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M. Zh. Minglibayev[1,2], T. M. Zhumabek[1]

[1] Al-Farabi Kazakh National University, Almaty, Kazakhstan;
[2] Fesenkov Astrophysical Institute, Almaty, Kazakhstan.
E-mail: minglibayev@gmail.com

ON THE RESTRICTED THREE-BODY PROBLEM

Abstract. The paper analytically investigates the classical restricted three-body problem in a special non-inertial central coordinate system, with the origin at center of forces. In this coordinate system, an analytical expression of the invariant of the centre of forces is given. The existence of the invariant of the centre of forces admits the correct division of the problem into two problems. The first is a triangular restricted three-body problem. The second is a collinear restricted three-body problem. In this paper the collinear restricted three-body problem is investigated. Using the properties of the invariant of centre of forces of the restricted three-body problem in the special non-inertial central coordinate system, the basic differential equations of motion for the collinear restricted three-body problem are obtained when three bodies lie on the same line during all motion. Differential equations of the collinear restricted three-body problem in the rotating non-inertial central coordinate system in pulsating variables are derived. New differential equations of motion for the collinear restricted three-body problem in three regions of possible location of the massless body with stationary solutions corresponding to the three Euler libration points have been derived. The circular collinear restricted three-body problem is investigated in detail. The corresponding Jacobi integrals are obtained. New exact non-stationary partial analytical solutions of the obtained new differential equations of motion of the collinear restricted three-body problem have been found for the considered case.
Keywords: restricted three-body problem, non-inertial coordinate system, libration points, exact non-stationary partial rectilinear solutions.

1. Introduction. The motion of a small natural or artificial celestial body in the gravitational field of two large celestial bodies (hereinafter the primary bodies) is well described by the mathematical model of the widely known limited three-body problem [1-9]. The problem has many applications. For arbitrary values of the main body masses, the problem has five known libration points - stationary exact partial solutions. Of these, three are Euler *collinear* solutions, when three bodies all the time moving on the same line. Mathematical conditions of the restricted three-body problem with masses m_1, m_2, m_3 can be written in the form [1-6]

$$m_2 << m_1, \; m_2 << m_3, \; m_3 \geq m_1, \tag{1.1}$$

where the mass of a vanishingly small body (a massless body further) is denoted m_2 by the term. Due to the lack of a general analytical solution in final form, many aspects of the problem have been studied by various qualitative and numerical methods [1-10]. The search for new exact partial analytical solutions seems to be relevant.

The present paper is a continuation of the works [11, 12]. The purpose of this paper is to investigate the collinear restricted three-body problem in a special non-inertial central coordinate system and to establish new exact particular analytic solutions. Using the properties of the invariant of center-of-forces for the restricted three-body problem in a special non-inertial central coordinate system, the basic differential equations of motion of the collinear restricted three-body problem when the three bodies lie on the same line all the time of motion are investigated.

Differential equations of the three-body collinear restricted problem in a rotating non-inertial central coordinate system in pulsating variables are derived. Three new differential equations of motion for the collinear restricted three-body problem in three regions of possible location of a massless body with stationary solutions corresponding to the three Euler libration points have been derived. The circular collinear restricted three-body problem is investigated in detail. The corresponding Jacob integrals are obtained. New exact non-stationary partial analytical solutions of the three new differential equations of motion of the collinear restricted three-body problem have been found in the case of the collinear circular restricted three-body problem .

