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әл-Фараби атындағы Қазақ ұлттық университетінің

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## ИЗВЕСТИЯ

НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК  
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## **CHOOSING A COMPRESSION STANDARD FOR TRANSMITTING A TELEVISION IMAGE**

**Abstract.** With the advent of the MPEG-4 Visual and H.264 compression standards, the role of the MPEG-2 compression standard has not diminished at the present time, since these standards are largely compatible, especially for information highways. Today, in a number of countries, the MPEG-2 compression standard is actually the main one for broadcasting, the functioning of which is based on the operation of DVB-T terrestrial digital television systems.

Due to the fact that the majority of the currently used STBs support the MPEG-2 standard, it will remain widespread for at least the next few years.

The ITU-T H.264 /MPEG-4 Part 10 AVC standard (abbreviated as H.264/AVC) is a promising new technology for encoding and compressing audiovisual information. The compression efficiency of the H.264/AVC standard is higher than the MPEG-2 standard with equal visual perception. The H.264/AVC standard was developed independent of the transport layer of the transmission systems used. Therefore, information delivery in the H.264/AVC standard can be carried out using any existing transmission and broadcasting systems, including: systems with IP protocols (including streaming delivery), transport streams of broadcasting systems in the MPEG-2 standard, as well as specific formats H.264 / AVC files for storage and processing on servers.

**Keywords:** standard, digital TV broadcasting, codec, signal, IP protocol.

The ITU-T H.264/MPEG-4 Part 10 AVC standard (abbreviated as H.264/AVC) is a promising new technology for encoding and compressing audiovisual information. The compression efficiency of the H.264/AVC standard is higher than the MPEG-2 standard with equal visual perception. The H.264/AVC standard was developed independent of the transport layer of the transmission systems used. Therefore, the delivery of information in the H.264/AVC standard can be carried out on any existing systems transmission and broadcasting, including: systems with IP protocols (including streaming delivery), transport streams of broadcasting systems in the MPEG-2 standard, as well as specific H.264/AVC file formats for their storage and processing on servers.

Thus, existing MPEG-2 digital TV broadcasting networks are directly suitable for transmitting to users H.264/AVC programs, the number of which in the multiplex will be greater.

To switch to broadcasting in the H.264/AVC standard, international standardization of terminal equipment is required: digital studios and subscriber receiver decoders. For transmission paths of transport streams, specific support of the H.264/AVC standard by statistical multiplexers, in which operations of decoding and compressing program information are performed, will be required. Advantages of the H.264/AVC standard. With the same image quality, the H.264/AVC standard requires approximately half the digital bit rate than MPEG-2. This allows you to either increase the number of standard definition television (SDTV) programs or transmit HDTV programs.

Unlike MPEG-2, the H.264/AVC standard allows you to transfer images with a lower resolution (1/16 standard resolution) at very low digital bit rates. This allows you to organize the transfer of low-definition TV programs to mobile phones, PDAs, TV receivers in cars, etc. The capabilities of the H.264/AVC standard for compressing images of various quality are shown in table 1.1.

Table 1.1 - H.264 / AVC Image Compression Capabilities

Application scenario	Resolution and frame rate	Transmission speed, kbps
Mobile content	176x144, 10-15 frame/s	50-60
Internet / Standard Definition	640x480, 24 frame /s	1000-2000
High definition	1280x720, 24p	5000-6000
Full high definition	1920x1080, 24p	7000-8000

In a number of European countries, a large number of transmission networks based on MPEG2 technology are currently deployed and operate. In such publicly accessible digital TV broadcasting networks with the set-top boxes and MPEG-2 decoders already installed, the transition to the H.264/AVC standard will be difficult for many years, while the existing MPEG-2 set-top boxes are actively used.

In addition, this will also impede the implementation of HDTV in the H.264/AVC standard. Possible alternatives: reloading new H.264/AVC decoding software, installing additional modules (decoding or converting standards) in the set-top boxes. The mildest alternative to replacing the MPEG-2 set-top boxes is the duplication of digital TV broadcasting in the MPEG-2 and H.264/AVC standards, but this is only possible if there are free radio frequencies released as the analog broadcast is turned off.

In paid digital TV broadcast networks, the situation is very different. In this case, broadcasters can coordinate the updating of decoders of their subscribers. But for financial reasons, this transfer can only begin after depreciation of the initial paid equipment, i.e. a few years after putting it into operation.

In all cases, users should be given full clarity regarding the prospects of the development of digital broadcasting and the possibilities of using the equipment.

Therefore, in order to minimize the cost of viewers during the first few years, without slowing down the implementation of HDTV, the digital set-top boxes offered to the population should have the options necessary to receive both current and future services, i.e., first of all, they must support the H.264/AVC standard, for standard definition television (HDTV) and high definition (HDTV).

The use of H.264 / AVC encoders in window devices currently requires the use of digital TV program transmission systems (SDTV and HDTV) in the ADSL standard with a bandwidth of 4 ... 6 Mbit / s.

It is believed that modern ADSL modems-decoders should support decoding in MPEG-2 and H.264/AVC standards using the same software, which will eliminate the complexity of the transition from MPEG-2 to H.264/AVC.

It should be borne in mind that when deploying mobile television networks of the DVB-H standard, all ongoing development and experiments are based on the H.264/AVC standard, as a more effective one.

Principles of building digital terrestrial television broadcasting systems in the H.264/MPEG-4 AVC standard.

Currently, there is an increased interest in encoding and digital broadcasting of TV programs in the H.264/MPEG-4 AVC standard in the world.

It is known that in the world there are also a number of proposals and experiments conducted on the use of the H.264/MPEG-4 standard instead of the MPEG-2 standard for encoding standard definition TV images.

In this regard, digital TV broadcasting systems that use the H.264/MPEG-4 AVC compression algorithm have certain development prospects. In such systems, the transmission of high-definition TV programs (HDTV) and an increased number of standard-definition programs is possible, which allows the audience to provide new services and make better use of the frequency resource. When switching to digital TV broadcasting, it is necessary to take into account all aspects of digital broadcasting of TV programs in the H.264/MPEG-4 AVC standard as an alternative to the MPEG-2 standard.

The transition to digital TV broadcasting is preferably carried out at the level of individual regions as they become available. You can, as an example, propose a distribution and broadcasting scheme for digital TV programs for a single region with a family number of about 300-350 thousand. The general idea is that the central program block is distributed via satellite channels in MPEG-2 format (which is most often the case at present).

But in the regional center these programs are converted to MPEG-4 format and multiplexed with other regional programs (both high and standard definition), originally encoded in MPEG-4 format. In principle, depending on the available resources, several digital multiplexes in MPEG-4 may be formed in the region, including high-definition and standard-definition programs. Then these multiplexes should be

distributed to the population on digital terrestrial and cable TV broadcasting networks. The transmitters of the regional single-frequency network of digital TV broadcasting can be downloaded via satellite channels (as shown in the diagram) or via fiber optic cables.

At the same time, it should be noted that in order to improve the quality of the final image received by the consumer, it is advisable to avoid re-compression of previously compressed digital signals, or, if possible, reduce the number of nodes in the distribution network of programs in which such compression occurs.

Under these conditions, it is advisable, given the technical ability, to build the entire program delivery network based on H.264/MPEG-4 AVC. In the absence of such a possibility, an alternative may be the use of optimized encoders that re-compress MPEG2-MPEG4 with minimal loss of image quality [1].

When using any compression methods for TV programs, quality is an important issue. Both MPEG2 and H.264/MPEG-4 are lossy compression technologies. This means that once lost due to compression, the image details cannot be restored. Modern compression technologies make it possible to achieve very high rates of decreasing the speed of the digital stream, but at the same time you can get an image that is significantly inferior in quality to the image of an analog TV under good reception conditions, especially for dynamic scenes.

Therefore, it is important to standardize the minimum bit rate for transmitting a standard quality program (SDTV) in both MPEG2 and MPEG4 standards. Practical experiments with the currently available encoder models show that to ensure high-quality broadcasting of one TV program in SDTV in the H.264/MPEG-4 standard, at least 2.5 Mbit/s is required, and in the MPEG2 standard - 4.5 Mbit/s. Using statistical multiplexing with a variable signal coding rate (VBR) allows this value to be slightly reduced without loss of quality, but only with careful equipment setup [2].

Thus, the introduction of H.264/MPEG-4 compression technology will increase the number of transmitted programs within a single multiplex (with a fixed reception of 64 QAM, 24 Mbps) from 4-5 to 9-10 programs of standard quality. Currently, in most cities the number of received analog TV programs does not exceed 10.

Thus, the use of modern equipment and technologies based on MPEG4 will make it possible to duplicate all programs accepted by the population using one multiplex, which will not only save the frequency resource, but also open the way to a two-step transition model from analog to digital broadcasting. At the same time, the first step within the region ensures the broadcasting of all TV programs in digital format, and the second step (after the distribution of digital set-top boxes among the population).

Switching to HDTV is facilitated if the receivers support all H.264/MPEG-4 profiles up to 1920×1080, 24/25p. In order to ensure the transition to HDTV, it is advisable to distribute only TV set-top boxes in the regions that allow receiving HDTV in the MPEG4 standard. In this case, it will be possible to transmit up to 3 HDTV programs in one multiplexed channel, and with the development of compression technologies, possibly more.

Recommendations for using the MPEG-4 compression standard may be as follows:

- a) the use of a more advanced compression standard H.264/MPEG-4 can significantly increase the efficiency of use of the radio frequency resource and facilitate the transition to digital broadcasting;
- b) when using equipment that supports H.264/MPEG-4, a painless transition from standard definition television to high-definition television can be made without the need for additional costs from the user;
- c) the transition to the technology of effective video compression is an objective necessity and is inevitable in the future, even if it is postponed at present. It is advisable for RCC countries that do not have a large fleet of MPEG2 receiving equipment use MPEG4 when introducing NCTV.

From the above we can see the superiority of the digital standard DVB-T over ATSC. Due to the fact that each of the sub-channels is narrow-band in COFDM modulation, the influence of reflected signals during multi-path reception is reduced. Which can not be said about the modulation of 8VSB standard ATSC [3. 4].

Table 1.2 shows the main technological capabilities of the two compression standards described above.

Table 1.2 - Comparison of MPEG2 and MPEG4 Standards

Parameters	MPEG-2	MPEG-4
Mode of operation	8K 64 QAM 5/6 1/8 27,65 Mbp/s	8K 64 QAM 5/6 1/8 27,65 Mbp/s
Mode of transmission	without statistical multiplexing	without statistical multiplexing
Information speed on 1 TV channel	5 Mbp/s	2 Mbp/s
Number of channels in one package	5	13
Number of channels in three packages	15	39
Mode of transmission	with statistical multiplexing	with statistical multiplexing
Information speed on 1 TV channel	3,5 Mbp/s	1,8 Mbp/s
Number of channels in one package	7	15
Number of channels in three packages	21	45

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### **ТЕЛЕВИЗИЯЛЫҚ КЕСКІН БЕРУ ҮШІН СЫҒЫМДАУ СТАНДАРТЫН ТАҢДАУ**

**Аннотация.** MPEG-4 Visual және H.264 сығымдау стандарттары пайда болғандықтан, қазіргі уақытта MPEG-2 сығымдау стандартының рөлі төмендей қойған жоқ, өйткені бұл стандарттар, әсіресе, ақпараттық магистральдармен үйлесе түседі. Бүгінде бірқатар елдерде MPV-2 сығымдау стандарты эфирлік сандық теледидар жүйесінің DVB-T жұмысына негізделген, хабар таратуда басты орын алады.

Қазіргі уақытта қолданыстағы абоненттік тіркемелердің көпшілігі (STB) көпшілігі MPEG-2 стандартын қолдайтындықтан, бірнеше жыл бойы кең таралып келді.

ITU-T H.264/MPEG-4 AVC 10-бөлігі (H.264/AVC қысқа) – аудиовизуалды ақпаратты кодтау мен сығымдаудың жаңа технологиясы. H.264/AVC стандартының сығылу тиімділігі бірдей көру қабілетке ие MPEG-2 стандартынан жоғары. H.264/AVC стандарты тарату жүйелерінің көлік қабатынан бөлек жасалған. Сондықтан H.264/AVC стандартындағы ақпаратты жеткізу кез-келген қолданыстағы тарату жүйелерін, соның ішінде, IP протоколы бар жүйелерді (ағын беруді қоса), MPEG-2 стандартындағы хабар тарату жүйелерінің көлік ағынын, сондай-ақ нақты форматтарды пайдалану арқылы жүзеге асырылуы мүмкін. H.264 / AVC файлдары серверде сақтауға және өңдеуге арналған.

