NEWS
OF THENATIONAL ACADEMY OF SCIENCES OF THE REPUBLIC OF KAZAKHSTAN
PHYSICO-MATHEMATICAL SERIES
ISSN 1991-346X **https://doi.org/10.32014/2020.2518-1726.89**
Volume 5, Number 333 (2020), 106 – 112

UDK 524.4; 524.82; 524.83; 524.85
IRSTI 41.29.15; 41.29.17; 41.29.25; 41.29.33

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INFLATION IN *F*(*R*, *T*) GRAVITATION WITH *f*-ESSENCE

Abstract. Modified theories of gravity have become a kind of paradigm in modern physics because they seem to solve several shortcomings of the standard General Theory of Relativity (GTR) related to cosmology, astrophysics and quantum field theory. The most famous modified theories of gravity are *F*(*R*) and *F*(*T*) theories of gravity. A generalization of these two modified theories and gravitations, which was first proposed by Myrzakulov Ratbay. In this paper, we study an inhomogeneous isotropic cosmological model with a fermion field f-essence whose action has the form $S_{RT} = \int d^4x\sqrt{-g}[F(R,T) + L_m]$, where *R* is the scalar of curvature, and *T* is the torsion scalar, and L_m is the Lagrangian f-essence. A particular case $F(R,T) = \alpha R + \beta T$ is studied in detail when parameters are obtained that describe the current accelerated expansion of the Universe. The type of Lagrangian *f*-essence of this model is determined. The presented results show that gravity with *f*-essence can describe inflation in the early evolution of the Universe. A modified *F*(*R*, *T*) gravity with *f*-essence is considered. Equations of motion were obtained and the inflationary period of the early Universe was considered. To describe the inflationary period, the form of the Hubble parameter and the slow-roll parameter were determined.

Key words: Universe, accelerated expansion, inflation, cosmology, *f*-essence, slow-roll parameter, Friedmann-Robertson-Walker metric, *F*(*R*, *T*)-gravity.

Introduction. The idea of expanding Einstein's theory of gravity is productive and economical in terms of several attempts that try to solve problems by adding new and, in most cases, unjustified new components to give a self-consistent picture of dynamics. The accelerated expansion of the Hubble flow observed today and the missing material of astrophysical large-scale structures are primarily included in these considerations. Both problems can be solved by changing the gravity sector, i.e. field equations. Philosophy is an alternative for adding new cosmic fluids (new components in the field equations), which should lead to cluster structures (dark matter) or to accelerated dynamics (dark energy) due to exotic equations of state. In particular, weakening the hypothesis that the gravitational Lagrangian should be a linear function of the scalar of Ricci curvature R, as in the Hilbert-Einstein formulation, one can take into account the effective action as a minimum extension when the gravitational Lagrangian is a general function.

The main and well-known examples of such modified theories of gravity are $F(R)$ theory of gravity and $F(T)$ theory of gravity. In this direction, the generalization of the theories of gravity $F(R)$ and $F(T)$ promises to be a promising and interesting version of modified theories of gravity. One of these generalizations of the $F(R)$ and $F(T)$ theories was first presented in [1] (see also [2-7]). There was shown that the torsion scalar, as well as the curvature scalar without a source of matter, leads to the formation of this union, the expansion of the Universe, inflation, and so on. This generalization includes all the properties of the $F(R)$ and $F(T)$ theory of gravity.

Astronomical observations suggest the universe can be viewed as homogeneous and isotropic for cosmological purposes that at very large scales, and so this is well described by the Friedmann-Robertson-Walker (FRW) metric. In this paper, we investigate the dynamics of the fermionic field of the f -essence in the framework of $F(R,T)$ gravity described by the FRW metric. The special case of $F(R,T)=\alpha R+\beta T$, where α and β are real constants, is investigated in detail. The necessary cosmological parameters have been obtained.

The expansion in the early time of space-time in the evolution of the Universe, namely inflation, is introduced to solve some cosmological problems. The discovery of a gravitational wave from a heavy celestial body and the observation of a black hole (BH) is a great success of Einstein's general relativity. However, from a cosmological point of view, some problems remain, such as flatness, horizon, monopole problems.

The theory of inflation is one of the elegant and simple solutions for these problems. An alternative approach is to modify Einstein's gravity.

The standard method for determining observable inflation is to perform detailed disturbance analysis [8]. However, the approach can be simplified by introducing slow-roll parameters [9], either when inflation occurs due to modification of standard gravitational actions, or due to scalar fields.