2. Differential equations of the collinear restricted three-body problem in a special non-inertial central coordinate system and the invariant of forces center. In [11] differential equations of the collinear restricted three-body problem in a special non inertial central coordinate system $Gxyz$ have been derived in the form are

$$\ddot{\vec{r}}_2 + f\left(\frac{m_3}{\Delta_{23}^3} + \frac{m_1}{\Delta_{21}^3}\right)\vec{r}_2 + f\left(\frac{km_3}{(1+k)\Delta_{23}^3} - \frac{m_1}{(1+k)\Delta_{21}^3}\right)\vec{r}_{31} = \vec{W} \tag{2.1}$$

$$\vec{W} = \vec{W}(t) = -f\frac{(m_1 - km_3)}{k+1}\frac{\vec{r}_{31}}{r_{31}^3} + 2\dot{\vec{r}}_{31}\frac{d}{dt}\left(\frac{1}{1+k}\right) + \vec{r}_{31}\frac{d^2}{dt^2}\left(\frac{1}{1+k}\right), \tag{2.2}$$

where $\vec{r}_{31} = \vec{r}_1 - \vec{r}_3$, f is the gravitational constant, $\vec{r}_1$, $\vec{r}_3$ are radius vectors of the primary bodies, $\vec{r}_2$ is the radius vector of a massless body, Δ_{ij} is the distance between the bodies, $k = r_3/r_1$. In deriving this differential equation (2.1), (2.2) the center-of-forces invariant in the form

$$\left[\frac{m_3}{\Delta_{23}^3}r_3 - \frac{m_1}{\Delta_{21}^3}r_1\right]\cdot\{r_2\sin\alpha\} = 0 \tag{2.3}$$

In this paper we investigate the collinear restricted three-body problem , considering only the case where

$$\{r_2\sin\alpha\} = 0\,. \tag{2.4}$$

It follows from this equality that in this case the three bodies lie on the same straight line all the time they are moving. This straight line is on a fixed plane of motion of the main two bodies described by the differential equation

$$\ddot{\vec{r}}_{31} = -f\frac{m_3 + m_1}{r_{31}^3}\vec{r}_{31}\,. \tag{2.5}$$

Equation (2.5) is the well-known equation of the classical two-body problem in a special non-inertial central coordinate system. From the integral

$$\vec{r}_{31}\times\dot{\vec{r}}_{31} = \vec{c}_{31} = \overrightarrow{const} \tag{2.6}$$

follows that the orbit is planar, without loss of generality, one can assume that the orbit of the two main body problem lies on the main plane Gxy . Therefore $z_2 = 0$, i.e. *the collinear restricted three-body problem is planar*.

The solution of the differential equation of motion of the two-body problem (2.5) in a special non-inertial central coordinate system will be written in the form [4-6]

$$r_{31} = r = \frac{p}{1+e\cos\theta}\ ,\ r^2\dot{\theta} = c_{31} = c = const \neq 0 \tag{2.7}$$

$$p = a\left(1-e^2\right),\ c^2 = \mu_{31}p\ ,\ \mu_{31} = f\left(m_1 + m_3\right) \tag{2.8}$$

$$x_{31} = r\cos\theta, \quad y_{31} = r\sin\theta, \quad z_{31} = 0\ \ ,\ r_{31}^2 = r^2. \tag{2.9}$$

In the case of $r_2 = 0$, a massless body lies at the origin of the coordinate system. If in equality (2.4)

$$r_2 \neq 0\,,\ \sin\alpha = 0 \tag{2.10}$$

then three combinations of locations of three bodies on the same line are possible.

In scalar form, equations (2.1) , (2.2) can be written as

$$\ddot{x}_2 + f\left(\frac{m_3}{\Delta_{23}^3} + \frac{m_1}{\Delta_{21}^3}\right)x_2 \ + f\left(\frac{km_3}{(1+k)\Delta_{23}^3} - \frac{m_1}{(1+k)\Delta_{21}^3}\right)x_{31} = W_x \quad , \tag{2.11}$$

$$W_x = -f\frac{(m_1 - km_3)}{k+1}\frac{x_{31}}{r_{31}^3} + 2\dot{x}_{31}\dot{s} + x_{31}\ddot{s}\,, \tag{2.12}$$

