NEWS
OF THENATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2020.2518-1726.60**
Volume 4, Number 332 (2020), 13 – 20

E.A. Abylkasymova[1], G.I. Beysenova[1], U.P. Suiinzhanova[2]

[1]Regional Social Innovation University, Shymkent, Kazakhstan;
[2]South KazakhstanState Pedagogical University, Shymkent, Kazakhstan.
E-mail: megaelmyra@mail.ru, gulia-74-74@mail.ru, only-kh@mail.ru

WEAK CONVERGENCE OF INTEGRAL CURVATURES OF CONVEX SURFACES

Abstract. The article contains a concentrated analysis of the existing information on the main problems of the theory of convex surfaces and differential geometry “in general” and is devoted to the problems of the reconstructing convex surfaces from the information about their curvature studied by the topological methods of the functional analysis.

The class of smooth surfaces in a bounded convex domain $G\subset E^2$ is considered. The concept of the R-area of a normal image is set forth. In the class K+(G), the Monge–Ampere equation is considered.

The paper considers the integrals of transverse Minkowski measures associated with the parallel surfaces. If the surface Φ is given by the explicit equation z = f (x, y), then for the integral curvatures of this surface transferred to the plane E^2, the inequalities $\varpi_k(\Phi, G)\leq\omega_k(\partial Z_\varphi, \partial Z_\varphi)$ for k = 0, 1, 2. Of these inequalities, inequalities follow

$$\varpi_0(\Phi, G)\leq 2\pi r^2+4\pi r\|\varphi\|_G,\ \varpi_1(\Phi, G)\leq 2\pi^2 r+4\pi\|\varphi\|_G,\ \varpi_2(\Phi, G)\leq 4\pi,$$

which are used in reasoning. We prove the weak convergence of the integral curvatures of the convex surfaces. The result obtained in the form of a theorem plays an important role in the proofs of the theorems on the existence of a convex hypersurface with a given combination of the integral conditional curvatures. For the first time, the conditional curvatures are taken in the most general form, as a given function of the integral conditional curvatures of the various orders. The integrand functions are the product of the continuous functions and the integral curvatures of the various orders.

Keywords: convex surface, convex surfaces in Euclidean space, Monge-Ampère equation, the cone of convex surfaces in the space of continuous functions, conditional curvature, integral curvature, restoration of surface.

1. Introduction. The existence and uniqueness of a surface with the predetermined geometrical characteristics is one of the important and urgent problems of the differential geometry “in general” and its various analytical and mechanical applications[15]. The formulation of this problem in the language of analysis leads to the boundary value problems for the elliptic or hyperbolic partial differential equations of the second order, and also, as I.Ya. Bakelman showed, to integral equations.[12],[13], [14]

The class of smooth surfaces in a bounded convex domain $G\subset E^2$ is considered. The concept of the R-area of a normal image is set forth[16]. In the class K+(G), the Monge–Ampere equation is considered.

A number of geometric problems, including the Minkowski problem of constructing a convex surface, are reduced to the simplest Monge–Ampere equations.[9] G. Minkowskiposed and solved the problem of constructing a convex surface, whose Gaussian curvature has a given normal function.

2. Setting of a problem. Integrals of transverse Minkowski measures. The papers [1], [3], [10] set forth the information from the theory of integrals of the transverse Minkowski measures for the convex bodies, and also presented in [2], [11] the concepts and properties of the integral curvatures of the various orders for the closed convex surfaces. In [4], some fundamental formulas of the theory of surfaces are considered.

Let *T* be a bounded convex body in the space E^3. Then $F=\partial T$ is a closed surface that homeomorphic to the sphere S^2.

Let P be the plane of support to F at the point $X_0 \in F$. We denote by Π such of the closed half-spaces E^3 with the boundary P, which contains the body T. If the origin of coordinates is placed in a point lying inside the body T, then the plane P and the half-space Π are specified analytically as follows:

$$P=\{X \in E^3 | \langle X, \nu\rangle = d\};\ \Pi = \{X \in E^3 | \langle X, \nu\rangle \leq d\}, \tag{1}$$

where ν is the unit outward normal to P, X is the current radius vector of points from P or from $\Pi$and d is the distance from the origin to the plane P.