Осылайша қолданыстағы MPEG-2 сандық телехабар тарату желілері H.264/AVC бағдарламаларын пайдаланушыларға тікелей таратуға жарамды әрі саны мультиплекста көбірек болады. H.264/AVC стандартында хабар таратуға ауысу үшін сандық студиялар мен абоненттік қабылдағыш декодері сынды терминалды жабдықты халықаралық стандарттау қажет. Көлік ағынының өткізу жолдарына бағдарламалық ақпараттарды декодтау және сығымдау операциялары орындалатын статистикалық мультиплексорларлы H.264/AVC стандартына қолдау көрсету керек. H.264/AVC стандартының артықшылығы, сурет сапасы бірдей болса, H.264/AVC стандарты MPEG-2-ге қарағанда сандық бит жылдамдығының жартысын қажет етеді. Бұл стандартты теледидар бағдарламаларының (SDTV) санын көбейтуге немесе HDTV бағдарламаларын таратуға мүмкіндік береді.

**Түйін сөздер:** стандартты, теледидарлық сандық хабар тарату, кодек, сигнал, IP протокол.

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### **ВЫБОР СТАНДАРТА КОМПРЕССИИ ПРИ ПЕРЕДАЧЕ ТЕЛЕВИЗИОННОГО ИЗОБРАЖЕНИЯ**

**Аннотация.** С появлением стандартов сжатия MPEG-4 Visual и H.264 роль стандарта компрессии MPEG-2 в настоящее время не уменьшилась, так как данные стандарты во многом совместимы, особенно это относится к информационным магистралям. На сегодня в ряде стран стандарт сжатия MPEG-2 является для вещания фактически основным, на функционировании которого основаны работа наземных систем цифрового телевидения DVB-T.



Вследствие того, что большинство используемых ныне абонентских приставок (STB) поддерживают стандарт MPEG-2, он, по крайней мере, в течение нескольких последующих лет останется широко распространенным.

Стандарт ITU-T H.264/MPEG-4 Part 10 AVC (сокращенно – H.264/AVC) является новой перспективной технологией кодирования и сжатия аудиовизуальной информации. Эффективность сжатия стандарта H.264/AVC выше, чем стандарта MPEG-2 при равном визуальном восприятии. Стандарт H.264/AVC был разработан как независимый от транспортного уровня используемых систем передачи. Поэтому доставка информации в стандарте H.264/AVC может осуществляться по любым действующим системам передачи и вещания, включая системы с IP-протоколами (в том числе для потоковой доставки), транспортные потоки систем вещания в стандарте MPEG-2, а также специфические форматы файлов H.264/AVC для их хранения и обработки на серверах.

Таким образом, существующие сети цифрового ТВ-вещания стандарта MPEG-2 напрямую подходят для передачи пользователям программ в стандарте H.264/AVC, число которых в мультиплексе будет больше.

Для перехода на вещание в стандарте H.264/AVC требуется стандартизация на международном уровне окончательного оборудования: цифровых студий и декодеров абонентских приемников. Для трактов передачи транспортных потоков потребуется специфическая поддержка стандарта H.264/AVC статистическими мультиплексорами, в которых производятся операции декодирования и сжатия информации программ. Преимущества стандарта H.264/AVC. При одинаковом качестве изображения стандарт H.264/AVC требует примерно вдвое меньшей скорости цифрового потока, чем MPEG-2. Это позволяет либо увеличить число ТВ программ стандартной четкости (SDTV), либо осуществлять передачу программ ТВЧ (HDTV).

**Ключевые слова:** стандарт, цифровое ТВ-вещание, кодек, сигнал, IP-протокол.

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**WAVE PROCESSES IN A FLAT BODY WITH  
A SYMMETRIC LOCATED RECTANGULAR CUTOUT**

**Abstract.** The work is devoted to the generalization of the difference method of spatial characteristics to the case of the plane problem of the propagation of waves in a rectangular region of finite dimensions with a symmetrically-located rectangular cutout at the lateral boundaries. Based on the numerical technique developed in this work, the calculated finite - difference relations of dynamic problems are obtained at the corner points of a rectangular cutout, where the smoothness of functions that is “familiar” to dynamic problems is violated. At these corner points, the first and second derivatives of the desired functions suffer a discontinuity of the first kind. The results of the study are brought to a numerical solution. The effect of stress concentration in the vicinity of the cutout was studied and it was shown that the impact of the cutout on the particle velocity distribution, on the stress distribution has a local character.

**Keywords:** elastic, velocity, stress, load, plane deformation, wave process, stress concentration, numerical solution.

**Introduction.** Prediction of dynamic wave processes in solids of finite dimensions, taking into account a number of attenuating factors (discontinuities in the boundary conditions, holes, cavities, cutouts, etc.) by mathematical modeling to determine the nature of possible damage, is, besides purely scientific interest, an important applied value determined by requests for engineering practice.

The majority of structures used in construction, engineering and other branches of technology are characterized by the presence of various discontinuities in the form of holes, recesses, notches, protrusions, etc., due to either manufacturing technology or operational requirements. Near such discontinuities, as is known, the phenomenon of local distortion of stresses and strains arises, usually called stress concentration. These effects make a significant change in the “average” stresses and are often the cause of failure. The qualitative and quantitative effects of stress concentration are influenced by a variety of reasons related to the geometry of the concentrator, the type of impact, and the real properties of the material. Therefore, the problem of stress concentrations is given great attention in modern technology, which is reflected in the almost infinite number of domestic and foreign studies, as well as in the special monographs and the reference manuals. In general, the number of works devoted to the dynamic problems, taking into account a number of weakening factors, is very small; far from all aspects of their working capacity under conditions of unsteady external loads are considered in them [1-19]. However, interest in these problems, due primarily to the importance of solving complex practical problems, is great, and further improvement of numerical methods in various modifications using increasingly sophisticated electronic computing equipment should lead to a significant development of this direction.

**The mathematical formulation of the problem.** Let's consider the basic equations of the problem of the dynamic theory of elasticity for a homogeneous isotropic elastic body in a Cartesian coordinate system. For convenience, the axes of the Cartesian coordinate system are denoted by  $Ox_i$  ( $i=1,2$ ), and the time by  $t$ . The components of the stress tensor are denoted by  $\sigma_{ij}$  ( $x_1, x_2, t$ ), the strain tensor by  $\varepsilon_{ij}$  ( $x_1, x_2, t$ ) ( $i, j = 1,2$ ) and, finally, the displacement vectors by  $u_i$  ( $x_1, x_2, t$ ) ( $i = 1,2$ ).

It is convenient to represent the equations of motion and Hooke's law in the form of a normal hyperbolic system:

$$\sigma_{ij,j} = \rho \partial^2 u_i / \partial t^2 \quad (i = 1, 2), \tag{1}$$

$$\sigma_{ij} = \lambda \theta \delta_{ij} + 2 \mu \varepsilon_{ij}, \tag{2}$$

where  $\rho$  is the density of the medium material,  $\lambda, \mu$  are the Lamé constants,  $\delta_{ij}$  is the Kronecker symbol,  $\theta$  is the volumetric strain equal to

$$\theta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}.$$

A comma with an index means the partial derivative with respect to the corresponding argument. The summation is performed over  $j$  ( $j = 1, 2$ ).

The transition to dimensionless variables is carried out according to the formulas [7]:

$$\begin{aligned} \bar{t} &= \frac{tc_1}{b}; & \bar{x}_i &= \frac{x_i}{b}; & v_i &= \frac{1}{c_1} \frac{\partial u_i}{\partial t}, \quad (i=1,2) \\ p &= \frac{\sigma_{11} + \sigma_{22}}{2\rho c_1^2}; & q &= \frac{\sigma_{11} - \sigma_{22}}{2\rho c_1^2}; \\ \tau &= \frac{\sigma_{12}}{\rho c_1^2}; & \gamma &= \frac{c_1}{c_2}, \end{aligned} \tag{3}$$

where  $b$  is the characteristic length,  $c_1, c_2$  are the velocities of the expansion and shear waves,  $\sigma_{11}, \sigma_{22}, \sigma_{12}$  are the components of the stress tensor, and  $\gamma$  is a constant parameter. In the future, the line over dimensionless parameters is omitted. Differentiating dependences (2) with respect to time, adding and subtracting the normal stresses  $\sigma_{11}$  and  $\sigma_{22}$ , the equations of motion (1) and physical relations (2) are written as a system of equations for the velocities of displacements  $v_1, v_2$  and three linear combinations  $p, q, \tau$  of the stress tensor components [7]:

$$\begin{aligned} v_1, t - p, 1 - q, 1 - \tau, 2 &= 0; & v_2, t - p, 2 + q, 2 - \tau, 1 &= 0; \\ \gamma^2 (\gamma^2 - 1) - 1 p, t - v_1, 1 - v_2, 2 &= 0; \\ \gamma^2 q, t - v_1, 1 + v_2, 2 &= 0; \\ \gamma^2 \tau, t - v_1, 2 - v_2, 1 &= 0. \end{aligned} \tag{4}$$

A comma with an index means the partial derivative with respect to the corresponding argument. This statement was chosen because such systems have been studied quite well by now, and in addition, when choosing velocities of displacements and stresses as dependent variables, the derivatives from the boundary conditions are excluded.

Find the functions  $v_1, v_2, p, q, \tau$ , that in a rectangular strip  $0 \leq x_1 \leq l, -L \leq x_2 \leq L$  with a symmetrically located rectangular cutout on the lateral boundaries (Fig.1) satisfy the equations (4), the initial conditions for  $t = 0$

$$v_1(x_1; x_2; 0) = v_2(x_1; x_2; 0) = p(x_1; x_2; 0) = q(x_1; x_2; 0) = \tau(x_1; x_2; 0) = 0 \tag{5}$$

and the following boundary conditions for  $t \geq 0$ :

$$v_1 = f(t), \quad v_2 = 0 \quad \text{at } x_1 = 0, \quad -L \leq x_2 \leq L, \tag{6}$$

$$p - q = 0, \quad \tau = 0 \quad \text{at } |x_2| = L, \quad 0 \leq x_1 \leq x_1^0 \quad \text{and} \quad x_1^1 \leq x_1 \leq l, \tag{7}$$

$$p + q = 0, \quad \tau = 0 \quad \text{at } x_1 = x_1^0, \quad x_2^1 \leq |x_2| \leq L \quad \text{and} \quad \text{at } x_1 = x_1^1, \quad x_2^1 \leq |x_2| \leq L, \tag{8}$$

$$p - q = 0, \quad \tau = 0 \quad \text{at } |x_2| = x_2^1, \quad x_0^1 \leq x_1 \leq x_1^1, \tag{9}$$

$$v_1 = v_2 = 0 \quad \text{at } x_1 = l, \quad |x_2| \leq L. \tag{10}$$

Here  $f(t)$  is a given function that varies in time according to the law of a continuously differentiable function, which at the beginning monotonically increases to the maximum value  $f(t_0)$ , and then monotonically decreases;  $x_1^0, x_1^1, x_2^1$  are the constant numbers defining the size of the cutout. Zero initial conditions (5) mean that a rectangular strip with a symmetrically positioned rectangular cutout at the lateral boundaries until the time instant  $t \leq 0$  is in an unperturbed state. The boundary condition (6) is the specification of the normal component of the particle velocity and the absence of the tangent component of the particle velocity at the strip boundary  $x_1 = 0$  for all time instants. The boundary conditions (7) mean that the corresponding points of the boundary  $|x_2| = L$  ( $0 \leq x_1 \leq x_1^0$  and  $x_1^1 \leq x_1 \leq l$ ) are free of the stress for all moments of time. The boundary conditions (8) - (9) mean that the contour of the rectangular cutout is free of stresses. The boundary conditions (10) correspond to the conditions of rigid fastening of the back of the border  $x_1 = l$ .

The task is to determine inside a rectangular region with a symmetrically located rectangular cutout on the lateral boundaries of the stress and velocity fields caused by the fronts of incident and repeatedly diffracted elastic waves at a time  $t > 0$ .

The solution of the system of equations (4) under initial (5) and boundary (6) - (10) conditions is found by the difference method of spatial characteristics. An algorithm for solving equations (4) based on the spatial characteristics method was developed in [7] for internal, boundary, and corner points and the corresponding calculated finite-difference relations were obtained. It is based on finite-difference relations obtained by integrating equations (4) along bicharacteristics and recorded at the nodal points into which the entire investigated area is divided (figure 1). When considering the boundary conditions (7) - (9), for each of the corner points B, G, P, D, C, S, Q, F (Fig.1) of the cutout, the pairing law of tangential stresses is fulfilled. This circumstance reduces the number of specified conditions at the corner point by one, and in order to obtain a closed system of equations, it is necessary to obtain one additional equation for each corner point B, G, P, D, C, S, Q, F of the cutout, respectively. The method of obtaining the additional equations for each of the corner points of the rectangular region was considered in [7]. However, all the resulting finite - difference relations can only be used in areas with a continuous change in all incoming parameters.

A feature of the considered body with a cutout is that at the corner points of a rectangular cutout (figure 1), the smoothness of functions that is "familiar" to dynamic problems is violated, i.e. at these points, the first and second derivatives of the desired functions suffer a discontinuity of the first kind. Therefore, they are considered as ordinary corner points of a rectangular cutout, and the method developed in [6] is generalized to obtain resolving equations at these singular points. When constructing a numerical solution for problem (4) - (10), it is assumed that the border of the strip and the contour of rectangular cutouts coincide with the line of nodes of the square grid that covers the studied area.