$$\ddot{y}_2 + f\left(\frac{m_3}{\Delta_{23}^3} + \frac{m_1}{\Delta_{21}^3}\right)y_2 + f\left(\frac{km_3}{(1+k)\Delta_{23}^3} - \frac{m_1}{(1+k)\Delta_{21}^3}\right)y_{31} = W_y\,, \tag{2.13}$$

$$W_y = -f\frac{(m_1 - km_3)}{k+1}\frac{y_{31}}{r_{31}^3} + 2\dot{y}_{31}\dot{s} + y_{31}\ddot{s} \quad , \tag{2.14}$$

where, for convenience, it has been renamed

$$s = 1/(1+k)\,. \tag{2.15}$$

The resulting differential equations of motion (2.11)-(2.15) will be called *the basic scalar differential equations of* motion for the collinear restricted three-body problem in the special non-inertial central coordinate system.

Note that the centre of force is always on the line connecting the two primary bodies. Therefore, any point lying on this line can be chosen as the centre of force. Next, we will analyse this fact and specify a specific point for determining the centre of force.

Note that in fact the system of equations (2.11) to (2.15) contains two unknowns $k(t)$ and $r_2(t)$. Therefore, in fact, we have two scalar equations with two unknowns.

3. Differential equations of motion of a collinear restricted three-body problem in a special non-inertial central rotating coordinate system in pulsating variables. Consider the basic equations of motion of the collinear restricted three-body problem (2.11)-(2.15) in a special non-inertial central coordinate system. Let us consider the general case $k \neq const$. In the solution (2.7)-(2.9) of the two-body problem $c_{31} = const \neq 0$, one can investigate elliptic, hyperbolic, parabolic restricted collinear three-body problems.

Let's move to a rotating coordinate system. Let the new axis $G\xi_{вр}$ pass through the points with masses of primary bodies and $m_1\,, m_3$. Since, we consider rectilinear motion along the axis $G\xi_{вр}$, so there is no $G\eta_{вр}$ motion along the axis. Hence, we obtain $\eta_{вр} = const = 0$. Therefore, the transition is done by formulae

$$x_2 = \xi_{вр}\cos\theta,\;\; y_2 = \xi_{вр}\sin\theta,\;\; r_2^2 = \xi_{вр}^2 = \rho_{вр}^2\,,\; r_{31}^2 = r^2 \tag{3.1}$$

where and $\theta = \theta(t)$ are defined $r = r(t)$ by the solution of the two-body problem (2.7)-(2.9).

Calculating, we obtain

$$\ddot{x}_2 = \left(\ddot{\xi}_{вр} - \dot{\theta}^2\xi_{вр}\right)\cos\theta - \left(2\dot{\theta}\dot{\xi}_{вр} + \ddot{\theta}\xi_{вр}\right)\sin\theta\,,\;\; \ddot{y}_2 = \left(2\dot{\theta}\dot{\xi}_{вр} + \ddot{\theta}\xi_{вр}\right)\sin\theta + \left(\ddot{\xi}_{вр} - \dot{\theta}^2\xi_{вр}\right)\cos\theta\,.$$

As a result, we obtain the transformed equations of motion in a rotating coordinate system as

$$\begin{aligned}&\left(\ddot{\xi}_{вр} - \dot{\theta}^2\xi_{вр}\right)\cos\theta - \left(2\dot{\theta}\dot{\xi}_{вр} + \ddot{\theta}\xi_{вр}\right)\sin\theta + f\left(\frac{m_3}{\Delta_{23}^3} + \frac{m_1}{\Delta_{21}^3}\right)\xi_{вр}\cos\theta + f\left(\frac{km_3}{(1+k)\Delta_{23}^3} - \frac{m_1}{(1+k)\Delta_{21}^3}\right)x_{31} = \\ &= -f\frac{(m_1 - km_3)}{k+1}\frac{x_{31}}{r_{31}^3} + 2\dot{s}\,\dot{x}_{31} + x_{31}\,\ddot{s}\,,\end{aligned} \tag{3.2}$$