To each plane of support P to F we assign the plane P_h, having the equation

$$P_h=\{X \in E^3 | \langle X, \nu\rangle=d+ h\}, \tag{2}$$

where $h \geq 0$ is a fixed number. We denote by Π_h the half-space given by the inequality

$$\Pi_h =\{X \in E^3 | \langle X, \nu\rangle \leq d+h\}. \tag{3}$$

Obviously, $P_h=\partial\Pi_h$. Denote by T_h the intersection of all half-spacesΠ_h,which are constructed along all possible support hyperplanes$P$ to F. Then T_h is a bounded convex body, and $F_h=\partial T_h$ is a closed convex surface geomeomorphic to the sphere S^2. Surfaces $F_h$are called parallel to surface F.

3. Research methods. The following formula is valid ([5], [1]):

$$V(T_h) = \sum_{k=0}^{3} C_3^k W_k(T) h^k, \tag{4}$$

where $h \geq 0$ is a fixed number, the numbers $W_k(T)$ that are factors in the coefficients of the polynomial on the right-hand side of (4) are called integrals of Minkowski transverse measures for a convex body T. Integrals of transverse Minkowski measures are the functionals on a class of the bounded convex bodies. We point out the properties of these functionals necessary for further.

1. $W_0(T)=V(T)$, $W_1(T)=\frac{1}{3}\sigma(\partial T)$.

2. For any k = 0..3, the functionals$W_k(T)$ on the class of convex bodies are non-negative, invariant with respect to motions in E^3, monotone, bounded, homogeneous degrees $(3 - k)$, continuous, and additive.

The formulation and proof of these properties are given in [5], Chapter VI, §1.

3. For convex bodies with a smooth C^2 boundary, in the sense of differential geometry, the functional Wk(T) can be represented as integrals on ∂T from elementary symmetric functions of order $(k - 1)$ of 3 principal curvatures ∂T.

For each fixed $h \geq 0$, the mapping is defined

χ_h: *subset* $F \rightarrow$ *subset* F_h,

which assigns $A_h \subset F_h$ to each set $A \subset F$ such that:

If P is the plane of support to the surface F, such that $A \cap P \neq \varnothing$, then the plane $P_h = \{X \in E^3 \mid \langle X, \nu\rangle = d+h\}$ is the plane of support to the surfaceF_h and $A_h \cap P_h \neq \varnothing$, and, on the contrary, if P_h is the plane of support to the surface F_h, then P is the plane of support to the surface F.

Obviously, χ_0 is the identity transformation.

Let F be a complete convex surface in E^3. As is known, the limit of a converging sequence of planes of support to a complete convex surface F is the plane of support to F. From this fact the following properties of the map χ_h follow:

1. χ_h translates the closed subsets into the closed subsets.
2. χ_h translates the bounded subsets into the bounded subsets.
3. χ_h translates the compact subsets into the compact subsets.
4. χ_h translates the Borel subsets into the Borel subsets.

It is known that, under the imposed conditions, the areas of the Borel subsets exist.

In [8] the formula is proved

$$\sigma(A_h) = \sum_{k=0}^{2} C_3^k \omega_k(F, A) h^k, \tag{5}$$

where $\omega_k(F,A)$ are non-negative completely additive functions on the ring of Borel sets of the surface F such that $\omega_k(F,F)=W_1(T)$.

In [5], [6], it was shown that for the convex bodies with a smooth C^2boundary of the function of the set $\omega_k(F,A)$, $k = 1,2$, there are integrals over the set A of the elementary symmetric functions of order k of the 2 principal curvatures F. Therefore, $\omega_k(F,A)$ can be considered as functions of a set associated with integrals of transverse Minkowski measures. We will call these functions the integral curvatures of order k of the convex surface F.

The following was proved in [7]: if the closed convex surfaces $F_m$converge to a closed convex surface F, then the completely additive non-negative functions of the sets ω_k (F_m, A_m) weakly converge to a completely additive non-negative function of the set ω_k (F, A).

We give a more detailed formulation of the last statement. Since the convex surface F is homeomorphic to the sphere S^2 and the convex surfaces $F_m$converge to F, not breaking its generality, we can assume that there is a closed ball contained simultaneously within all $F_m$and F. Let's transfer the origin of coordinates to the center of this ball; the values of functions do not change from this.

Let θ be an arbitrary point of the sphere S^2. We obtain that the surfaces $F_m$and F are given respectively by the equations Φ_μ: $\rho=\rho_\mu(\theta)$, Φ: $\rho=\rho(\theta)$, the convergence of F_m to F means the uniform convergence of the functions $\rho_m(\theta)$ to $\rho(\theta)$ on S^2.