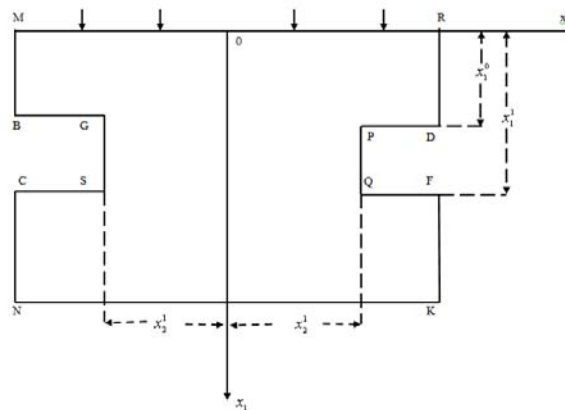


Figure 1 - The study area

**Analysis of the calculation results.** A square grid is applied to an elastic body in the form of a rectangular strip with a rectangular cutout, at the nodes of which the values of the velocity components  $v_1, v_2$  of displacements and stress  $p, q, \tau$  are determined. It is assumed that the boundaries of the body coincide with the line of nodes of the square grid that covers the studied area (figure 1).

The computational process is carried out in time steps. The time step  $k$  is selected in accordance with the necessary stability conditions [7]

$$\left(\frac{k}{h}\right)^2 \leq \min\left\{\frac{\gamma^2}{\gamma^2+1}, \frac{\gamma^2}{2(\gamma^2-1)}\right\} \quad (11)$$

that are used by the explicit finite - difference calculation scheme. In this way, the values of the sought quantities are calculated at any point of a rectangular strip with a rectangular cut at time  $t = t_0 = k\Delta t$ .

To obtain results at the next time step  $t = (k + 1) \Delta t$ , it is enough to take the found values as the initial data and repeat the calculations. For the numerical implementation of the developed finite-difference scheme and the solution of non-stationary problems of the mechanics of a deformable solid, a calculation method and algorithm have been created and based on them a complex of calculation programs in Fortran-90 has been developed for high-speed personal computers.

Numerical results are given for the rectangular region  $0 \leq x_1 \leq 100 \cdot h$ ,  $|x_2| \leq 100 \cdot h$ . As an example, an I-beam construction with a deep symmetrically positioned side cutout was considered with the following values of the initial data:

$$f(t) = A \cdot t \cdot e^{-st}, \quad A=1, \quad s = 0.2, \quad k = 0.025, \quad h = 0.05, \quad x_1^0 = 20 \cdot h, \quad x_1^1 = 80 \cdot h, \quad |x_2^1| = 10 \cdot h, \quad \nu = 0.3, \\ \gamma = 1.87.$$

When analyzing the calculation results, the symmetry property of the problem with respect to the axis  $x_2 = 0$ , at the points of which  $v_2 = \tau = 0$  was used. Therefore, the obtained results are presented only for positive values  $x_2 (x_2 \geq 0)$ .

It is noteworthy that, as the compression wave (longitudinal wave) propagates, the velocity of which  $c_1 = \sqrt{\frac{\lambda+2\mu}{\rho}}$ , at the corner points of the strip and cutout, zones of diffraction perturbations arise. The emergence of these zones significantly complicates the construction of analytical solutions. From the physical point of view, the presence of these zones is due to lateral unloading in a direct compression wave, which, in turn, leads to a rotation of the velocity vector at internal points of the medium. The reflected longitudinal and transverse waves over time undergo multiple reflections from all boundaries, and the wave pattern is greatly complicated.

In the study of the complex stress state of an elastic medium, an analysis of the behavior of the individual components of the stress tensor is insufficient to conclude dangerous stress concentrations in terms of strength. In these cases, common criteria are often used, based on the application of the main normal and maximum tangential stresses. In the accepted dimensionless form, they can be written as

$$\sigma_{1,2} = p \pm \sqrt{q^2 + \tau^2}, \quad \tau_{\max} = \sqrt{q^2 + \tau^2}. \quad (12)$$

In this regard, Figures 2–4 in the plane  $x_1/h \cdot x_2/h$  for the time instant  $t = 500 \cdot k$  show, respectively, the isolines of the main normal  $\sigma_1 = const$ ,  $\sigma_2 = const$  and maximum tangential  $\tau_{\max} = const$  stresses, which demonstrate the stress state characteristic of an I-beam structure with a deep lateral cutout. The previous notation is accepted: solid, dash-dotted and dashed lines mark the contours of compressive, zero and tensile stresses, respectively. The numbers around the lines correspond to the values of the stresses on the isolines, and near the points - to the extreme values of the stresses. It can be noted that the level of the main normal  $\sigma_1$ ,  $\sigma_2$  and maximum tangential  $\tau_{\max}$  stresses, the nature of their distribution is complex due to the superposition of reflected, diffracted, and interference waves and constantly changes in time, which is typical for dynamic problems.

First of all, it should be noted that most of the area behind the cutout (lower shelf) is not loaded for a long period of time (figure 2–4). The stress level in the "stagnant" zone is extremely low. The main bearing part of the I-beam structure is its wall. The intermediate values by the stress level are observed in the area located directly under the load (upper shelf). The lack of symmetry with respect to the midline

$x_1 = 50 \cdot h$  of the cutout can be explained by the different nature of the specified boundary conditions on the surfaces  $x_1 = 0$  and  $x_1 = 100 \cdot h$ .

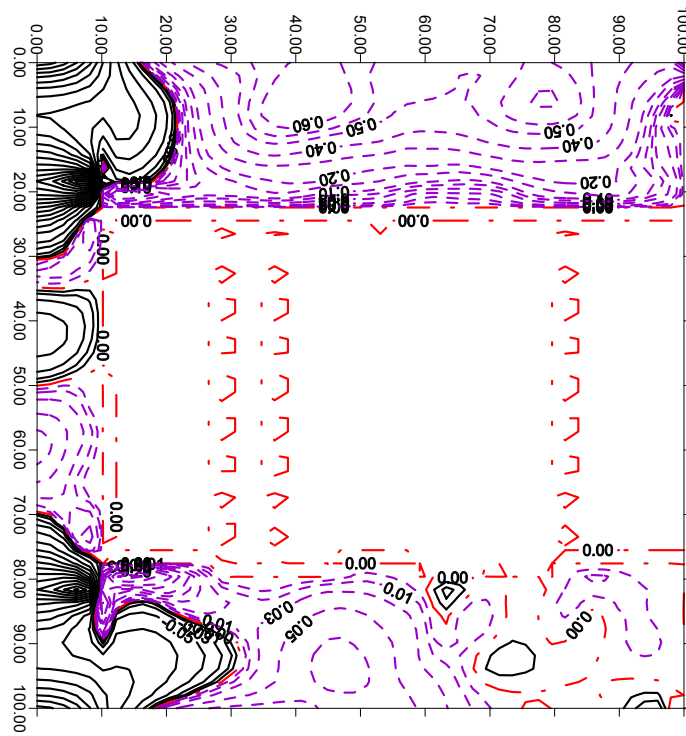


Figure 2 - Isolines of maximum stresses  $\sigma_1 = const$  at time  $t = 500 \cdot k$

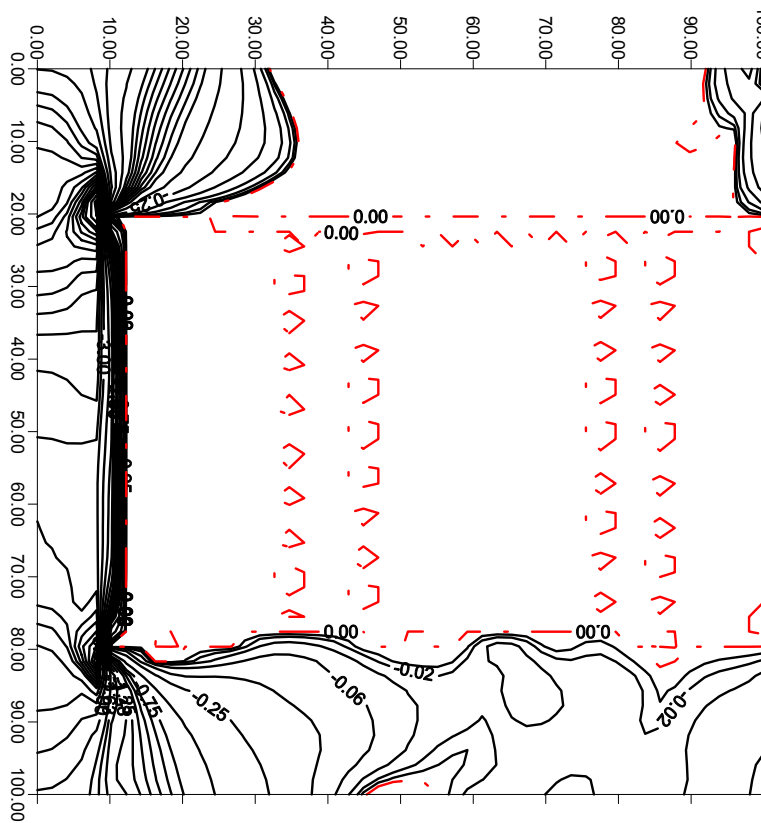


Figure 3 - Isolines of minimum stresses  $\sigma_2 = const$  at time  $t = 500 \cdot k$

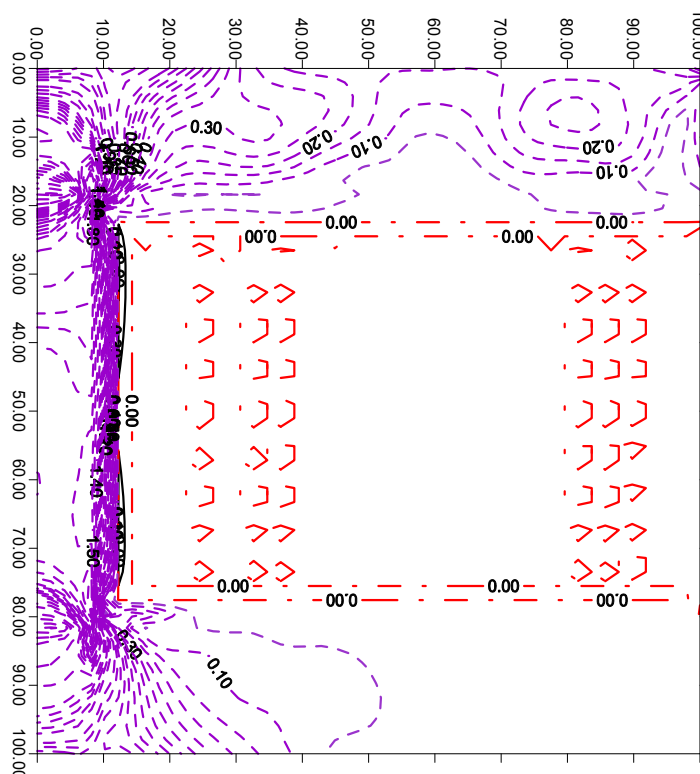


Figure 4 - Isolines of maximum tangential stresses  $\tau_{\max}$  at time  $t = 500 \cdot k$

The analysis of maximum stresses  $\sigma_1$  shows that in the I-beam design, six characteristic subregions can be distinguished (figure 2). At the time in question, in the subregions 1, 3 and 5 the maximum stresses  $\sigma_1$  are compressive, and in the subregions 2, 4 and 6 they are tensile. Moreover, each subregion is bordered by an isoline of zero stresses. The maximum compressive stress  $\sigma_1$  is achieved at some distance from the corner points  $(P, Q)$ , and the minimum compressive  $\sigma_2$  (figure 3) and maximum tangential stresses  $\tau_{\max}$  (figure 4) are at the corner points  $(P, Q)$ . The nature of the change in the main normal  $\sigma_1$ ,  $\sigma_2$  and maximum tangential  $\tau_{\max}$  stresses in the region before the cutout is explained by the propagation of diffracted waves emanating from the corner points  $(P, D, R)$ .

The maximum values of the main normal  $\sigma_1$ ,  $\sigma_2$  and maximum tangential  $\tau_{\max}$  stresses can be compared with their permissible limit values. This will evaluate the performance of this construction.

The developed mathematical model for solving plane dynamic problems of the theory of elasticity can be used to analyze the propagation of dynamic disturbances in a strip with a rectangular cross section of finite size and a cutout of complex geometric shape. In addition to the identified and discussed physical phenomena, the results obtained demonstrate the effectiveness of the developed computational algorithms.

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**СИММЕТРИЯЛЫ ОРНАЛАСҚАН ТЕСІКТЕРІ БАР ДЕНЕДЕГІ ТОЛҚЫНДЫ ӨРІС**

**Аннотация.** Бірқатар босаңсыту факторы (шекаралық шарттардағы үзіліс, тесік, қуыс, кесік, т.с.с.) негізінде конструкция элементтерінің динамикалық әрекетін болжаудың инженерлік тәжірибе сұраныстары арқылы анықталатын теориялық әрі қолданбалы мәні бар.