F(R,T) gravity

The action has the following form [1] (see also [2-17]):

$$S_{RT}=\int d^4x\sqrt{-g}[F(R,T)+L_m], \tag{1}$$

where L_m is the matter Lagrangian, R is the curvature scalar, T is the torsion scalar

$$R=g^{\mu\nu}R_{\mu\nu},\, T=S_\rho{}^{\mu\nu}T^\rho{}_{\mu\nu}. \tag{2}$$

This generalization was first proposed in [1], and these properties were investigated in [2-17]. In general, $G^\lambda{}_{\mu\nu}$ has the form

$$G^\lambda{}_{\mu\nu}=e_i{}^\lambda\partial_\mu e^i{}_\nu+e_j{}^\lambda e^i{}_\nu\omega^j{}_{i\mu}=\Gamma^\lambda{}_{\mu\nu}+K^\lambda{}_{\mu\nu}, \tag{3}$$

where $\Gamma^\lambda{}_{\mu\nu}$ - Levi-Civita connection, and $K^\lambda{}_{\mu\nu}$ - contorsion tensor. We write the metric in the standard form as

$$ds^2=g_{ij}dx^idx^j. \tag{4}$$

Orthonormal tetrad components $e_i(x^\mu)$ are related to the metric tensor as follows

$$g_{\mu\nu}=\eta_{ij}e^i_\mu e^j_\nu. \tag{5}$$

The orthonormalized condition is given as

$$\eta_{ij}=g_{\mu\nu}e^\mu_i e^\nu_j. \tag{6}$$

Here's $\eta_{ij}=diag(-1,1,1,1)$ where we used the relation

$$e^i_\mu e^\mu_j=\delta^i_j. \tag{7}$$

Basics of f -essence

Briefly describe some of the basics fermion model f-essences indicated in [7]. The action of the model has the form [7]

$$S_f=\int d^4x\sqrt{-g}[R+2K(Y,\psi,\bar{\psi})], \tag{8}$$

where ψ and $\bar{\psi}=\psi^\dagger\gamma^0$ denote the fermionic field and its conjugation, respectively, the cross means complex conjugation and R is the Ricci scalar. K is the density of the Lagrangian of the fermionic field, the canonical kinetic term of which has the form

$$Y = \frac{1}{2} i[\bar{\psi}\Gamma^{\mu} D_{\mu}\psi - (D_{\mu}\bar{\psi})\Gamma^{\mu}\psi]. \tag{9}$$

Here $\Gamma^{\mu} = e^{\mu}_{a}\gamma^{a}$ is the generalized Dirac-Pauli matrices satisfying the Clifford algebra

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}, \tag{10}$$

where the brackets denote the anti-commutation relation. e^{μ}_{a} denotes a tetrad, while the covariant derivative is given by

$$D_{\mu}\psi = \partial_{\mu}\psi - \Omega_{\mu}\psi, \quad D_{\mu}\bar{\psi} = \partial_{\mu}\bar{\psi} + \bar{\psi}\Omega_{\mu}. \tag{11}$$

The fermionic coupling Ω_{μ} is determined by the formula

$$\Omega_{\mu} = -\frac{1}{4} g_{\rho\sigma}[\Gamma^{\rho}_{\mu\delta} - e^{\rho}_{b}\partial_{\mu}e^{b}_{\delta}]\Gamma^{\sigma}\Gamma^{\delta}, \tag{12}$$

where $\Gamma^{\rho}_{\mu\delta}$ Christoffel symbols.

Fermionic fields are considered here as classically commuting fields. By the classical fermionic field, we mean a set of four complex space-time functions that transform in accordance with the spinor representation of the Lorentz group. The existence of such fields have of decisive importance in our work, because, despite the fact that fermions are described by quantized fermion fields which that do not have a classical limit, we assume that such classical fields exist and use them as matter fields associated with gravity.

***F(R,T)* gravity with f -essence**

Now let's consider $F(R,T)$ gravity with f _essence. Its action is given as

$$S = \int d^4x\sqrt{-g}[F(R,T) + 2K(Y,\psi,\bar{\psi}], \tag{13}$$

where R and T are curvature and torsion scalars, respectively, expressed as

$$R = g^{\mu\nu}R_{\mu\nu}, T = S_{\rho}{}^{\mu\nu}T^{\rho}{}_{\mu\nu}. \tag{14}$$

FRW metric case

To simplify, this section will make the next two to simplify the problem, namely, we assume that:

1) F is of the form

$$F = \alpha R + \beta T, \tag{15}$$

where α and β are some positive constants.