Let H be an arbitrary Borel set from S^2. Denote by A and Am the Borel sets of convex hypersurfaces$F$ and F_m, which are obtained as a result of the central projection of the set H from the origin of coordinates on these surfaces. Put now

$$\varpi_k(F, H) = \omega_k(F, A),\ \varpi_k(F_m, H) = \omega_k(F_m, A_m),\ k = 0, 1, 2.$$

Obviously, $\varpi_k(F,H)$ and $\varpi_k(Fm,H)$ are completely additive nonnegative functions of sets on the sphere S^2. We will call these set functions the integral curvatures of order k for convex surfaces F and F_m transferred to the sphere S^2.

The weak convergence of the functions of the sets $\varpi_k(F_m,H)$ to $\varpi_k(F,H)$ according to [7] means that for any continuous function $f(\theta)\in C(S^2)$, the equality

$$\lim_{m\to\infty} \oiint_{S^2} f(\theta)\varpi_k(F_m, dH) = \oiint_{S^2} f(\theta)\varpi_k(F, dH). \tag{6}$$

As is known, see [1], [9], a necessary and sufficient condition for the weak convergence of $\omega_k(F_m,A_m)$ to $\omega_k(F,A)$, provided that F_m converges to F, is that if $H\subset S^2$ is a closed set, then

$$\overline{\lim_{m\to\infty}}\, \overline{\omega}_k(F_m, H) \le \overline{\omega}_k(F, H) \tag{7}$$

and if $H\subset S^2$ is an open set, then

$$\underline{\lim}_{m\to\infty}\, \overline{\omega}_k(F_m, H) \le \overline{\omega}_k(F, H) \tag{8}$$

The functions of the sets $\omega_k(\Phi, B)$ of convex surfaces given by explicit equations. Let $\Phi_z\in K(G)$ be the surface and let M be a convex compact subset of G. Without loss of generality, we can assume that $\Phi_z\in K^-(G)$. Then the distance $\delta M = dist(M, \partial G)$ is positive. Let denote by Φ_M the graph of the function $z_M(x, y) = z(x, y)/\delta M$.

Let h be an arbitrary real number satisfying the condition $h<h_M=\inf_M z(x,y)$.

Let $T_M(\Phi, h) = conv(F_M,H(M,h))$. It's obvious that

$$\partial T_M(F, h) = F_M\cup H(M, h) \cup Z_M,$$

where Z_M is the part of the cylindrical surface with the guide ∂M, forming - segments, parallel to the Z axis, located between ∂F_M and $\partial H(M, h)$. Set

$$\omega_k(z_M, A)=\omega_k(\partial T_M(\Phi,h), A), \tag{9}$$

where $A\subset F_M$ is a Borel subset. Note that the number $\omega_k(\partial T_M(\Phi,h), A)$does not depend on the number h.

If $M \subset M_1$ are two compact convex subsets of G, then for any $h \in (-\infty, h_M)$, obviously, the equality

$$\omega_k(\Phi_M, A) = \omega_k(\Phi_M, A), \tag{10}$$

where $A \subset F_M$ is an arbitrary Borel subset. From (10) it follows that in the left-hand side of formula (9), the F_M surface can be replaced by F.

Thus, for $\Phi \in K^-(G)$and any Borel set $A \subset \Phi$ such that the orthogonal projection A^1 of the set A on E^2 is removed from ∂G by a positive distance, we have

$$\omega_k(\Phi, A) = \omega_k(\partial T_m(\Phi, h), A) \tag{11}$$

where $T_M(\Phi, h) = conv\ (F_M, N_M)$,

M is any compact convex subset of G such that $A_1 \subset M$,

h is any number satisfying the inequality $h < h_M = inf_M z\ (x, y)$.

Formula (10) implies an important corollary:

1) Let $M \subset G$ be a compact convex subset. Then, on the ring of Borel subsets A of the F_M surface, the integral curvatures $\omega_k(\Phi, A)$, $k=0,1,2,$ represent nonnegative completely additive functions of a set of bounded variation.

2) For any Borel set $A \subset \Phi$ we set

$$\omega_k(\Phi, A) = sup_{M \subset G}\, \omega_k(\Phi, A \cap \Phi_M) \tag{12}$$

over all convex compact subsets of $M \subset G$. The number $\omega_k(\Phi, A)$ is always non-negative, it can also take the value $+\infty$.