Құрылыста, машина жасауда және техниканың басқа да салаларында дайындау технологиясы немесе тасымалдау талаптарына байланысты болатын тесік, ойма (қырнау), ойық, шығынқы жерлер, т.с.с. тегістікті бұзушылықтар байқалады. Мұндай тұтассыздық маңайында әдетте кернеу концентрациясы деп аталатын кернеулер мен деформациялардың жергілікті бұрмалау құбылысы пайда болады. Бұл эффектілер «орташа» кернеулерге айтарлықтай өзгерістер енгізеді және бүліну себебі ретінде аталады. Кернеу концентрациясының сапалық және сандық эффектілеріне концентратор геометриясы, әсер ету түрі, материалдың нақты

касиеттерімен байланысты әртүрлі себептер ықпал етеді. Сондықтан заманауи техникада кернеулер концентрациясы мәселесіне үлкен назар аударылуда, бұл отандық және шетелдік зерттеулердің, сондай-ақ арнайы монографиялар мен анықтамалық құралдардың көбісінде көрініс тапты. Мұндай облыстар үшін шекаралық мәселелерді шешу тиімді сандық әдістерді құрастырусыз мүмкін емес. Сондықтан тұтас ортадағы стационар емес толқынды қозғалысты зерттеу мәселелерінің ауыр центрі есептеу нәтижелерін жақсартуға мүмкіндік беретін айырымдық схемаларды жасау және жетілдіруге ығысып отыр. Алайда қазіргі уақытқа дейін сандық әдістер арқылы әртүрлі конфигурациялы біртекті шектелген денелердегі динамикалық серпімділік теориясының шешілген мәселелері салыстырмалы түрде аз. Осылайша практикаға деген қажеттілік осы жұмыстың зерттеу пәніне айналып отырған тұтас орта динамикасының өзекті ғылыми және практикалық мәселелерінің шеңберін анықтайды.

Жұмыста мәселенің сызықтық қойылымында бүйір шекараларында симметриялы орналасқан тікбұрышты қимасы бар тікбұрышты облыста динамикалық ұйытқудың таралуы туралы есеп қарастырылады. Толқындық процесс сыртқы динамикалық жүктемені тікбұрышты облыстың беттік шекарасына қою арқылы туындайды, ал облыстың бүйір шекаралары кернеуден бос жатады. Тікбұрышты облыстың төменгі шекарасы қатты бекітілген. Тікбұрышты ойындының контуры кернеуден бос. Мәселе бүйір шекараларда тікбұрышты ойындысы симметриялы түрде орналасқан тікбұрышты облыстың ішінде құлап жатқан және  $t > 0$  уақыт моментінде бірнеше рет дифракцияланған серпімді толқындардың шебі арқасында туындаған кернеу мен жылдамдық өрісін анықтауға тіреледі.

Бүйір шекараларда тікбұрыш ойындысы симметриялы түрде орналасқан соңғы өлшемді тікбұрышты облыс үшін бастапқы және шекаралық шарттары берілген аралас есепті сандық әдіспен шешу үшін кеңістік сипаттамалар әдісіне негізделген айқын соңғы айырымдық схема құрылды. Ол бүкіл зерттеліп отырған облыс бөлінетін түйін нүктелерде жазылған биосипаттамалар бойындағы теңдеулер жүйесін интегралдау арқылы алынған ақырлы-айырымдық қатынастарға негізделген. Жұмыста тәуелді айнымалылар ретінде кернеудің орын ауыстыру жылдамдықтары мен тензор векторының компоненттері қабылданады, себебі кернеу мен орын ауыстыру жылдамдықтары механизмдерді зерттеу жағынан серпімді деформация кезіндегі динамикалық үдерістерді анықтайтын негізгі физикалық шамалар болып саналады. Есептің бұлайша қойылуы мұндай жүйелердің қазіргі кезде жеткілікті түрде терең және жан-жақты зерттелгендігінен, сонымен қатар орын ауыстыру жылдамдықтары мен кернеулерді таңдау кезінде шекаралық шарттардан туындылар ескерілмейтіндіктен таңдалды, ал бұл осы мәселені сандық әдіспен шешуде пайдалану үшін өте маңызды.

Тікбұрышты ойығы бар қарастырылып отырған дененің ерекшелігі тікбұрышты ойықтың бұрыштық нүктелерінде динамикалық мәселелер үшін «әдеттегі» функцияның тұтастығы бұзылады, яғни бұл нүктелерде қарастырылған функциялар мен олардың бірінші және екінші ретті туындылары бірінші текті үзіліске ұшырайды. Біз зерттеп отырған мәселеде осындай ерекше нүктелер үшін есептеу әдісі құрастырылмаған. Сондықтан олар тікбұрышты ойықтың кәдімгі бұрыштық нүктелері ретінде қарастырылады және осындай ерекше нүктелерде қарастырылған функцияларды табуға арналған шешуші теңдеулерді алуға арналған әдіс ұсынылып отыр. Шекаралық шарттар үзілісінің маңайындағы динамикалық кернеу концентрациясы зерттелді. Зерттеу нәтижелері сандық шешімге дейін келтірілді.

Зерттеу нәтижелері қатаң физикалық және математикалық негізді тірек етеді. Алынған нәтижелердің дәлелі шешімін табатын мәселенің қатаң математикалық тұжырымы мен белгілі сандық әдістерді қолдану, қажетті орнықты шарттарды орындау мен есептеу әдісінің орнықтылығын сандық әдіспен тексеру, сандық есептеу мен Рэль-Ламба есебінің аналитикалық шешімінің сәйкес келуі, алынған нәтижелердің қойылған есептің физикалық мазмұнымен сәйкес келуін тірек етеді.

Серпімділік теориясының жазық динамикалық есептерін шығаруға арналып құрастырылған математикалық модель арқылы өлшемді тікбұрышты көлденең қимасы және күрделі геометриялық формалы ойығы бар жолақтағы динамикалық ұйытқудың таралуын талдауда пайдаланылуы мүмкін. Алынған нәтижелер айқындалған және талқыланған физикалық құбылыстардан басқа құрастырылған есептеу алгоритмдерінің тиімділігін де көрсетеді.

**Түйін сөздер:** серпімді, жылдамдық, кернеу, күш, жазық деформация, толқын үдерісі, кернеу концентрациясы, сандық шешім.

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### **ВОЛНОВЫЕ ПРОЦЕССЫ В ПЛОСКОМ ТЕЛЕ С СИММЕТРИЧНО РАСПОЛОЖЕННЫМ ПРЯМОУГОЛЬНЫМ ВЫРЕЗОМ**

**Аннотация.** Прогнозирование динамического поведения элементов конструкций с учетом ряда ослабляющих факторов (разрывы в граничных условиях, отверстия, полости, вырезы и т.д.) имеет не только теоретическое, но и прикладное значение, определяемое запросами инженерной практики.



Для большинства конструкций, применяемых в строительстве, машиностроении и других отраслях техники, характерно наличие различных нарушений сплошности в виде отверстий, выточек, пазов, выступов и т.п., обусловленных либо технологией изготовления, либо эксплуатационными требованиями. Возле таких несплошностей возникает, как известно, явление местного искажения напряжений и деформаций, называемое обычно концентрацией напряжений. Эти эффекты вносят существенное изменение в «средние» напряжения и нередко являются причиной разрушения. На качественные и количественные эффекты концентрации напряжений влияют разнообразные причины, связанные с геометрией концентратора, видом воздействия, реальными свойствами материала. Поэтому проблеме концентраций напряжений уделяется в современной технике огромное внимание, что нашло отражение в практически необозримом количестве отечественных и зарубежных исследований, а также специальных монографиях и справочных руководствах. Решение краевых задач для таких областей невозможно без разработки эффективных численных методов. Поэтому центр тяжести проблемы исследования нестационарных волновых движений в сплошных средах все больше смещается в сторону разработки и совершенствования разностных схем, позволяющих улучшать результаты расчетов. Однако до настоящего времени сравнительно мало решенных задач динамической теории упругости в ограниченных телах с неоднородностями различной конфигурации с использованием численных методов.

Таким образом, потребности практики определяют круг актуальных научных и практических задач динамики сплошной среды, которые стали предметом исследования настоящей работы.

В работе в линейной постановке рассматривается задача о распространении динамических возмущений в прямоугольной области конечных размеров с симметрично-расположенным прямоугольным вырезом на боковых границах. Волновой процесс вызывается прикладыванием внешней динамической нагрузки на лицевой границе прямоугольной области, а боковые границы области свободны от напряжений. Нижняя граница прямоугольной области жестко закреплена. Контур прямоугольного выреза свободен от напряжений. Задача заключается в определении внутри прямоугольной области с симметрично-расположенным прямоугольным вырезом на боковых границах полей напряжений и скоростей, вызванных фронтами падающих и многократно дифрагированных упругих волн в момент времени  $t > 0$ .

Для численного решения смешанной задачи с начальными и граничными условиями для прямоугольной области конечных размеров с симметрично-расположенным прямоугольным вырезом на боковых границах построена явная конечно-разностная схема, основанная на методе пространственных характеристик. Он основан на конечно – разностных соотношениях, полученных интегрированием системы уравнений вдоль бихарактеристик и записанных в узловых точках, на которые разбивается вся исследуемая область. В работе в качестве зависимых переменных принимаются компоненты вектора скоростей перемещений и тензора напряжений, поскольку напряжения и скорости перемещений являются основными физическими величинами, представляющими интерес с точки зрения изучения механизмов, определяющих динамические процессы при упругом деформировании. Эта постановка выбрана потому, что такие системы изучены в настоящее время достаточно глубоко и основательно и, кроме того, при выборе скоростей перемещений и напряжений в качестве основных переменных, исключаются производные из граничных условий, что чрезвычайно важно при использовании численных методов решения задачи.

Особенностью рассмотренного тела с прямоугольным вырезом является то, что в угловых точках прямоугольного выреза нарушается «привычная» для динамических задач гладкость функций, т.е. в этих точках искомые функции и их первые и вторые производные терпят разрыв первого рода. Для таких особых точек, которые имеют место в исследуемой нами задаче, метод расчета не разработан. Поэтому они рассматриваются как обычные угловые точки прямоугольного выреза. Был предложен метод получения разрешающих уравнений для нахождения искомых функций в этих особых точках. Исследована концентрация динамических напряжений в окрестности разрыва граничных условий. Результаты исследования доведены до численного решения.

Результаты исследований базируются на строгих физических и математических основах. Достоверность полученных результатов обусловлена строгой математической формулировкой решаемой задачи и применением известных численных методов решения, выполнением необходимых условий устойчивости и численной проверкой устойчивости метода расчета, совпадением результатов численных расчетов с результатами точного аналитического решения задачи Рэлея – Ламба, соответствием полученных результатов физическому содержанию поставленной задачи.

Разработанная математическая модель решения плоских динамических задач теории упругости может быть использована для анализа распространения динамических возмущений в полосе с прямоугольным поперечным сечением конечного размера и вырезом сложной геометрической формы. Кроме выявленных и обсужденных физических явлений, полученные результаты демонстрируют эффективность разработанных расчетных алгоритмов.

**Ключевые слова:** упругость, скорость, напряжение, нагрузка, плоская деформация, волновой процесс, концентрация напряжений, численное решение.

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**THE CORRESPONDENCE OF THE EREZ-ROSEN  
 SOLUTION WITH THE HARTLE-THORNE SOLUTION  
 IN THE LIMITING CASE OF  $\sim Q$  AND  $\sim M^2$**

**Abstract.** The link between exterior solutions to the Einstein gravitational field equations such as the exact Erez-Rosen metric and approximate Hartle-Thorne metric is established here for the static case in the limit of linear mass quadrupole moment ( $Q$ ) and second order terms in total mass ( $M$ ). To this end, the Geroch-Hansen multipole moments are calculated for the Erez-Rosen and Hartle-Thorne solutions in order to find the relationship among the parameters of both metrics. The coordinate transformations are sought in a general form with two unknown functions in the corresponding limit of  $\sim Q$  and  $\sim M^2$ . By employing the perturbation theory, the approximate Erez-Rosen metric is written in the same coordinates as the Hartle-Thorne metric. By equating the radial and azimuthal components of the metric tensor of both solutions the sought functions are found in a straightforward way. It is shown that the approximation  $\sim Q$  and  $\sim M^2$ , which is used throughout the article, is physical and suitable for solving most problems of celestial mechanics in post-Newtonian physics. This approximation does not require the use of the Zipoy-Voorhees transformation, which is a necessary strict mathematical requirement in the  $\sim Q$  approximation, i.e. when no other approximations are made. This implies that the explicit form of the coordinate transformations depends entirely on the approximation that is adopted in each particular case. The results obtained here are in agreement with the previous results in the literature and can be applied to different astrophysical goals. The paper pursues not only pure scientific, but also academic purposes and can be used as an auxiliary and additional material to the special courses of general theory of relativity, celestial mechanics and relativistic astrophysics.