2) consider only the FRW space-time

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2). \tag{16}$$

Then action (13) takes the form

$$S = \int dx^4\sqrt{-g}[\alpha R + \beta T + 2K(Y,\psi,\bar{\psi})]. \tag{17}$$

Next important suggestion is that for the FRW metric, R and T take the forms

$$\begin{aligned} R &= u + 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = u + 6\left(\dot{H} + 2H^2\right) = u + R_S, \\ T &= v - 6\frac{\dot{a}^2}{a^2} = v - 6H^2 = v - T_S, \end{aligned} \tag{18}$$

where u and v are some functions from $a, a, \frac{d^j a}{dt^j}, R, T$ and so on. Here

$$\begin{aligned} R_S &= 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = 6\left(\dot{H} + 2H^2\right), \\ T_S &= -6\frac{\dot{a}^2}{a^2} = -6H^2. \end{aligned} \tag{19}$$

Next step for our model can be rewritten as

$$S = \int dx^4 \sqrt{-g}[\alpha R_r + \beta T_w + \overline{L}_m], \qquad (20)$$

where

$$\overline{L}_m = \alpha u + \beta v + L_m = \alpha u + \beta v + 2K(Y,\psi,\overline{\psi}) = 2\overline{K}(Y,\psi,\overline{\psi}). \qquad (21)$$

For the FRW metric, the Dirac matrices of the curved space-time Γ^{μ} have the form

$$\Gamma^0 = \gamma^0,\ \Gamma^j = a^{-1}\gamma^j,\ \Gamma^5 = -i\sqrt{-g}\Gamma^0\Gamma^1\Gamma^2\Gamma^3 = \gamma^5,\ \Gamma_0 = \gamma^0,\ \Gamma_j = a\gamma^j (i = 1,2,3). \qquad (22)$$

Hence we get

$$\Omega_0 = 0,\ \Omega_j = \frac{1}{2}\dot{a}\gamma^j\gamma^0 \qquad (23)$$

and

$$Y = \frac{1}{2}i\left(\overline{\psi}\gamma^0\dot{\psi} - \dot{\overline{\psi}}\gamma^0\psi\right). \qquad (24)$$

Finally, note that the gamma matrices that we write in the Dirac basis are of the form

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},\quad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix},\quad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad (25)$$

where $I = diag(1,1)$ and σ^k are Pauli matrices having the following forms

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \qquad (26)$$

We rewrite action (20) in the following form

$$S = \int dt \sqrt{-g} L, \qquad (27)$$

where L is the point-like Lagrangian. To find the equations of motion of the model, we use the Euler-Lagrange equations. In our case, the Euler-Lagrange equations are defined as follows

$$\frac{\partial L}{\partial a} - \frac{d}{dt}\frac{\partial L}{\partial \dot{a}} = 0,\quad \frac{\partial L}{\partial R} - \frac{d}{dt}\frac{\partial L}{\partial \dot{R}} = 0,\quad \frac{\partial L}{\partial T} - \frac{d}{dt}\frac{\partial L}{\partial \dot{T}} = 0,\quad \frac{\partial L}{\partial \psi} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\psi}} = 0,\quad \frac{\partial L}{\partial \overline{\psi}} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\overline{\psi}}} = 0. \qquad (28)$$

Accordingly, the condition is given as

$$E_L = \frac{\partial L}{\partial \dot{a}}\dot{a} + \frac{\partial L}{\partial \dot{R}}\dot{R} + \frac{\partial L}{\partial \dot{T}}\dot{T} + \frac{\partial L}{\partial \dot{\psi}}\dot{\psi} + \frac{\partial L}{\partial \dot{\overline{\psi}}}\dot{\overline{\psi}} - L = 0. \qquad (29)$$

Finally, the equations of motion for our model take the form

$$\begin{aligned} &3H^2 - \overline{\rho} = 0, \\ &2\dot{H} + 3H^2 + \overline{p} = 0, \\ &\overline{K}_Y\dot{\psi} + 0.5(3H\overline{K}_Y + \dot{\overline{K}}_Y)\psi - i\gamma^0\overline{K}_{\overline{\psi}} = 0, \\ &\overline{K}_Y\dot{\overline{\psi}} + 0.5(3H\overline{K}_Y + \dot{\overline{K}}_Y)\overline{\psi} + i\overline{K}_{\psi}\gamma^0 = 0, \end{aligned} \qquad (30)$$

where

$$\overline{\rho} = Y\overline{K}_Y - \overline{K},\qquad \overline{p} = \overline{K}. \qquad (31)$$

where $\dot{H} = \dot{a}/a$ is the Hubble parameter, $a(t)$ is the scale factor, and $\dot{a}(t)$ is cosmic time derivative t.