3) For any $\Phi \in K(G)$, the integral curvatures $\omega_k(\Phi, A)$, $k = 0,1,2$, are completely additive, non-negative functions on the ring of Borel sets F. If Φ is a bounded set in E^3, then these functions have limited variation.

For any $\Phi \in K\ (G)$ we set

$$\varpi_k(\Phi, A^1) = \omega_k(\Phi, A),\ k = 0, 1, 2, \tag{13}$$

where A is a Borel subset on Φ, and A^1 is the orthogonal projection of A onto the plane E^2. Obviously, $A^1$is a Borel subset of G. It follows directly from the definition that $\varpi_k(\Phi, B)$ are the non-negative completely additive functions on the ring of Borel subsets of G, which take finite values for sets B remote from ∂G by a positive distance.

These set functions will be called the integral curvatures of the surface Φ transferred to the plane E^2.

We consider the question of the weak convergence of these integral curvatures. We give a well-known literature definition of the weak convergence of the completely additive functions of sets defined on the Borel subsets of the domain G. It is said that the sequence of the completely additive functions of the sets $\mu_m(B)$ weakly converges to a completely additive function of the set $\mu(B)$, if for any $f \in C(G)$ with a compact support $M \subset G$, an equality holds

$$\lim_{m \to \infty} \iint_G f(x, y)\mu_m(dB) = \iint_G f(x, y)\mu(dB).$$

The following theorem is true.

4. Results of the research. Theorem 1. Let the convex surfaces $\Phi_m \in K(G)$ converge to the convex surface $\Phi \in K\ (G)$, then for all $k = 0,1,2$ the integral curvatures $\varpi_k(\Phi_m, B)$ of the hypersurfacesΦ_m, transferred to E^2, weakly converge to the integral curvature $\varpi_k(\Phi, B)$, transferred to E^2.

Proof. For definiteness, we assume that $\Phi_m \in K^+\ (G)$ and $\Phi \in K^+\ (G)$. Since Since, under parallel shifts of Φ along the Z axis, the integral curvatures $\varpi_k(\Phi, B)$ of the surface Φ transferred to E^2 do not change, without loss of generality, we can assume that all surfaces $\Phi$and Φ_m lie under the plane E^2.

Now let $f \in C(G)$ with compact support $M' \subset G$. Obviously, $dist(M', \partial G) > 0$. Therefore, one can find a compact convex set M such that $G \supset M \supset M'$. Obviously, $dist(M, \partial G)$ is also positive. We introduce the convex surfaces Φ_M and $\Phi_{M,m}$. Since they are uniformly bounded in E^3, one can find a point (x,y,h) in

E^3 such that $(x,y)\in M$, and h is a sufficiently large number, and the projections of the surfaces Φ_M and $\Phi_{M,m}$ from the points (x,y,h) on the plane E^2 are contained in some compact convex set N satisfying the condition $M \subset N \subset G$.

Obviously, $dist(M, \partial G) > 0$. Without loss of generality, we can assume that all surfaces Φ_M and $\Phi_{M,m}$ lie above the plane $z = h$.

Now suppose that the S_1^2 is the sphere centered at the point (x,y,h) of the radius 1. Without loss of generality, we can assume that S_1^2 lies under all the surfaces Φ_M and $\Phi_{M,m}$.. Let S_{1+}^2 be the open hemisphere S_1^2, the equatorial plane of which is parallel to the E^2 plane. Denote by $\pi: S_{1+}^2\to E^2$ the central projection of S_{1+}^2 to the plane E^2. Obviously, π is a diffeomorphism.

Suppose $N'=\pi^{-1}(N)$, then N' is a convex compact subset on S_1^2, contained inside S_{1+}^2. We introduce the function on S_1^2

$$g(\theta)=\begin{cases} f(\pi(\theta)), & \text{if } \theta\in N', \\ 0, & \text{if } \theta\in S_1^2 \setminus N'. \end{cases}$$

It is easy to see that $g(\theta)\in C(S_1^2)$. By construction, this function is such that it is zero on S_1^2 outside the set $\pi^{-1}(M')$. Note that $\pi^{-1}(M^{/}) \subset \pi^{-1}(M) \subset \pi^{-1}(N)$.

We now construct the convex bodies.