$$\begin{aligned}&\left(\ddot{\xi}_{вр} - \dot{\theta}^2\xi_{вр}\right)\sin\theta + \left(2\dot{\theta}\dot{\xi}_{вр} + \ddot{\theta}\xi_{вр}\right)\cos\theta + f\left(\frac{m_3}{\Delta_{23}^3} + \frac{m_1}{\Delta_{21}^3}\right)\xi_{вр}\sin\theta + f\left(\frac{km_3}{(1+k)\Delta_{23}^3} - \frac{m_1}{(1+k)\Delta_{21}^3}\right)y_{31} = \\ &= -f\frac{(m_1 - km_3)}{k+1}\frac{y_{31}}{r_{31}^3} + 2\dot{s}\,\dot{y}_{31} + y_{31}\,\ddot{s}\,.\end{aligned} \tag{3.3}$$

Multiplying (3.2) by ($+\cos\theta$) and (3.3) by ($+\sin\theta$) then summing them we get

$$\ddot{\xi}_{вр}-\dot{\theta}^2\xi_{вр}+f\left(\frac{m_3}{\Delta_{23}^3}+\frac{m_1}{\Delta_{21}^3}\right)\xi_{вр}+f\left(\frac{km_3}{(1+k)\Delta_{23}^3}-\frac{m_1}{(1+k)\Delta_{21}^3}\right)r=W_\xi \tag{3.4}$$

$$W_\xi=-f\frac{m_1-km_3}{k+1}\frac{r}{r^3}+2\dot{r}\dot{s}+r\ddot{s} \tag{3.5}$$

Further, by multiplying (3.2) by (- $\sin\theta$) and (3.3) by ($+\cos\theta$) then summing them we obtain

$$\left(2\dot{\theta}\dot{\xi}_{вр}+\ddot{\theta}\xi_{вр}\right)+2\dot{s}\;r\dot{\theta}=0 \tag{3.6}$$

As a consequence of the centrality of forces (integral of areas) we have $\left(2\dot{\theta}\dot{\xi}_{вр}+\ddot{\theta}\xi_{вр}\right)=0$. Therefore, given that $\dot{\theta}\neq 0$, $r\neq 0$ from equality (3.6) we obtain

$$\dot{s}=0. \tag{3.7}$$

Consequently, given the notation (2.15) we have

$$k=const\,. \tag{3.8}$$

Thus, we have established that the origin of the non-inertial central coordinate system is a fixed point on the segment r_{31} . As noted above, any point lying on a straight line connecting two primary bodies can be chosen as the centre of forces - the origin of the non-inertial central coordinate system. Therefore, it becomes clear that the barycenter of two bodies may also be chosen as centre of force ($k=m_1/m_3=const$). Next, for convenience, let us consider a non-inertial central coordinate system with $k=const>0$. For certain cases, let us define certain values $k=const$.

Accordingly, the distances between the bodies are

$$\Delta_{21вр}=\left[\left(\xi_{вр}-\xi_1 r\right)^2\right]^{1/2}=\left|\xi_{вр}-\frac{1}{1+k}r\right|,\ \Delta_{23вр}=\left[\left(\xi_{вр}-\xi_3 r\right)^2\right]^{1/2}=\left|\xi_{вр}+\frac{k}{1+k}r\right|, \tag{3.9}$$

$$\xi_1=1/(1+k)=const\ ,\ \xi_3=-k/(1+k)=const\ . \tag{3.10}$$

Next, move to a pulsating coordinate ξ with a new independent variable using the formulas θ

$$\xi_{вр}=r\xi,\ \ dt=\left(r^2/c\right)d\theta, \tag{3.11}$$

where $\theta=\theta(t)$ and $r=r(t)$ are defined $c=const\neq 0$ by the solution of the two-body problem (2.7)-(2.9). By calculating, we obtain