**Keywords:** exact and approximate solutions of Einstein's gravitational field equations, Erez-Rosen metric, Hartle-Thorne metric, coordinate transformations, quadrupole moment, Geroch-Hansen multipole moments, perturbation method.

**1. Introduction.**

There are a plenty of exact and approximate solutions to the Einstein field equations (EFE) in the literature [1,2]. Most of the solutions are pure mathematical and only some of them are physical and usually used in a various realistic astrophysical context. We here focus on the exterior exact Erez-Rosen (ER) [3] and approximate Hartle-Thorne (HT) [4,5] solutions, which are very well established and widely exploited. The Erez-Rosen solution mainly involved in the description of the exterior gravitational field of a deformed astrophysical object. Instead the Hartle-Thorne solution is used to study both interior and exterior fields of slowly rotating and slightly deformed astrophysical objects in the strong field regime. In connection with this, it is interesting to show how to find the relationship between these solutions in the limiting static case with a small deformation.

The metric of a nonrotating mass with a quadrupole moment has been obtained by Erez and Rosen in 1959 [3] with a use of Weyl method [6]. This metric was also analyzed by applying the spheroidal coordinates, which are adapted to characterize the gravitational field of non-spherically symmetric bodies. Later corrected for several numerical coefficients by Doroshkevich (1966) [7], Winicour et al. (1968) [8] and Young and Coulter (1969) [9]. The physical properties of the ER metric were investigated by Zeldovich and Novikov [10] and later by Quevedo and Parkes [11]. More general solutions involving multipole moments were obtained by Quevedo, Quevedo and Mashhoon (QM) [12–15].

In the article dated back 1967 Hartle put forward his approach for studying physical properties of slowly rotating relativistic stars [4]. Physical quantities that describe the equilibrium configurations of rotating stars such as the change in mass, gravitational potential, eccentricity, binding energy, change in moment of inertia, quadrupole moment, etc. were proportional to the square of the star's angular velocity  $\Omega^2$ . Hartle and Thorne tested the formalism for different equations of state of relativistic objects [5]. From that moment this solution is widely known as the Hartle-Thorne (HT) solution. Unlike other solutions of the Einstein equations, the Hartle-Thorne solution has an internal counterpart [4,16], which makes it more practical for investigation the equilibrium structure and physical characteristics of relativistic compact objects such as white dwarfs, neutron stars and hypothetical quark stars [17–21]. Relatively recently this solution was extended up to  $\Omega^4$  approximation [22].

The main objective of this article is to find the relationship between Hartle-Thorne solution and Erez-Rosen solution and show their equivalence in the limiting static case with a small deformation. We adopted the signature of the line elements for this article as  $(+ - - -)$  and used geometrical units  $G = c = 1$ .

It should be emphasized that the relationship between the ER and HT solutions has been established by Mashhoon and Theiss in 1991 [23], involving the Zipoy-Voorhees transformation in the limiting static case for small deformation. In addition Frutos-Alfaro and Soffel has shown that in the limit of  $\sim Q$  and  $\sim M^2$  for static case one can find the relationship between the two metrics without involving the Zipoy-Voorhees transformation [24]. In [25] we revisited the derivation by Mashhoon and Theiss providing all technical details in an instructive way. However, in this work we revisit the results of Frutos-Alfaro and Soffel [24], justifying physical significance, providing technical details. The paper pursues pure scientific and academic purposes.

The work is organized as follows. We review the main properties of the ER solution in section 2. The main physical characteristics of the exterior Hartle-Thorne solution are discussed in section 3. Section 4 is devoted to the computation of the multipole structure of the solutions. The linearized, up to the first order in mass quadrupole moment  $Q$  and to the second order in total mass  $M$  the Erez-Rosen solution is considered in section 5. The Hartle-Thorne solution in the limit of  $\sim Q$  and  $\sim M^2$  is considered in section 6. Using the perturbation method, the coordinate transformations are sought in section 7. Finally, we summarize our conclusions and discuss about future prospects.

## 2. The Erez-Rosen metric

The Erez-Rosen metric is an exact exterior solution with mass ( $m$ ) and quadrupole ( $q$ ) parameters that describes the gravitational field of static deformed objects in the strong field regime [26]. It belongs to the Weyl class of static axisymmetric vacuum solutions in prolate spheroidal coordinates  $(t, x, y, \varphi)$ , with  $x \geq 1$  and  $-1 \leq y \leq 1$ :

$$ds^2 = e^{2\psi} dt^2 - m^2 e^{-2\psi} \left[ e^{2\gamma} (x^2 - y^2) \left( \frac{dx^2}{x^2-1} + \frac{dy^2}{1-y^2} \right) + (x^2 - 1)(1 - y^2) d\varphi^2 \right], \quad (1)$$

where the metric functions  $\psi$  and  $\gamma$  depend on the spatial coordinates  $x$  and  $y$ , only, and  $m$  represents the mass parameter.

The solution found by Erez and Rosen has the following form [27]

$$\psi = \frac{1}{2} \ln \left( \frac{x-1}{x+1} \right) + \frac{1}{2} q (3y^2 - 1) \left[ \frac{1}{4} (3x^2 - 1) \ln \left( \frac{x-1}{x+1} \right) + \frac{3}{2} x \right] \quad (2)$$

and

$$\gamma = \frac{1}{2} (1 + q)^2 \ln \left( \frac{x^2-1}{x^2-y^2} \right) - \frac{3}{2} q (1 - y^2) \left[ x \ln \left( \frac{x-1}{x+1} \right) + 2 \right] + \quad (3)$$

$$\begin{aligned}
& + \frac{9}{16} q^2 (1 - y^2) \left[ x^2 + 4y^2 - 9x^2 y^2 - \frac{4}{3} \right. \\
& + x \left( x^2 + 7y^2 - 9x^2 y^2 - \frac{5}{3} \right) \ln \left( \frac{x-1}{x+1} \right) \\
& \left. + \frac{1}{4} (x^2 - 1)(x^2 + y^2 - 9x^2 y^2 - 1) \ln^2 \left( \frac{x-1}{x+1} \right) \right]
\end{aligned}$$

where  $q$  is the quadrupole parameter.

### 3. The exterior Hartle-Thorne solution

The general form of the exterior approximate HT metric [4, 5] in spherical  $(t, R, \theta, \phi)$  coordinates is given by:

$$\begin{aligned}
ds^2 = & \left( 1 - \frac{2M}{R} \right) \left[ 1 + 2k_1 P_2(\cos \theta) - 2 \left( 1 - \frac{2M}{R} \right)^{-1} \frac{J^2}{R^4} (2\cos^2 \theta - 1) \right] dt^2 \\
& - \left( 1 - \frac{2M}{R} \right)^{-1} \left[ 1 - 2 \left( k_1 - \frac{6J^2}{R^4} \right) P_2(\cos \theta) - 2 \left( 1 - \frac{2M}{R} \right)^{-1} \frac{J^2}{R^4} \right] dR^2 \\
& - R^2 [1 - 2k_2 P_2(\cos \theta)] (d\theta^2 + \sin^2 \theta d\phi^2) + \frac{4J}{R} \sin^2 \theta dt d\phi
\end{aligned} \quad (4)$$

where

$$k_1 = \frac{J^2}{MR^3} \left( 1 + \frac{M}{R} \right) + \frac{5Q - J^2/M}{8M^3} Q_2^2(x) \quad (5)$$

$$k_2 = k_1 + \frac{J^2}{R^4} + \frac{5Q - J^2/M}{4M^2 R} \left( 1 - \frac{2M}{R} \right)^{-\frac{1}{2}} Q_2^1(x) \quad (6)$$

and

$$Q_2^1(x) = (x^2 - 1)^{\frac{1}{2}} \left[ \frac{3x}{2} \ln \left( \frac{x+1}{x-1} \right) - \frac{3x^2 - 2}{x^2 - 1} \right]$$

$$Q_2^2(x) = (x^2 - 1) \left[ \frac{3x}{2} \ln \left( \frac{x+1}{x-1} \right) - \frac{3x^3 - 5x}{(x^2 - 1)^2} \right]$$

are the associated Legendre functions of the second kind, being  $P_2(\cos \theta) = (1/2)(3\cos^2 \theta - 1)$  the Legendre polynomial, and  $x = \frac{R}{M} - 1$ . The constants  $M$ ,  $J$  and  $Q$  are the total mass, angular momentum and quadrupole moment of a rotating object, respectively. Note, that according to Hartle  $Q > 0$  for oblate and  $Q < 0$  for prolate objects.

In order to obtain the HT solution for static objects, we set  $J = 0$  in the general form (4) and obtain

$$\begin{aligned}
ds^2 = & \left( 1 - \frac{2M}{R} \right) [1 + 2k_1 P_2(\cos \theta)] dt^2 - \left( 1 - \frac{2M}{R} \right)^{-1} [1 - 2k_1 P_2(\cos \theta)] dR^2 \\
& - R^2 [1 - 2k_2 P_2(\cos \theta)] (d\theta^2 + \sin^2 \theta d\phi^2)
\end{aligned} \quad (7)$$

Where

$$k_1 = \frac{5Q}{8M^3} Q_2^2(x) \quad (8)$$

$$k_2 = k_1 + \frac{5Q}{4M^2 R} \left( 1 - \frac{2M}{R} \right)^{-\frac{1}{2}} Q_2^1(x) \quad (9)$$

Hence we can find the function  $\psi(x, y)$  for the static HT metric from a simple relation

$$g_{tt} = e^{2\psi} \quad (10)$$

The solution found for  $\psi$  has the following form

$$\psi = \frac{1}{2} \ln \left[ \left( \frac{x-1}{x+1} \right) \left( 1 - \frac{5Q(3y^2-1)(10x-6x^3+3(x^2-1)^2 \ln \frac{x+1}{x-1})}{16M^3(1-x^2)} \right) \right] \quad (11)$$

where  $y = \cos \Theta$ . Finally, for the function  $\gamma$  it is not possible to find an explicit expression, unless we consider the approximate version of the ER metric. This will be done in Sec. 5.

#### 4. Geroch-Hansen multipole moments

Using the original definition formulated by Geroch [28], the calculation of multipole moments is quite laborious. Fodor et al. [29] found a relation between the Ernst potential [30,31] and the multipole moments which facilitates the computation. In the case of static axisymmetric space-times, the Ernst potential is defined as

$$\xi(x, y) = \frac{1-e^{2\psi}}{1+e^{2\psi}} \quad (12)$$

The idea is that the multipole moments can be obtained explicitly from the values of the Ernst potential on the axis by using the following procedure. On the axis of symmetry  $y = 1$ , we can introduce the inverse of the Weyl coordinate  $z$  as

$$\tilde{z} = \frac{1}{z} = \frac{1}{mx} \quad (13)$$

If we introduce the inverse potential as

$$\tilde{\xi}(\tilde{z}, 1) = \frac{1}{z} \xi(\tilde{z}, 1) \quad (14)$$

The multipole moments can be calculated as

$$M_n = m_n + d_n, \quad m_n = \frac{1}{n!} \frac{d^n \tilde{\xi}(\tilde{z}, 1)}{d\tilde{z}^n} \quad (15)$$

where  $d_n$  must be determined from the original Geroch definition (e.g. Refs. [15]). For the Erez-Rosen metric, the Geroch-Hansen multipole moments read

$$M_0 = m, \quad M_2 = \frac{2}{15} qm^3 \quad (16)$$

where  $M_0$  is the monopole moment and  $M_2$  is the quadrupole moment.

For the Hartle-Thorne metric, we obtain

$$M_0 = M, \quad M_2 = -Q \quad (17)$$

As one can see from the Geroch-Hansen definition of multipole moments the quadrupole moment of the HT metric has an opposite sign, which is due to the use of a different convention.