$T_N(\Phi, h) = conv\ (\Phi_N, H(N, h)),\ T_N(\Phi_m, h) = conv\ (\Phi_{N,m}, H(N, h))$.

Let B be an arbitrary Borel subset of the set M. Let, further, A and A_m be the Borel subsets of surfaces Φ and Φ_m, whose orthogonal projections are the set B. Then

$\varpi_k(\Phi,B)=\omega_k(\Phi_N,A)=\omega_k(\Phi_M,A),\ \varpi_k(\Phi_m,B)=\omega_k(\Phi_{N,m},A)=\omega_k(\Phi_{M,m},A),$

$k=0, 1, 2, m=1, 2,\dots$

According to the above, we also have

$$\varpi_k(\Phi,B)=\omega_k(\Phi_N,A)=\omega_k(\Phi_M,A)= \varpi_k(\partial T_N(\Phi, h), \pi^{-1}(A)),$$
$$\varpi_k(\Phi_m,B)=\omega_k(\Phi_{N,m},A_m)=\omega_k(\Phi_{M,m},A_m)= \varpi_k(\partial T_N(\Phi_m, h), \pi^{-1}(A)). \quad (14)$$

Formulas (14) establish the relations between the integral curvatures of a fixed order k of the convex surfaces Φ and Φ_m, transferred respectively to the E^2 plane and the S_1^2 sphere. From these formulas and the definition of the function g, the properties of the functions f and g, and the fact that $\pi: S_{1+}^2\to E^2$ is a diffeomorphism, we obtain

$$\iint_G \varpi_k(\Phi, dB) = \oiint_{S_1^2} g\,\varpi_k(\Phi, dH),$$
$$\iint_G \varpi_k(\Phi_m, dB) = \oiint_{S_1^2} g\,\varpi_k(\Phi_m, dH).$$

Using the weak convergence of the functions of the sets $\varpi_k(\Phi_m, dH)$ to the function of the set $\varpi_k(\Phi, dH)$, we obtain

$$\lim_{m\to\infty} \oiint_{S_1^2} g\,\varpi_k(\Phi_m, dH) = \oiint_{S_1^2} g\,\varpi_k(\Phi, dH).$$

From here we have

$$\lim_{m\to\infty} \iint_G f\varpi_k(\Phi_m, dB) = \iint_G f\varpi_k(\Phi, dB).$$

Since the continuous function f, which vanishes outside a certain compact set in G, is chosen arbitrarily, the functions of the set $\varpi_k(\Phi_m, B)$ converge weakly to the function of the set $\varpi_k(\Phi_m, B)$. The theorem is proved.

5. Conclusions. Thus, we investigated the integrals of transverse Minkowski measures associated with parallel surfaces. The weak convergence of integral curvatures of convex surfaces is proved. The result obtained in Theorem 1 is new and plays an important role in the proofs of the existence theorems for a convex hypersurface with a given combination of integral conditional curvatures.

Э.А. Абылкасымова[1], Г.И. Бейсенова[1], Ү.П. Сүйінжанова[2]

[1]Аймақтық әлеуметтік-инновациялық университеті, Шымкент, Қазақстан;
[2]Оңтүстік Қазақстан мемлекеттік педагогикалық университеті, Шымкент, Қазақстан

ДӨҢЕС БЕТТІҢ ИНТЕГРАЛ ҚИСЫҒЫНЫҢ ӘЛСІЗ ЖИНАҚТАЛУЫ

Аннотация. Мақалада дөңес бет теориясы мен «тұтастай» дифференциалды геометрияның негізгі проблемалары туралы талдау жасалған және функционалдық талдаудың топологиялық әдістері арқылы меңгерілген қисықтар туралы ақпараттарға, дөңес беттерді қалпына келтірудің мәселелері іздестірілген.

Шектелген дөңес $\mathbf{G} \subset E^2$ аймағындағы тегіс беттер класы қарастырылады. Қалыпты кескіннің R-ауданы туралы түсінік берілген. K+(G) класындағы Монж-Ампер теңдеуі қарастырылады.

