соңында, Ньютон гравитациясында Чандрасекар мен Салпитердің күй теңдеуін қолдана отырып, бірқалыпты айналатын ақ ергежейлі жұлдыздардың тепе-тең конфигурацияларын зерттелді және масса-радиус, масса-орталық тығыздық қатынастарын тұрғызылды. Айналудың әсерлері массаның барлық мәндерінде ақ ергежейлі жұлдыздардың құрылымы үшін маңызды.

Түйін сөздер: ақ ергежейлі жұлдыздар, жалпы салыстырмалық теориясы, шекті температура, ядролық құрамы, айналу.

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О НЕКОТОРЫХ ЭФФЕКТАХ В СТРУКТУРЕ БЕЛЫХ КАРЛИКОВ

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Аннотация. Рассмотрено влияние общей теории относительности, конечных температур, ядерного состава и вращения, которые вносят существенный вклад в структуру белых карликов. Во-первых, построены соотношения масса-радиус, масса-центральная плотность и профили плотности белого карлика с полной массой 1,415 $M_{\odot}$ как в ньютоновской гравитации, так и в общей теории относительности, которые четко показывают, что общие релятивистские эффекты важны для массивных белых карликов около предела массы Чандрасекара, следовательно, в сильных гравитационных полях. Во-вторых, горячие белые карлики изучены в рамках общей теории относительности. Вычислены основные параметры белых карликов, такие как центральная плотность, давление, масса, радиус и т. д. Показано, что влияние конечных температур играет ключевую роль в маломассивных белых карликах. В-третьих, холодные белые карлики исследуются в рамках общей теории относительности с использованием уравнения состояния Салпитера. В заключение исследованы равновесные конфигурации равномерно вращающихся белых карликов, используя уравнения состояния Чандрасекара и Салпитера в ньютоновской гравитации и построены зависимости масса-радиус, масса-центральная плотность. Показано, что эффекты вращения необходимы в структуре белых карликов во всем диапазоне масс.

Ключевые слова: белые карлики, общая теория относительности, конечная температура, ядерный состав, вращение.

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