#### 5. The approximated Erez-Rosen solution in the limit of $\sim Q$ and $\sim M^2$

In order to obtain the approximated ER metric we express  $m$  and  $q$  in terms of  $M$  and  $Q$  by equating relations (16) and (17), respectively as follows

$$m = M, \quad q = -\frac{15}{2} \frac{Q}{M^3} \quad (18)$$

and find its limit in  $\sim Q$  and  $\sim M^2$  by expanding in Taylor series keeping only  $Q$  and  $M^2$  and neglecting  $QM^2$  terms. Taking into account  $x = \frac{r}{m} - 1$  and  $y = \cos \theta$ , the final result is written in spherical-like coordinates  $(t, r, \theta, \phi)$

$$\begin{aligned}
ds^2 = & \left(1 - \frac{2M}{r} + \frac{2QP_2(\cos \theta)}{r^3} + \frac{2MQP_2(\cos \theta)}{r^4}\right) dt^2 \\
& - \left(1 + \frac{2M}{r} + \frac{4M^2}{r^2} - \frac{2QP_2(\cos \theta)}{r^3} - \frac{2MQ(5P_2^2(\cos \theta) + 11P_2(\cos \theta) - 1)}{3r^4}\right) dr^2 \\
& - r^2 \left(1 - \frac{2QP_2(\cos \theta)}{r^3} - \frac{2MQ(5P_2^2(\cos \theta) + 5P_2(\cos \theta) - 1)}{3r^4}\right) d\theta^2 \\
& - r^2 \sin^2 \theta \left(1 - \frac{2QP_2(\cos \theta)}{r^3} - \frac{6MQP_2(\cos \theta)}{r^4}\right) d\phi^2
\end{aligned} \tag{19}$$

### 6. The Hartle-Thorne solution in the limit of $\sim Q$ and $\sim M^2$

In order to obtain the HT metric for static objects we set  $J = 0$  and find its limit in  $\sim Q$  and  $\sim M^2$ , taking into account  $x = \frac{R}{M} - 1$  and  $y = \cos \theta$ . So the HT metric in standard spherical coordinates  $(t, R, \theta, \phi)$  reads

$$\begin{aligned}
ds^2 = & \left(1 - \frac{2M}{R} + \frac{2QP_2(\cos \theta)}{R^3} + \frac{2MQP_2(\cos \theta)}{R^4}\right) dt^2 - \left(1 + \frac{2M}{R} + \frac{4M^2}{R^2} - \frac{2QP_2(\cos \theta)}{R^3} - \frac{10MQP_2(\cos \theta)}{R^4}\right) dR^2 \\
& - R^2 \left(1 - \frac{2QP_2(\cos \theta)}{R^3} - \frac{5MQP_2(\cos \theta)}{R^4}\right) (d\theta^2 + \sin^2 \theta d\phi^2)
\end{aligned} \tag{20}$$

### 7. Coordinate transformations from the Erez-Rosen metric to the Hartle-Thorne metric

To obtain the correspondence between the ER solution, with coordinates  $(t, r, \theta, \phi)$ , and the HT solution, with coordinates  $(t, R, \theta, \phi)$ , both solutions must be written in the same coordinates. Therefore, we search for a coordinate transformation of the following form:

$$r \rightarrow R + \frac{MQ}{R^3} f_1(\theta) \quad \theta \rightarrow \theta + \frac{MQ}{R^4} f_2(\theta) \tag{21}$$

where  $f_1(\theta)$  and  $f_2(\theta)$  are the sought unknown functions. In view of  $\sim Q$  and  $\sim M^2$  approximation the functions  $f_1$  and  $f_2$  depend only on  $\theta$ . The total differentials of the coordinates are given by:

$$dr = \frac{\partial r}{\partial R} dR + \frac{\partial r}{\partial \theta} d\theta = \left(1 - \frac{3MQ}{R^4} f_1(\theta)\right) dR + \frac{MQ}{R^3} \left(\frac{\partial f_1(\theta)}{\partial \theta}\right) d\theta \tag{22}$$

$$d\theta = \frac{\partial \theta}{\partial R} dR + \frac{\partial \theta}{\partial \theta} d\theta = \left(-\frac{4MQ}{R^5} f_2(\theta)\right) dR + \left(1 + \frac{MQ}{R^4} \frac{\partial f_2(\theta)}{\partial \theta}\right) d\theta \tag{23}$$

These expressions should be plugged in the approximated ER solution (19). Then, only terms  $\sim Q$  and  $\sim M^2$  must be retained. We thus obtain the approximated ER metric in the same coordinates as the HT solution with the same parameters

$$\begin{aligned}
ds^2 = & \left(1 - \frac{2M}{R} + \frac{2QP_2(\cos \theta)}{R^3} + \frac{2MQP_2(\cos \theta)}{R^4}\right) dt^2 \\
& - \left(1 + \frac{2M}{R} + \frac{4M^2}{R^2} - \frac{2QP_2(\cos \theta)}{R^3} - \frac{2MQ(5P_2^2(\cos \theta) + 11P_2(\cos \theta) - 9f_1(\theta) - 1)}{3R^4}\right) dR^2 \\
& - R^2 \left(1 - \frac{2QP_2(\cos \theta)}{R^3} - \frac{2MQ(5P_2(\cos \theta)(1+P_2(\cos \theta)) + 3(f_1(\theta) - f_2'(\theta)) - 1)}{3R^4}\right) d\theta^2 \\
& + \left(\frac{2MQ(f_1'(\theta) - 4f_2(\theta))}{R^3}\right) dR d\theta \\
& - R^2 \sin^2 \theta \left(1 - \frac{2QP_2(\cos \theta)}{R^3} - \frac{2MQ(3P_2(\cos \theta) + f_1(\theta) + f_2(\theta) \cot \theta)}{R^4}\right) d\phi^2
\end{aligned} \tag{24}$$

Furthermore, by equating the corresponding  $g_{RR}$  components of the metric tensor of both approximated ER (24) and HT (20) solutions, written in  $(t, R, \Theta, \phi)$  coordinates, we find the expression for function  $f_1(\Theta)$  as follows

$$f_1(\Theta) = \frac{5P_2^2(\cos \Theta) - 4P_2(\cos \Theta) - 1}{9} \quad (25)$$

Analogously, by comparing only the azimuthal components of the metric tensor  $g_{\phi\phi}$  of both solutions, we find the function  $f_2(\Theta)$  as

$$f_2(\Theta) = \frac{1}{6}(2 - 5P_2(\cos \Theta)) \cos \Theta \sin \Theta \quad (26)$$

To this end, if we plug these functions into the mixed component of the metric tensor  $g_{R\Theta}$  of the approximated ER solution (24), written in  $(R, \Theta)$  coordinates,  $g_{R\Theta}$  vanishes, as we expected.

**8. Conclusion.** We explored the Erez-Rosen and Hartle–Thorne metrics (in the absence of rotation) in the limit  $\sim Q$  and  $\sim M^2$  by using the perturbation method. The approximation that we used throughout the paper is physical and convenient to solve most problems of celestial mechanics in the post-Newtonian Physics. We showed that the approximate Erez–Rosen line element coincides with the Hartle–Thorne solution in the considered limit.

The use of Geroch and Hansen invariant definition of the multipole moments helped us to calculate the corresponding mass monopole and quadrupole moments and establish the interconnection among the parameters of both solutions.

In addition, we have showed that the explicit form of the coordinate transformations in the limit  $\sim Q$  and  $\sim M^2$  do not require the use of the Zipoy-Voorhees transformation in view of Ref. [32] (see also Ref. [25] for details).

Due to the recent results [33–36], it will be interesting to find the connection between the Erez–Rosen and Zipoy-Voorhees (q-metric) solutions. This will be the issue of future studies.

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### **$\sim Q$ ЖӘНЕ $\sim M^2$ ЖУЫҚТАУЫНДА ЭРЕЗ-РОЗЕН ЖӘНЕ ХАРТЛ-ТОРН МЕТРИКАЛАРЫНЫҢ СӘЙКЕСТІГІ**

**Аннотация.** Бұл жұмыста гравитациялық өріс үшін Эйнштейн теңдеулерінің сыртқы шешімі, атап айтқанда Эрез-Розен және Хартл-Торн метрикалары арасына байланыс орнатылды. Эрез-Розен метрикасы Эйнштейн теңдеулерінің дәл шешімі болып саналады және статикалық, осьтік симметриялы астрофизикалық нысандардың сыртқы гравитациялық өрісін сипаттайды, ал Хартл-Торн метрикасы Эйнштейн теңдеулерінің жуық шешімі болып есептеледі, ол баяу айналатын және аздап деформацияланған астрофизикалық нысандардың ішкі және сыртқы гравитациялық өрісін сипаттайды. Мақалада алға қойылған мақсатқа жету үшін Хартл-Торнның сыртқы метрикасы тек статикалық жағдайда қарастырылды.

Жоғарыда көрсетілген шешімдер арасында қатынас орнату үшін олар бірдей жуықтау және айналу болмағанда, дәлірек айтқанда, сызықтық квадрупольдік момент  $Q$  және толық масса квадраты  $M^2$  жуықтауында қарастырылды. Хартл-Торн метрика координаталарында Эрез-Розен метрикасын жазу үшін алдымен Героч-Хансен мультипольдік моменттері есептеледі, инвариантты моменттерді анықтау екі метрика параметрлері арасында байланыс орнатуға мүмкіндік берді. Содан кейін Эрез-Розен метрикасы Хартл-Торн метрикасының параметрлері  $Q$ ,  $M$  арқылы жазылды және оның жуық өрнегі  $\sim Q$  және  $\sim M^2$  шегінде алынды.

Әрі қарай,  $f_1$  және  $f_2$  екі белгісіз функциялары бар жалпы түрдегі координаталық түрлендіруді қолдану арқылы қарастырылған жуықтауда Эрез-Розен метрикасы Хартл-Торн метрикасының координатасында



жазылды. Осыдан кейін екі шешімнің метрикалық тензорының радиалды және азимуталды құраушыларын теңестіре отырып,  $f_1$  және  $f_2$  функциялары айқындалды. Осылайша Эрез-Розен мен Хартл-Торн шешімі арасындағы байланысты анықтайтын координаталық түрлендірудің жуық өрнегі анықталды.

Мақалада қолданылған  $\sim Q$  және  $\sim M^2$  жуықтауы физикалық және постньютондық физикада аспан механикасының көптеген мәселелерін шешуге қолайлы екенін атап өткен жөн. Бұл жуықтау Зипой-Вурхис түрлендіруіне жүгінбеуге мүмкіндік береді. Себебі Зипой-Вурхис түрлендіруі  $\sim Q$  жуықтауында, яғни, басқа ешқандай жуықтау ескерілмеген кезде қажетті қатаң математикалық талап болып саналады. Бұл координаталық түрлендірудің айқын түрі толығымен әрбір нақты жағдайда қолданылатын жуықтауға байланысты болады дегенді білдіреді.

Қорытындылай келе, Эрез-Розен және Хартл-Торн метрикалары  $\sim Q$  және  $\sim M^2$  жуықтауында статикалық жағдай үшін ұйытқу теориясының әдістерін қолдану негізінде зерттеу жасалды. Эрез-Розеннің сызықтық элементінің қарастырылған жуықтауда Хартл-Торн шешімімен сәйкес келетіні көрсетілді.

Осылайша жұмыста алынған нәтижелер әдебиетте алынған белгілі нәтижелермен сәйкес келеді және оларды әртүрлі астрофизикалық мәселелерге қолдануға болады. Мақала тек қана ғылыми емес, сонымен бірге академиялық мақсаттарды да көздейді және жалпы салыстырмалылық теориясы, аспан механикасы және релятивистік астрофизика бойынша арнайы курстарда көмекші және қосымша құрал ретінде пайдалануға болады.

**Түйін сөздер:** гравитациялық өріс үшін Эйнштейн теңдеулерінің дәл және жуық шешімдері, Эрез-Розен метрикасы, Хартл-Торн метрикасы, координаттық түрлендіру, квадрупольдік момент, Героч-Хансен мультипольдік моменттері, ұйытқу теориясының әдістері.

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## СООТВЕТСТВИЕ МЕТРИК ЭРЕЗА-РОЗЕНА И ХАРТЛА-ТОРНА В ПРИБЛИЖЕНИИ $\sim Q$ И $\sim M^2$

**Аннотация.** В данной работе установлена взаимосвязь между внешними решениями уравнений Эйнштейна для гравитационного поля, а именно метрикой Эреза-Розена и метрикой Хартла-Торна. Метрика Эреза-Розена является точным решением уравнений Эйнштейна и описывает внешнее гравитационное поле статических, аксиально-симметричных астрофизических объектов, в то время как метрика Хартла-Торна является приближенным решением уравнений Эйнштейна, которое описывает как внешние, так и внутренние гравитационные поля медленно вращающихся и слегка деформированных астрофизических объектов. Для достижения цели статьи рассмотрена только внешняя метрика Хартла-Торна в статическом случае.

Чтобы установить взаимосвязь между перечисленными метриками, они были рассмотрены в одинаковом приближении и в отсутствие вращения, точнее, в пределе линейного квадрупольного момента  $Q$  и квадрата полной массы  $M^2$ . Чтобы записать метрику Эреза-Розена в координатах метрики Хартла-Торна, сначала были вычислены мультипольные моменты Героча-Хансена, которые позволяют установить взаимосвязь между параметрами обеих метрик посредством вычисления инвариантных моментов. Затем метрика Эреза-Розена была записана через параметры метрики Хартла-Торна  $Q$ ,  $M$  и был получен её приближенный вид в пределе  $\sim Q$  и  $\sim M^2$ .

Далее, используя координатные преобразования, записанные в общем виде с двумя неизвестными функциями  $f_1$  и  $f_2$ , метрика Эреза-Розена была записана в координатах метрики Хартла-Торна для рассматриваемого приближения. После этого, приравняв радиальные и азимутальные компоненты метрических тензоров двух рассматриваемых решений, были найдены искомые функции  $f_1$  и  $f_2$ . Таким образом, был найден приближенный вид координатных преобразований, которые определяют взаимосвязь между решениями Эреза-Розена и Хартла-Торна.