Мақалада параллель беттерге қатысты Минковскийдің көлденең өлшем интегралдары қарастырылады. Егер Φ беті анық z=f(x,y) теңдеуімен берілсе, онда E2 жазықтығына берілген осы беттің интегралдық қисықтары үшін мұндағы k=0, 1, 2 болғанда мына теңсіздік $\varpi k(\Phi, G) \leq \omega k(\partial Z\varphi, \partial Z\varphi)$ ақиқат болып саналады. Бұл теңсіздіктен кейін болжау барысында қолданылатын келесі теңсіздік шығады:

$$\varpi_0(\Phi, G) \leq 2\pi r^2 + 4\pi r\|\varphi\|_G,\ \varpi_1(\Phi, G) \leq 2\pi^2 r + 4\pi\|\varphi\|_G,\ \varpi_2(\Phi, G) \leq 4\pi$$

F дөңес беті S2 сферасына гомеоморфты және Fm дөңес беттері F-ге ортақтықты жоғалтпай жинақталады, сондықтан барлық Fm және F ішінде бір мезгілде тұйық шар бар деп болжауға болады. Координатаның бастапқы мәндерін осы шардың ортасына әкелгенде, функцияның мәндері өзгермейді.

θ - S^2 сферасындағы еркін нүкте деп алайық. F_m және F беттері тиісінше мынадай теңдулермен F_m: $\rho=\rho_m(\theta)$,F: $\rho=\rho(\theta)$ беріледі. S^2 сферасындағы F_m-нің F-ке жинақталуыρ_m (θ) функциясының $\rho(\theta)$-ге біркелкі жинақталуын білдіреді.

H – S^2 сферасының еркін борель көпмүшелігі деп алайық. A және A_m борель көпмүшелігі арқылы дөңес гипербеттерінің F және F_m, H көпмүшелікті орталық жобалауы осы беттегі бастапқы координатасынан алынады. Енді k=0, 1, 2 болғанда төмендегідей формулаға келтіреміз:

$$\varpi_k(F, H) = \omega_k(F, A),\ \varpi_k(F_m, H) = \omega_k(F_m, A_m),$$

Көріп тұрғанымыздай, $\varpi_k(F,H)$ және $\varpi_k(F_m,H)$ – S^2 сферасында көпмүшеліктің толықтай аддитивтік теріс емес функциясы болып саналады. Көпмүшеліктің бұл функцияларын біз әрі қарай S^2 сферасына тасымалданған F және F_m, дөңес беттері үшін k ретіндегі интегралдық қисықтар деп айтамыз. Осы интегралдық қисықтардың әлсіз жинақталуы туралы мәселе қарастырылды. Борель ішкі көпмүшелігінің G аймағында берілген көпмүшелік функциясының толықтай аддитивті әлсіз жинақталуын анықтау туралы анықтамалар келтірілді.

$\mu_m(B)$ көпмүшелігінің толықтай аддитивті функциялар тізбегі $\mu(B)$ көпмүшелігінің толықтай аддитивті функциясымен әлсіз жинақталады деп айтылады, барлығына $f \in C(G)$ ықшамды тасымалдаушы $M \subset G$ болса, онда мынадай теңдікті ұсынады:

$$\lim_{m\to\infty} \iint_G f(x,y)\mu_m(dB) = \iint_G f(x,y)\mu(dB).$$

Келесі теореманы тұжырымдаймыз: $\Phi_m \in K(G)$ дөңес беті $\Phi \in K$ (G) дөңес бетімен жинақталатын болса, онда барлығына k=0,1,2 болғанда, E^2 -ге берілген Φ_m гипербетінің $\varpi_k(\Phi_m,B)$ интегралдық қисығы E^2-ге берілген $\varpi_k(\Phi,B)$ интегралдық қисығы арқылы әлсіз жинақталады. Дөңес беттердің интегралдық қисықтардың әлсіз жинақталуы дәлелденді.

Теорема түрінде алынған нәтиже интегралды шартты қисықтардың комбинациясы арқылы берілген дөңес гипербеттердің туралы теоремаларды дәлелдеуде мағызды рөл атқарады. Түрлі реттегі интегралды шартты қисықтардың берілген функциясы ретінде алғашқы рет оның жалпылама түрдегі шартты қисық сызықтары алынды. Ішкі интегралды функциялар үздіксіз функциялардың және түрлі ретті интегралдық қисықтардың туындысы болып саналады.

Түйін сөздер: дөңес бет, Евклид кеңістігіндегі дөңес беттер, Монж-Ампер теңдеуі, үздіксіз функциялар кеңістігіндегі дөңес беттердің конусы, шартты қисық, интегралдық қисық, бетті қалпына келтіру.