$$\dot{\xi}_{вр}=\dot{\theta}\left(r'\xi+r\xi'\right),\ \ddot{\xi}_{вр}=\dot{\theta}\left(\dot{\theta}'\left(r'\xi+r\xi'\right)+\dot{\theta}\left(r''\xi+2r'\xi'+r\xi''\right)\right),$$

where the dashed line denotes the differentials in the variable θ . Given the relations $2r'\dot{\theta}+\dot{\theta}'r=0$, $\dot{\theta}r''+\dot{\theta}'r'-\dot{\theta}r=-\left(c/p\right)$ differential equations of motion (3.4)-(3.5) in pulsating variables

$$\xi''=\mu_{31}\frac{r}{c^2}\left\{\xi-\frac{f}{\mu_{31}}\left(\frac{m_3}{\Delta_{23}^3}+\frac{m_1}{\Delta_{21}^3}\right)\xi-\frac{f}{\mu_{31}}\left(\frac{km_3}{\Delta_{23}^3}-\frac{m_1}{\Delta_{21}^3}\right)\frac{1}{1+k}+\frac{B_2}{\mu_{31}}\right\}, \tag{3.12}$$

where dimensionless values are indicated

$$\Delta_{21}=\Delta_{21\xi}=\left|\xi-\frac{1}{1+k}\right|,\ \xi_1=\frac{1}{1+k}=const,\ \Delta_{23}=\Delta_{23\xi}=\left|\xi+\frac{k}{1+k}\right|,\ \xi_3=-\frac{k}{1+k}=const.$$

Substituting in equation (3.12) the solutions of the two-body problem (2.7) - (2.9) we finally obtain the differential equation of motion of the collinear restricted three-body problem in a special non-inertial rotating coordinate system in pulsating variables

$$\xi'' = \frac{1}{1+e\cos\theta}\left\{\xi - \frac{1}{m_1+m_3}\left(\frac{m_3}{\Delta_{23}^3}+\frac{m_1}{\Delta_{21}^3}\right)\xi - \frac{1}{m_1+m_3}\left(\frac{km_3}{\Delta_{23}^3}-\frac{m_1}{\Delta_{21}^3}\right)\frac{1}{1+k}+B\right\}, \tag{3.13}$$

$$B = \frac{B_2}{\mu_{31}} = \frac{km_3 - m_1}{(k+1)(m_1+m_3)} = \frac{k-\nu}{(k+1)(1+\nu)} = const\ ,\ \nu = \frac{m_1}{m_3}\ . \tag{3.14}$$

Relationships (3.13)-(3.14) can be written in a more convenient form

$$\xi'' = \frac{1}{1+e\cos\theta}\left\{\xi - (1-\mu)\cdot\frac{\xi+\left(k/(k+1)\right)}{\left|\xi+\left(k/(k+1)\right)\right|^3} - \mu\cdot\frac{\xi-\left(1/(k+1)\right)}{\left|\xi-\left(1/(k+1)\right)\right|^3} + (1-\mu) - \frac{1}{1+k}\right\}, \tag{3.15}$$

where dimensionless values are indicated

$$\mu = \frac{m_1}{m_1+m_3}\ ,\ 1-\mu = \frac{m_3}{m_1+m_3}\ ,\ 0<\mu\le\frac{1}{2}\,. \tag{3.16}$$

The differential equation (3.15) is called *the basic differential equation of the collinear restricted three-body problem* in a special non-inertial rotating coordinate system in pulsating variables.

As noted above, a massless body can be placed in three different positions on a straight line connecting the two primary bodies. It is necessary to consider each position of the massless body separately.