From the second equation of system (30) can be seen that, according to the rules of differentiation, if expressions from the spirit of the sides of equality depend on different arguments, then this equality will be constant

$$2\dot{H}(a) + 3H(a)^2 = -\overline{K}(Y,\psi,\overline{\psi}) = const.$$

To study inflation is enough to determine the form of the Hubble parameter, which has the form:

$$H = \frac{1}{6}\frac{\left(\left(12\sqrt{3}\sqrt{\frac{4C_1^3+27e^{2t}C_2}{C_2}}+108e^t\right)C_2^2\right)^{1/3}}{C_2} - \frac{2C_1}{\left(\left(12\sqrt{3}\sqrt{\frac{4C_1^3+27e^{2t}C_2}{C_2}}+108e^t\right)C_2^2\right)^{1/3}}. \quad (32)$$

Inflation. Let us consider an inflationary model with a minimum kinetic term, where the behavior of the system is described by the FRW and Dirac equations (30). To describe the evolution of the background is convenient to introduce the functions of the Hubble flow ε_n, which are defined by the formula

$$\varepsilon_{n+1} \equiv -\frac{d\ln|\varepsilon_n|}{dN}, \quad n \geq 0, \quad (33)$$

where $\varepsilon_0 \equiv H_{ini}/H$ and $N \equiv \ln(a_{ini}/a)$ are the amount of e-folding. By definition, inflation is a phase of accelerated expansion $\ddot{a}/a > 0$ or, equivalently $\varepsilon_1 < 1$. As a consequence, the end of inflation is determined by the condition $\varepsilon_1 = 1$. On the other hand, the conditions of slow-rolling down (or the slow-rolling approximation) refer to the situation when all ε_n satisfy $\varepsilon_n << 1$. For our case, the inflation solution evolves with the (positive) Hubble flow functions

$$\varepsilon_1 = -\frac{\dot{H}}{H^2}, \quad (34)$$

$$\varepsilon_2 = -\frac{2\dot{H}}{H^2} + \frac{\ddot{H}}{H\dot{H}}, \quad (35)$$

where C_1 and C_2 are integration constants. Figure 1 shows the typical behavior of the slow-roll parameter for different C_1 and C_2 values.

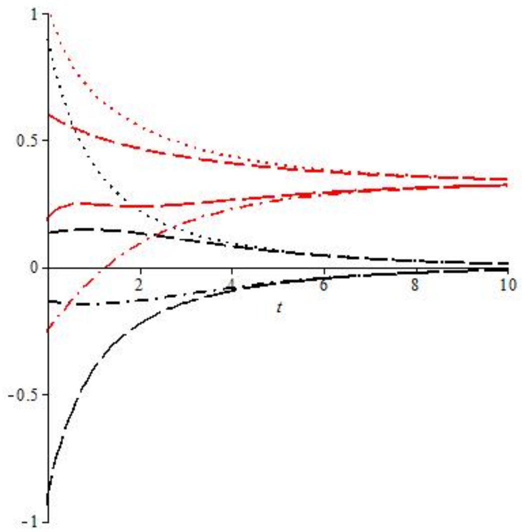

Figure 1 - Graphical analysis of ε_1 (black lines) and ε_2 (red lines). Dotted line: $C_1 > 0$ and $C_2 > 0$; dotted line: $C_1 < 0$ and $C_2 > 0$; long dotted line: $C_1 < 0$ and $C_2 < 0$; dot-dotted line: $C_1 > 0$ and $C_2 < 0$

Conclusion. In this work we have considered a modified $F(R,T)$ gravity with *f*-essence. Equations of motion were obtained and the inflationary period of the early Universe was considered. To describe the inflationary period, we determined the form of the Hubble parameter and the slow-roll parameter. As can be seen from the graph for our model, the slow-roll parameter does not satisfy the inflation conditions given in [9]. Thus, the given $F(R,T)$ gravity model with *f*-essence cannot describe viable inflation.

Acknowledgement. This work was supported by the Ministry of Education and Science of RK Grant No. AP08052197.