Э.А.Абылкасымова[1], Г. И. Бейсенова[1], Ү.П. Сүйінжанова[2]

[1]Региональный социально-инновационный университет, Шымкент, Казахстан;
[2]Южно-Казахстанский государственный педагогический университет, Шымкент, Казахстан

СЛАБАЯ СХОДИМОСТЬ ИНТЕГРАЛЬНОЙ КРИВИЗНЫ ВЫПУКЛЫХ ПОВЕРХНОСТЕЙ

Аннотация. Статья содержит концентрированный анализ существующей информации об основных проблемах теории выпуклых поверхностей и дифференциальной геометрии «в целом» и посвящена задачам восстановления выпуклых поверхностей по информации об их кривизне, изучаемым топологическими методами функционального анализа.

Рассматривается класс гладких поверхностей в ограниченной выпуклой области $G\subset E^2$. Излагается понятие R-площади нормального изображения. В классе$K^+(G)$ рассматривается уравнение Монжа-Ампера.

В работе рассматриваются интегралы поперечных мер Минковского, связанные с параллельными поверхностями. Если поверхность Φ задана явным уравнением z=f(x,y), то для интегральной кривизны этой поверхности, перенесенных на плоскость E^2, справедливы неравенства $\varpi_k(\Phi, G)\leq\omega_k(\partial Z_\varphi, \partial Z_\varphi)$ при k=0, 1, 2.

Из этих неравенств следуют неравенства

$$\varpi_0(\Phi, G)\leq 2\pi r^2+4\pi r\|\varphi\|_G,\ \varpi_1(\Phi, G)\leq 2\pi^2 r+4\pi\|\varphi\|_G,\ \varpi_2(\Phi, G)\leq 4\pi,$$

которые используются в рассуждениях.

Так как выпуклая поверхность F гомеоморфна сфере S^2и выпуклые поверхностиF_mсходятся к F, то не нарушая общности, можно считать, что существует замкнутый шар, содержащийся одновременно внутри всех F_m иF. Перенесем начало координат в центр этого шара, от этого значения функций не изменятся.

Пусть θ – произвольная точка сферы S^2. Получаем, что поверхности F_m иF задаются соответственно уравнениями F_m: $\rho=\rho_m(\theta)$,F: $\rho=\rho(\theta)$, сходимость F_mк F означает равномерную сходимость функций $\rho_m(\theta)$ к $\rho(\theta)$ на S^2.

Пусть H – произвольное борелевское множество из S^2. Обозначим через A и A_mборелевские множества выпуклых гиперповерхностей F иF_m, которые получаются в результате центрального проектирования множества H из начала координат на эти поверхности. Положим теперь

$$\varpi_k(F, H)= \omega_k(F, A),\ \varpi_k(F_m, H)= \omega_k(F_m, A_m),\ k=0, 1, 2.$$

Очевидно, $\varpi_k(F,H)$ и $\varpi_k(F_m,H)$ являются вполне аддитивными неотрицательными функциями множеств на сфере S^2. Эти функции множеств мы называем далее интегральными кривизнами порядка k для выпуклых поверхностей Fи F_m, перенесенных на сферу S^2.

Рассмотрен вопрос о слабой сходимости этой интегральной кривизны. Приведен известное из литературы определение слабой сходимости вполне аддитивных функций множеств, заданных на борелевских подмножествах области G.

Говорят, что последовательность вполне аддитивных функций множеств $\mu_m(B)$ слабо сходится к вполне аддитивной функции множества $\mu(B)$, если для всякой $f\in C(G)$ с компактным носителем $M\subset G$, имеет место равенство

$$\lim_{m\to\infty}\iint_G f(x,y)\mu_m(dB)=\iint_G f(x,y)\mu(dB).$$

Справедлива следующая теорема: пусть выпуклые поверхности $\Phi_m\in K(G)$ сходятся к выпуклой поверхности $\Phi\in K$ (G), тогда при всех k=0,1,2 интегральная кривизна $\varpi_k(\Phi_m,B)$ гиперповерхностей Φ_m, перенесенная на E^2, слабо сходится к интегральной кривизне $\varpi_k(\Phi,B)$, перенесенной на E^2.

Доказана слабая сходимость интегральной кривизны выпуклых поверхностей. Результат, полученный в виде теоремы, играет важную роль в доказательствах теорем существования выпуклой гиперповерхности с заданной комбинацией интегральной условной кривизны. Впервые условная кривизна взята в самом общем виде как заданная функция от интегральных условных кривизн различных порядков. Подынтегральные функции являются произведением непрерывных функций и интегральной кривизны различных порядков.