4. New differential equations of motion of the collinear restricted three-body problem in three regions of possible motion and non-stationary exact partial analytical solutions of the collinear circular restricted three-body problem. Let us consider the first case when a massless body is between two primary bodies during the whole period of motion - the region of possible motion L_1 . Let us denote by the distance from the body of smaller mass to the massless body as follows

$$\frac{1}{1+k} - \xi = z\,. \tag{4.1}$$

Then we get

$$\xi = \frac{1}{1+k} - z\,, \tag{4.2}$$

$$\xi - \frac{1}{1+k} = -z\ ,\ \xi + \frac{k}{1+k} = 1-z\,,\ \left|\xi - \frac{1}{1+k}\right| = z\ ,\ \left|\xi + \frac{k}{1+k}\right| = 1-z\,. \tag{4.3}$$

As a result, given (4.1), (4.2), (4.3), from equation (3.15), we obtain

$$\begin{aligned} z'' &= \frac{1}{1+e\cos\theta}\left\{z + (\mu-1) - \frac{\mu-1}{(1-z)^2} - \frac{\mu}{z^2}\right\} = \\ &= \frac{1}{1+e\cos\theta}\cdot\left\{\frac{1}{(1-z)^2 z^2}\left[z^5 - (3-\mu)z^4 + (3-2\mu)z^3 - \mu z^2 + 2\mu z - \mu\right]\right\} \end{aligned} \tag{4.4}$$

Equation (4.4) describes changes of length of segment (4.1) in a rectilinear restricted three-body problem in the region of possible motion where the known stationary solution - libration point - L_1 is located. Exactly analogously, we obtain differential equations of motion of the collinear restricted three-body problem in the remaining two cases when a massless body lies in the regions of possible motion L_2 and L_3 .

In the case of

$$e = 0\ ,\ r_{31} = r = a = const \tag{4.5}$$

i.e. in the circular collinear restricted three-body problem , the differential equation (4.4) becomes autonomous, hence the Jacobi integral takes place. Therefore, in case (4.5), differential equation (4.4) reduces to quadrature. A detailed study of the found exact new partial non-stationary analytical solutions of the collinear three-body bounded problem will be done in a separate paper.

5. Conclusion. The basic differential equations of motion for the collinear restricted three-body problem when three bodies lie on the same line during all motion have been investigated. New differential equations of the restricted collinear problem of three bodies in the rotating non-inertial central coordinate system in pulsating variables were derived. New differential equations of the three-body collinear restricted problem in three regions of possible motion of the massless body on the straight line joining the primary bodies have been derived. In these three regions of possible motion, there are three stationary solutions of the differential equations, the Euler libration points. The obtained new differential equations for the motion of the collinear restricted three-body problem that describe the circular, elliptic, parabolic and hyperbolic collinear restricted three-body problem.

In the case of the circular collinear restricted three-body problem, the derived differential equations of motion become autonomous and hence integrable. New exact non-stationary partial analytical solutions are established.

In the future, a detailed analysis of the obtained new exact non-stationary partial analytical solutions of the collinear restricted three-body problemis planned. The basic idea of this paper and preliminary results were stated in [11,15].

М. Ж. Минглибаев[1,2], Т. М. Жұмабек[1]

[1]әль-Фараби атындағы ҚазҰУ, Алматы, Қазақстан;
[2] В. Г. Фесенков атындағы Астрофизика институты, Алматы, Қазақстан