Следует заметить, что приближение  $\sim Q$  и  $\sim M^2$ , которое было использовано на протяжении всей статьи, является физическим и подходящим для решения большинства задач небесной механики в постньютоновской физике. Данное приближение позволяет не прибегать к преобразованию Зипоя-Вурхиса, которое является необходимым строгим математическим требованием в приближении  $\sim Q$ , т.е. когда не учитываются другие приближения. Это означает, что явная форма преобразования координат полностью зависит от приближения, которое используется в каждом конкретном случае.

В завершение следует отметить, что метрики Эреза-Розена и Хартла-Торна были исследованы для статического случая в приближении  $\sim Q$  и  $\sim M^2$  путем использования методов теории возмущений. Было показано, что приближенный линейный элемент Эреза-Розена совпадает с решением Хартла-Торна в рассматриваемом пределе.

Таким образом, результаты, полученные в этой статье, находятся в хорошем согласии с известными результатами из литературных источников и могут быть применены к различным астрофизическим задачам. Статья преследует не только чисто научные, но и академические цели, так как может быть использована в качестве вспомогательного и дополнительного материала для специальных курсов по общей теории относительности, небесной механике и релятивистской астрофизике.

**Ключевые слова:** точные и приближенные решения уравнений Эйнштейна для гравитационного поля, метрика Эреза-Розена, метрика Хартла-Торна, координатные преобразования, квадрупольный момент, мультипольные моменты Героча-Хансена, методы теории возмущений.

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**TWO PROPERTIES OF EXISTENTIALLY CLOSED  
COMPANIONS OF STRONGLY MINIMAL STRUCTURES**

**Abstract.** The proposed article studies some properties of existentially closed companions of strongly minimal structures. A criterion for the existential closedness of an arbitrary strongly minimal structure is found in the article and it is proved that the existentially closed companion of any strongly minimal structure is itself strongly minimal. It also follows from the resulting description that all existentially closed companions of a given strongly minimal structure form an axiomatizable class whose elementary theory is complete and model-complete and, therefore, coincides with its inductive and forcing companions.

This is the reason for the importance of the work done and the high international significance of the theorems obtained in it. Another equally important consequence of this research is the discovery of an important subclass of strongly minimal theories. It should be noted that a complete description of this class of theories is an independent and extremely important task.

It is known that natural numbers with the following relation are an example of a strongly minimal structure in which the existential type of zero is not minimal. Then the method used in the proof of the last theorem shows that the existentially closed companion of this structure are integers with the following relation.

**Keywords:** Existentially closed companions, strongly minimal structures, forcing companions.

**1 Introduction.** The theory of existential closedness appeared in the middle of the twentieth century in the works of one of the recognized classics of model theory, Abraham Robinson [1], [2], as well as in [3] – [6]. It is currently one of the two most significant and developed areas of modern model theory. In the previous works of Nurtazin A.T. [7] - [10], the most basic form of the concept of companion theory, widely known in the theory of existential closure, is introduced and studied. In [7], [8], a criterion for the countable categoricity of this companion theory was found and some properties of existentially closed and forcing companions were studied.

Informally, any companion of a given structure is constructed from the same end structures as the original structure. It is known that all companions of this structure form an axiomatizable class, and all existentially closed ones in this class are contained in it [10]. It is natural to assume that a companion class containing some strongly minimal structure is quite simple and has a number of additional properties. In [11], statements about companion classes containing a strongly minimal structure were formulated. This paper describes some properties of companion completions of an arbitrary strongly minimal theory and provides proofs of the statements from [10].

Over the past fifty years in model theory, starting with the work of Michael Morley [12] and continuing in the works of Shelach [13] and Laskar and Poizat [14], all the most significant results are associated with their classification by degrees of stability that arose during this time. Moreover, totally transcendent (or  $\omega$ -stable) theories were recognized as the most convenient for study. Recall that in these theories, all formulas have ranks and degrees, which respectively are countable ordinals and natural numbers. At the same time, all the successes in the study of these theories are primarily associated with the possibility of using the transfinite induction method in evidence. Historically, the initial impulse in the development of this direction was Los's hypothesis about uncountably categorical theories. At the same time, the core in the class of uncountably categorical theories turned out to have a single rank and degree,

the so-called "strongly minimal theories." This paper studies existentially closed companions of strongly minimal theories.

In the work, the criterion of existential closedness of an arbitrary strongly minimal theory was found. It turned out that in this case the existentially closed companion of such a structure is axiomatized and coincides with its inductive and forcing companions. And its elementary theory is complete and model-complete and, therefore,  $\forall\exists$ - is axiomatizable.

Strongly minimal theories are basic in the study of an important and one of the most studied classes of theories at present - the theory of uncountable categoricity. Meanwhile, the available literature completely lacks both estimates of their quantum complexity and estimates of the complexity of the relations defined in them. The study proposed here shows that the concept of "companion" proposed in [7], [8], [9] can be successfully applied to solve these issues.

### 1. Existential closedness criterion of a strongly minimal structure

In this section, we will find conditions of existential closedness of an arbitrary strongly minimal structure.

**THEOREM 1.** For an arbitrary strongly minimal structure, the following three conditions are equivalent:

- 1)  $M$  is existentially closed.
- 2) The existential type of any of its tuples is maximal.
- 3) The elementary theory  $T = Th(M)$  is model complete.

**Proof.**

$1 \rightarrow 2$ . By the existential closedness criterion from [11], this structure is strongly minimal if and only if the existential types of all its tuples are maximal.

$2 \rightarrow 3$ . Let the existential type of any tuple from  $M$  be maximal. Then, according to the criterion from [12], the structure  $M$  is existentially closed. Recall that any elementary substructure of a structure  $M$ , including a simple model  $M_0$  of a theory  $T$ , is also existentially closed. To prove the model completeness of the theory  $T$ , it is sufficient to show that the usual inclusion  $M < N$  between any two models of this theory is actually elementary. First, we note that the models  $M$  and  $N$  are isomorphic to some  $M_i$  and  $M_j$  from the elementary chain  $M_0 < M_1 < \dots < M_n < \dots$  of all countable models of the theory  $T$ . An uncomplicated reasoning shows that simple inclusion  $M_i < M_j$  determines the isomorphism of an algebraic closure of an empty subset of the structure  $M_i$  to the algebraic closure of an empty subset of the structure  $M_j$ . If at the same time  $a_1, \dots, a_i$  are algebraically independent in the structure  $M_i$ , then they are also algebraically independent in  $M_j$  and can be extended to the basis  $\{a_1, \dots, a_i, a_{i+1}, \dots, a_j\}$  of this structure. This determines the elementary inclusion  $M_i$  in  $M_j$  and with precision to automorphisms of these structures coincides with the original inclusion  $M_i < M_j$ . This completes the proof of the model completeness of the theory  $T$ .

$3 \rightarrow 1$ . It follows from the model completeness of the theory  $T$  that any of its models (including  $M$ ) is existentially closed.

The theorem is proved.

Informally, any companion of a given structure consists of the same finite substructures as the original structure. It is known that all companions of this structure form an axiomatizable class, and all existentially closed ones in this class are contained in it. It is natural to assume that a companion class containing some strongly minimal structure is quite simple and has a number of additional properties. We will show in the next section that the existentially closed companion of a strongly minimal structure is also strongly minimal and is model complete.

### 2. An existentially closed companion of a strongly minimal structure

Informally, any companion of a given structure consists of the same finite substructures as the original structure. It is known that all companions of this structure form an axiomatizable class, and all

existentially closed in this class are contained in it. It is natural to assume that a companion class containing some strongly minimal structure is quite simple and has a number of additional properties. Here we show that the existentially closed companion of a companion containing a strongly minimal structure is itself strongly minimal.

For certainty, we assume that the given companion class  $\mathcal{C}$  contains a countable saturated strongly minimal structure  $M$ . It is known from [7] that in any existentially closed structure from a companion class  $\mathcal{C}$ , the existential type of any tuple is maximal. By induction on the lexicographic order on pairs of natural numbers  $(m, n)$  we describe a procedure that allows to construct a sequence  $M_0^0 < M_1^0 < \dots < M_n^0 < \dots < M_0^m < M_1^m < \dots < M_n^m < \dots$ , of isomorphic  $M$ , the union  $N$  of which will be a strong minimal structure and an existentially closed companion  $M$ .

Let the strongly minimal structure  $M = M_0^0$  is not existentially closed. Then it contains tuples  $a^0, a^1, \dots, a^n, \dots$ , whose existential types  $p_0(x^0), p_1(x^1), \dots, p_n(x^n), \dots$  are not maximal. Using the compactness theorem, it is easy to prove the existence of an embedding of a structure  $M$  in itself such that the existential type  $q_0(x^0)$ , of the image of the tuple  $a^0$  is maximal.

Continuing further, one can obtain an increasing chain of strongly minimal structures  $M_0^0 < M_1^0 < \dots < M_n^0 < \dots$ , isomorphic to  $M$ , in the union  $M_0^1$  of which the existential types  $q_0(x^0), q_1(x^1), \dots, q_n(x^n), \dots$  of tuples  $a^0, a^1, \dots, a^n, \dots$  are maximal. Next, the process continues similarly. Then, in the union  $N = \bigcup M_n^m$ , the existential types of all tuples are maximal.

To prove the strong minimality of the structure  $N$  we first note that in the process of constructing the structure  $N$  one can include intermediate steps used in [8] when expanding a given countable existentially closed structure into a homogeneous one. Recall that in the countable homogeneous existentially closed structure constructed in this case, the correspondence between any two tuples of elements having the same existential type continues to an automorphism of the homogeneous structure itself. Since over any finite set, all subsets, except one, allocated by maximum existential types, are finite, then these subsets themselves are allocated over this subset by finite formulas. At the same time, all elements satisfying infinite existential formulas are translated over the finite subset by structure automorphisms chosen by us and, therefore, satisfy the same infinite formulas. This proves the strong minimality of the structure  $N$ . Now we show that any two existentially closed companions of the same minimal structure are elementary equivalent. For this we assume that there is a pair of existentially closed companion structures  $N_1$  and  $N_2$  of the structure  $M$ . However, without limiting generality, we can assume that the inclusion  $N_1 < N_2$  is enabled. Just as in the proof of the Los-Sushko theorem, the pair under consideration can be easily completed to the  $\omega$ -chain  $N_1 < N_2 < N_3 < \dots < N_{2n} < N_{2n+1} < \dots$ , in which the subchains  $N_1 < N_3 < \dots < N_{2n+1} < \dots$  and  $N_2 < \dots < N_{2n} < \dots$  are elementary. But then the union  $N$  of the built chain turns out to be a common elementary extension for each of the structures  $N_1$  and  $N_2$ , and the inclusion is elementary. We formulate the result as a separate statement.

**THEOREM 2.** If a given companion class has a strongly minimal structure, then any of its existentially closed companions is also strongly minimal. A complete and model-complete strongly minimal theory.

**Conclusion.** The results obtained in this paper link the three concepts of "model completeness, strong minimality, and existential closedness" that are currently most studied in modern model theory. This is the reason for the importance of the work done and the high international significance of the theorems obtained in it. Another equally important consequence of this research is the discovery of an important subclass of strongly minimal theories, which are strongly minimal model complete theories. It should be noted that a complete description of this class of theories is an independent and extremely important task.

It is known that natural numbers with the following relation are an example of a strongly minimal structure in which the existential type of zero is not minimal. Then the method used in the proof of the last theorem shows that the existentially closed companion of this structure are integers with the following relation.

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### **ЭКЗИСТЕНЦИАЛДЫ ТҰЙЫҚТАЛҒАН СЕРІКТЕСТЕРДІҢ АСА МИНИМАЛДЫ ҚҰРЫЛЫМЫНЫҢ ЕКІ ҚАСИЕТІ**

**Аннотация.** Алпыс жылдан астам тарихы бар экзистенциалды тұйықталу теориясы қазіргі модельдер теориясының дамыған бөлімдерінің бірі болып саналады және онда жеке позиция алады. Қазіргі уақытта алгебра мен модельдер теориясының классикалық бөлімдеріне экзистенциалды тұйықталу теориясының әдістерін және француз логигі Фрезенің революциялық идеяларын және экзистенциалды тұйықталудың жаңа ыңғайлы өлшемдерін, модельдің толықтығы мен жағдайын іздеу – «мәжбүр болу – модель» маңызды саналады. Қазіргі уақытта экзистенциалды және позитивті экзистенциалды тұйықталу теориясын және мәжбүрлі және индуктивті серіктес сияқты маңызды ұғымдарды зерттеуді одан әрі дамытуға мүмкіндік туады.

Зерттеуде өте аз құрылымдардың тұйықталу серіктесінің кейбір қасиеттері қарастырылған. Сондай-ақ, кез-келген күшті минималды құрылымның экзистенциалды тұйықталу өлшемі анықталды және кез-келген күшті минималды құрылымның экзистенциалды тұйықталу серіктесінің өте аз екендігі дәлелденді. Сондай-ақ, алынған сипаттамадан белгілі бір минималды құрылымның барлық тұйықталған серіктестері аксиоматизацияланатын сынып құрайды, олардың элементарлық теориясы толық және модельдік болып саналады, сондықтан оның индуктивті және мәжбүрлі серіктестерімен сәйкес келеді. Бейресми түрде берілген құрылымның кез-келген серіктесі бастапқы құрылымдағыдай ақырғы құрылымдардан жасалады. Белгілі құрылымның барлық серіктесі аксиоматизацияланатын сыныпты құрайтыны белгілі және сыныптағының барлығы экзистенциалды түрде тұйық. Біршама минималды құрылымды бар серіктес-сынып жеткілікті қарапайым және бірқатар қосымша қасиеті бар деп болжауға болады.