Ключевые слова: выпуклая поверхность, выпуклые поверхности в Евклидовом пространстве, уравнение Монжа-Ампера, конус выпуклых поверхностей в пространстве непрерывных функций, условная кривизна, интегральная кривизна, восстановление поверхности.

Information about authors:

Abylkasymova E.A., cand.ph.-math.sc., senior teacher of the chair “Math, physics and technique specialty”,Regional Social Innovation University, megaelmyra@mail.ru, https://orcid.org/0000-0002-4416-5638;

Beysenova G.I., cand.ped.sc., dean of the faculty of “Physics-Math and Business”, Regional Social Innovation University, gulia-74-74@mail.ru, https://orcid.org/0000-0003-0545-0652;

Suiinzhanova U.P., masterof Engineering of the chair of Physics, South Kazakhstan State Pedagogical University, only-kh@mail.ru, https://orcid.org/0000-0001-7267-2912

REFERENCES

[1] Hadviger G. Lectures on volume, surface area and isometry. Moscow, Science, 1966. 320 p.

[2] Buseman G. Convex surfaces. Moscow. Science, 1964. 320 p.

[3] Buseman G. Geodesic geometry. Moscow: Fismatgiz, 1962. 222 p.

[4] Bliev N.K., Myrzakul K., Rahimov F.K., Serikbaev N. S. On the geometry of Heisenberg ferromagnets. News of the National Academy of Sciences of the Republic of Kazakhstan, series physico-mathematical, 2002,No. 2, p. 10-15.

[5] Cantor B.E. Restoration of smooth surfaces from given curvature functions// Issues of differential geometry as “a whole”.Leningrad, 1983. p. 155-164.

[6] Federer H. Curvature measures. Trans. Amer. Math. Soc. 93, 1959, p. 418-491.

[7] Verner A.L. About external curvature of convex surfaces in spaces of constant curvature.Izv.vuzov, 1960, No. 1, seriiamatem., p. 58-68.

[8] Krasnoselsky M.A. Positive equations of operational solutions. Moscow,Fizmatgiz, 1962. 418p.

[9] Abylkasymova E.A., Koshanov M.,Koshanova G., Tasқaraev A.Monge-Ampere equation and the main challenge of differential geometry in general. Collection of abstracts of International scientific conference “Actual issues of partial differential equations” dedicated to 60th anniversary of professor Tungatarov A.B. Astana, 2008. p. 16-17.

[10] Pogorelov A.V. External geometry of convex surfaces. Moscow, Science, 1969. 540 p.

[11]TaskaraevA., Abylkasymova E.A. Surfaces with given conditional curvatures. Collection of papers of international scientific-practical conference dedicated to the 70th anniversary of the president of Kazakhstan university of friendship, Academician Kuatbekov A.M. Shymkent, 2012. p. 449-451.

[12] Kopzhasarova A.A., Besbayev E.A., Abylkasymova E.A., Shaldanbayev A. Sh.Spectral resolutions of solution of Volterra nonlocal boundary value problems of a wave equation. News of the National Academy of Sciences of the Republic of Kazakhstan.Series of physico-mathematical sciences.2016, Volume 5, Number 309, p.203-213.http://road.issn.org/issn/2518-1726, ISSN 1991-346X (print).

[13]BakelmanI.Ya.Geometric methods for solving elliptic equations. Moscow, Science, 1964.

[14] Abylkasymova E.A. The recovery of minimal and convex surfaces from the sum of integral conditional curvatures. Studies inintegro-differential equations, 2010, 42. p.168-173, Byshkek. ISSN0130-6553

[15]AsanovaA.T., AlikhanovaB.Zh., NazarovaK.Zh. Well-posedness of a nonlocal problem with integral conditions for third order system of the partial differential equation. News of the National Academy of Sciences of the RepublicofKazakhstan.Series of physic-mathematical sciences. 2018, Volume 5, No. 321, p. 33-41, DOI:10.32014/2018.2518-1726.5

[16]AldashevS.A., MaikotovM.N. Dirichlet problem in a cylindrical area for one class of multidimensional elliptic-parabolic equations .News of the National Academy of Sciences of the Republic of Kazakhstan. Series of physic-mathematical sciences. 2019, Volume 5, No.327. p.89–97. http://doi.org/10.32014/2019.2518-1726.62