ШЕКТЕЛГЕН ҮШ ДЕНЕ МӘСЕЛЕСІНЕ

Аннотация. Жұмыста аналитикалық түрде классикалық шектелген үш дене мәселесі басы мәселенеің күштер центрінде болатын арнаулы инерциалды емес централды санақ жүйесінде зерттелінген. Бұл санақ жүйесінде күштер центрінің инвариантының аналитикалық түрі берілген. Күштер центрінің инвариантының бар болуы мәселені дұрыс екіге бөліп қарастыруды мүмкін етеді. Біріншісі үшбұрышты шектелген үш дене мәселесі. Екіншісі коллинеар шектелеген үш дене мәселесі. Бұл жұмыста коллинеарлық шектелген үш дене мәселесі зерттелінген. Арнаулы инерциалды емес централды санақ жүйесінде шектлеген үш дене мәселесінің күштер центрінің инвариантын қасиеттерін қолдана отыра, үш дене бір түзудің бойында жатқанда коллинеарлық шектелген үш дене мәселесінің базалық дифференциалдық теңдеулері зерттелінді. Коллинеарлы шектлеген үш дене мәселесінің айналмалы инерциалды емес централды санақ жүйесінде пульсацiиялаушы айнымалыларында дифференциалдық теңдеулері алынды. Үш мүмкін болатын массасы өте аз дененің орналасу аймағы үшін коллинеарлы шектелген үш дене мәселесінің қозғалысының жаңа дифференциалдық теңдеулері алынды және олардың стационарлық шешімдері Эйлердің үш либрация нүктелеріне сәйкес келеді. Коллинеарлы шеңберлік шектлеген үш дене мәселесі терең зерттелінді. Сәйкес Якоби интегрлы алынды. Коллинеарлы шеңберлік шектелген үш дене мәселесінің қозғалысының жаңа дифференциалдық теңдеулерінің жаңа нақты бейстационар дербес аналитикалық шешімдері алынды.

Түйін сөздер: шектелген үш дене мәселесі, инерциалды емес санақ жүйесі, либрация нүктелері, нақты бейстационар дербес түзу сызықты шешімдер.

М. Дж. Минглибаев[1,2], Т. М. Жумабек[1]

[1]КазНУ им. аль-Фараби, Алматы, Казахстан;
[2]Астрофизический институт им. В. Г. Фесенкова, Алматы, Казахстан

К ОГРАНИЧЕННОЙ ЗАДАЧЕ ТРЕХ ТЕЛ

Аннотация. В работе аналитически исследована классическая ограниченная задача трех тел в специальной неинерциальной центральной система координат, с началом в центре сил исследуемой задачи. В этой системе координат приведены аналитическое выражение инварианта центра сил. Наличие инварианта центра сил допускает корректное разделение задачи на две задачи. Первая треугольная ограниченная задача трех тел. Вторая коллинеарная ограниченная задача трех тел. В настоящей работе исследована коллинеарная ограниченная задача трех тел. Используя свойства инварианта центра сил ограниченной задачи трех тел в специальной неинерциальной центральной система координат, исследованы базовые дифференциальные уравнения движения коллинеарной ограниченной задачи трех тел, когда три тела все время движения лежат на одной и той же прямой. Выведены дифференциальные уравнения коллинеарной ограниченной задачи трех тел в вращающейся неинерциальной центральной системе координат в пульсирующих переменных. Получены новые дифференциальные уравнения движения коллинеарной ограниченной задачи трех тел, в трех областях возможного расположения безмассового тела, стационарные решения которых соответствует трем точкам либрации Эйлера. Подробно исследована круговая коллинеарная ограниченная задачи трех тел. Получены соответствующие интегралы Якобы. Установлены новые точные нестационарные частные аналитические решения полученных новых дифференциальных уравнений движения коллинеарной ограниченной задачи трех тел, в рассмотренном случае.

Ключевые слова: ограниченная задача трех тел, неинерциальная система координат, точки либрации, точные нестационарные частные прямолинейные решения.

Information about authors:

Minglibayev Mukhtar Zhumabekovich, Doctor of Physical and Mathematical Sciences, Professor of al-Farabi Kazakh National University, Chief Researcher of Fesenkov Astrophysical Institute, Almaty, Kazakhstan; minglibayev@gmail.com; https://orcid.org/0000-0002-8724-2648;

Zhumabek Torebek Mukhtaruly, PhD student at al-farabi Kazakh National University, Almaty, Kazakhstan; torebekzhumabek@hotmail.com; https://orcid.org/0000-0001-6865-8309

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