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f-ЭССЕНЦИЯСЫ БАР *F*(*R*,*T*) ГРАВИТАЦИЯСЫНДАҒЫ ИНФЛЯЦИЯ

Аннотация. Жаңартылған гравитациялық теориялары қазіргі физикада өзгеше парадигманың түріне айналды, өйткені олар космология, астрофизика және кванттық өріс теориясымен байланысты стандартты жалпы салыстырмалылық теориясының бірнеше кемшіліктерін шешкен болуы мүмкін. Гравитациялықтың ең танымал модификацияланған теориялары – $F(R)$ және $F(T)$. Осы екі теорияның жалпыланған түрін алғаш рет Мырзақұлов Ратбай ұсынған. Бұл жұмыста біз *f*-эссенция фермиондық өрісі бар біртекті емес изотропты космологиялық модельді зерттейміз, әсерін былай көрсетеміз: $S_{RT}=\int d^4x\sqrt{-g}[F(R,T)+L_m]$, мұнда R – қисық скаляры және T – ширату скаляры, L_m – *f*-эссенция лагранжианы. Әлемнің үдетілген кеңею жағдайын сипаттайтын параметрлер алынған. Нақты жағдай $F(R,T)=\alpha R+\beta T$ қарастырылып, толықтай зерттеледі. Бұл модельдің *f*-эссенция лагранжианың түрі анықталған. Ұсынылған нәтижелер *f*-эссенциясы бар $F(R,T)$ гравитациясы әлемнің ерте инфляциялық эволюциясын сипаттайтындығын көрсетеді.

Жұмыста фермиондық өрісі бар және канондық емес кинетикалық мүшесі (*f*-эссенция) бар космологиялық модель қарастырылған. Модельдің кейбір нақты шешімдері табылған және біз осындай гравитациялық-фермиондық өзара әрекеттесудің әлемнің бақыланатын үдемелі ұлғаю әсерін тексереміз. Әрі қарай, осы модельдердің космологиялық қосымшалары талқыланады. Сондай-ақ, *f*-эссенция моделінде Эйнштейннің статикалық әлемі секілді шешім жоқ. *f*-эссенциясы бар модификацияланған $F(R,T)$ гравитациясы қарастырылған. Қозғалыс теңдеулері алынды және алғашқы Әлемнің инфляциялық кезеңі қарастырылған. Инфляциялық кезеңді сипаттау үшін Хаббл параметрінің формасы және баяу оралу параметрі анықталған.

Түйін сөздер: әлем, үдемелі ұлғаю, инфляция, космология, *f*-эссенция, баяу сырғу параметрі, Фридман-Робертсон-Уокер метрикасы, $F(R,T)$-гравитациясы.

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ИНФЛЯЦИЯ В *F*(*R*,*T*) ГРАВИТАЦИИ С *f*-ЭССЕНЦИЕЙ

Аннотация. Модифицированные теории гравитации стали своего рода парадигмой в современной физике, поскольку они, кажется, решают несколько недостатков стандартной Общей теории относительности (ОТО), связанной с космологией, астрофизикой и квантовой теорией поля. Наиболее известные модифицированные теории гравитации являются $F(R)$ и $F(T)$ теории гравитации. Обобщение этих двух модифицированных теорий $F(R)$ и $F(T)$ гравитации впервые было предложено Мырзакуловым Ратбаем. В данной работе исследована неоднородная изотропная космологическая модель с фемионным полем *f*-эссенцией, действие которой имеет вид $S_{RT}=\int d^4x\sqrt{-g}[F(R,T)+L_m]$, где R – скаляр кривизны, а T – скаляр кручения, L_m - лагранжиан *f*-эссенции. Детально изучается частный случай при $F(R,T)=\alpha R+\beta T$, получены параметры, описывающие текущее ускоренное расширение Вселенной. Определён вид лагранжиана *f*-эссенции данной модели. Представленные результаты показывают, что $F(R,T)$ гравитация с *f*-эссенцией может описывать инфляцию в ранней эволюции Вселенной.

В этой работе рассмотрена космологическая модель с фермионным полем и с неканоническим кинетическим членом (*f*-эссенция). Найдены некоторые точные решения модели и проверяем влияние такого гравитационно-фермионного взаимодействия на наблюдаемое ускоренное расширение Вселенной. Далее, обсуждаются космологические применения точных моделей. Также модель *f*-эссенции не имеет решения типа статической Вселенной Эйнштейна. Рассмотрено модифицированная $F(R,T)$ гравитация с *f*-эссенцией. Было получены уравнения движения и рассмотрен инфляционный период ранней Вселенной. Для описания инфляционного периода, были определены вид параметра Хаббла и параметр медленного скатывания.

Ключевые слова: Вселенная, ускоренное расширение, инфляция, космология, *f*-эссенция, параметр медленного скатывания, метрика Фридмана-Робертсона-Уокера, $F(R,T)$ - гравитация.

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