Еркін минималды теорияның экзистенциалды тұйықталу өлшемі анықталды. Бұл жағдайда мұндай құрылымның тұйықталу серігі аксиоматизацияланатын және оның индуктивті және мәжбүрлі серіктерімен сәйкес келетіні айқындалды. Күшті минималды теориялар маңызды және ең көп зерттелген теориялардың бірі – шексіз категориялық теория болып саналады. Сонымен бірге, қолжетімді әдебиеттерде олардың кванторлық санының күрделілігін бағалау да, олардағы қатынастардың күрделілігін бағалау мүлде жоқ. Мұнда ұсынылған зерттеулер «серіктес» ұғымын осы мәселелерді шешуде сәтті қолдануға болатындығын көрсетеді.

Жалпы индуктивті және экзистенциалды тұйықталған серіктестер теориясын дамыту индуктивті теория, классикалық құрылымдар мен теорияларға арналған модельдерді зерттеумен қатар жүруі керек.

**Түйін сөздер:** экзистенциалды тұйықталған серіктес, аса минималды құрылым, форсинг-серіктес.

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### **ДВА СВОЙСТВА ЭКЗИСТЕНЦИАЛЬНО ЗАМКНУТЫХ КОМПАЬОНОВ СИЛЬНО МИНИМАЛЬНЫХ СТРУКТУР**

**Аннотация.** Имеющая более чем шестидесятилетнюю историю теория экзистенциальной замкнутости, являясь одним из наиболее развитых разделов современной теории моделей, занимает в ней обособленное положение. В настоящее время актуальным становится поиск методов теории экзистенциальной замкнутости классическим разделам алгебры и теории моделей и использования для достижения этого революционных идей французского логика Фрэсе и найденные автором данной статьи А.Т.Нуртазиным новых удобных критериев экзистенциальной замкнутости, модельной полноты и условия – быть форсинг-моделью. В настоящее время становится возможным дальнейшее развитие теории экзистенциальной и позитивной экзистенциальной замкнутости и изучение важных сопутствующих понятий таких, как форсинг – и индуктивный компаньоны.

В исследовании рассматриваются некоторые свойства экзистенциально замкнутых компаньонов сильно минимальных структур. Также найден критерий экзистенциальной замкнутости произвольной сильно минимальной структуры и доказано, что экзистенциально замкнутый компаньон любой сильно минимальной структуры сам сильно минимален. Из полученного описания также следует, что все экзистенциально замкнутые компаньоны данной сильно минимальной структуры образуют аксиоматизируемый класс, элементарная теория которого полна и модельно полна и, следовательно, совпадает с её индуктивным и форсинг-компаньонами. Неформально любой компаньон данной структуры строится из тех же конечных структур, что и исходная структура. Известно, что все компаньоны данной структуры образуют аксиоматизируемый класс, а все экзистенциально замкнутые в этом классе содержатся в нём. Естественно предположить, что компаньон-класс, содержащий некоторую, сильно минимальную структуру достаточно прост и обладает рядом дополнительных свойств.

В работе найден критерий экзистенциальной замкнутости произвольной сильно минимальной теории. Оказалось, что в этом случае экзистенциально замкнутый компаньон такой структуры аксиоматизируем и совпадает с его индуктивным и форсинг-компаньонами. Сильно минимальные теории являются базисными при исследовании важного и одного из наиболее изученных в настоящее время класса теорий – теории несчётной категоричности. Между тем в имеющейся литературе совершенно отсутствуют как оценки их кванторной сложности, так и оценки сложности определяемых в них отношений. Предлагаемое здесь исследование показывает, что для решения этих вопросов можно успешно применить предложенное понятие «компаньон».

Развитие общей теории индуктивных и экзистенциальных замкнутых компаньонов должно сопровождаться исследованием индуктивных теорий и моделей для классических структур и теорий.

**Ключевые слова:** экзистенциально замкнутые компаньоны, сильно минимальная структура, форсинг-компаньон.

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## **CLASSIFICATION OF SEISMIC PHASES BASED ON MACHINE LEARNING**

**Abstract.** In the course of recent years, progresses in sensor innovation has lead to increments in the interest for automated strategies for investigating seismological signals. Fundamental to the comprehension of the components creating seismic signals is the information on the phases of seismic waves. Having the option to indicate the kind of wave prompts better performing seismic forecasting frameworks. In this article, we propose another strategy for the characterization of seismic waves quantification from a three-channel seismograms. The seismograms are isolated into covering time windows, where each time-window is mapped to a lot of multi-scale three-dimensional unitary vectors that portray the direction of the seismic wave present in the window at a few physical scales. The issue of arranging seismic waves gets one of ordering focuses on a few two-dimensional unit circles. We take care of this issue by utilizing kernel based machine learning that are remarkably adjusted to the geometry of the circle. The grouping of the seismic wave depends on our capacity to gain proficiency with the limits between sets of focuses on the circles related with the various kinds of seismic waves. At each signal scale, we characterize a thought of vulnerability connected to the order that considers the geometry of the dissemination of tests on the circle. At long last, we join the grouping results acquired at each scale into a unique label.

**Key words:** Seismology, machine learning, AI method, seismic waves, time window, three-channel seismograms, polarization content.

### **1 Introduction**

Seismology is a field of study centered on expanding the comprehension of the Earth's internal structure through the investigation of Earth's inner changes through time. There is a solid correlation between seismology and the sound wave analysis [1-7]. A seismic wave is produced at a source, moves through a medium, and could be monitored by recording tools. Correspondingly, a sound wave is created by a source (for example a car signal), moves through the air, and could be heard by the human ear. The investigation of such signals can give data on the area of the source and the medium through which the wave has moved.

In the course of recent years, the examination of seismic signals has prompted a more profound comprehension of the development of our planet, permitted countries to investigate domains for underground regular assets, and given information valuable to alleviating the impacts of seismic earthquakes on the human populace [8-11]. The core interest of this development are techniques for examining seismic signals to obtain data valuable in relieving the impacts of earthquakes on society. These techniques may prompt advances in Early Seismic Warning Systems (ESWS) innovation and give an alternate point of view on the investigation of seismic waves. Specifically, we present a methodology for the classifying seismic phases utilizing AI methods.

## 2 Feature Extraction

Given the data set it may seem that it is advantageous to work with a changed adaptation of that information. In these cases, it is possible to consider measures taken from the information to be *features*. Features are helpful to cast the data from an alternate approach and guaranteeing that what is being studied is depicted by just its most appropriate segments. A single illustration of the benefit of extracting features from data originates from the investigation of musical genres. In the aforementioned case the processing file consists of fragments from melodic soundtracks coming from structure distinctive musical genres, for example, rock, traditional, electronic, hip-hop, and so on. The objective might be to take a melodic soundtrack for which the genre isn't known and decide in which kind it best "fits". Rather than working with the melodic time arrangement, it is more valuable to work with features that depict the track, for example, lumber, beat, recurrence substance, and zero intersections, to give some examples. By changing the information, it is at times more obvious structures in the information that lead to better characterization results.

Our examination of the seismogram is performed on a sliding time windows of the three- segment seismogram  $[X_E(t) X_N(t) X_Z(t)]^T$ ,  $t = 0, 1, \dots$  (see Fig. 1). We structure the matrix  $\mathbf{X}$  by gathering  $T$  tests of the seismogram and stacking them into a  $T \times 3$  grid

$$\mathbf{X} = \begin{bmatrix} X_E(t) & X_N(t) & X_Z(t) \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ X_E(t+T-1) & X_N(t+T-1) & X_Z(t+T-1) \end{bmatrix} \quad \mathbf{t} = \mathbf{0}, \mathbf{1}, \dots \quad (1)$$

It is important to understand however: the matrix  $\mathbf{X}$  is actually a function of the time  $t$  at which we separate the time window. To mitigate the notations when there is no uncertainty, we decide not to make this reliance unequivocal. At the point when we think about two unique occasions  $t$  and  $t'$ , or two distinct seismograms, we use subscripts to separate between the time windows, for example  $\mathbf{X}_1, \mathbf{X}_2$ . Overall, we use subscripts all through this proposal to demonstrate that the matching vectors, or networks, have been separated from various seismograms or at various occasions.

### 2.1 Geometric Polarization Analysis

In order to describe the polarization content in the time window  $\mathbf{X}$  (1), we propose to break down the seismic waveform  $\mathbf{X}$  into a number of components that describe the Earth movement at various scales. For each scale, we obtain the primary vector of the Earth movement at that scale and utilize this data as the classifier input. In this segment of the article, we show the multi-scale examination of the matrix  $\mathbf{X}$ .

We decompose every one of the three sections of  $\mathbf{X}$  with a  $l$ -level stationary wavelet transform (where  $l \leq \log_2(T)$ ). The stationary wavelet decomposition is a redundant transform: we get  $l \times T$  coefficients for every one of the three orientations of the seismogram. Luckily, there exists a quick algorithm to obtain the stationary wavelet decomposition: the "a trou" calculation [26].

Figure 1 shows a seismic signal (upper left), its spectrogram (bottom left), and the stationary wavelet transform coefficients (right). The stationary wavelet transform can identify the second and third seismic waves, while the spectrogram scarcely changes when the waves show up (see Fig. 1-bottom left). Since seismograms can be estimated with extremely high accuracy utilizing few wavelet coefficients ([3, 16]), the wavelet transform is more qualified than a brief timeframe Fourier transform to identify seismic rushes of little plentifulness, as appeared in this model. Left:  $Z$  channel of a seismic signal (top) and spectrogram (bottom). The appearance times of three seismic waves are set apart by vertical bars. Note that the second and third waves scarcely cause any adjustments in the spectrogram. Right: stationary wavelet transform of the waveform given in left. The magnitude is color based and shown as a component of time, from fine scale (top) to coarse scale (bottom).

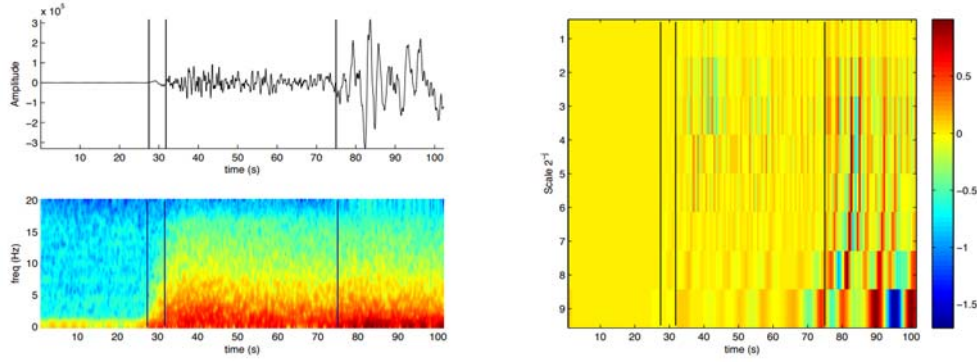


Figure 1 - A seismic signal

The result of the stationary wavelet examination of  $\mathbf{X}$  at scale  $j$  can be shown as a  $T \times 3$  network  $\mathbf{W}^j$  given by

$$\mathbf{W}^j = \begin{bmatrix} W_E^j(0) & W_N^j(0) & W_Z^j(0) \\ \vdots & \vdots & \vdots \\ W_E^j(T-1) & W_N^j(T-1) & W_Z^j(T-1) \end{bmatrix} \quad j = 1, \dots, l \quad (2)$$

where every column is the stationary wavelet transform of the matching column in  $\mathbf{X}$ . We pick  $l$  to be the most extreme scale permitted by the size of the signal ( $T$ ) and the size of the channel. The matrix  $\mathbf{W}^j$  encodes the movement of the Earth – estimated between time  $t$  and  $(t + T - 1)$  – in every one of the three orientations at scale  $j$ .

For the purposes of discovering the orientation related to the main energy at scale  $j$ , we process the singular value decomposition of  $\mathbf{W}^j$ ,

$$\mathbf{W}^j = \begin{bmatrix} \mathbf{U}^{j,1} & \mathbf{U}^{j,2} & \mathbf{U}^{j,3} \end{bmatrix} \begin{bmatrix} \sigma^{j,1} & 0 & 0 \\ 0 & \sigma^{j,2} & 0 \\ 0 & 0 & \sigma^{j,3} \end{bmatrix} \begin{bmatrix} [\mathbf{V}^{j,1}]^T \\ [\mathbf{V}^{j,2}]^T \\ [\mathbf{V}^{j,3}]^T \end{bmatrix} \quad (3)$$

where  $\sigma^{j,1} \geq \sigma^{j,2} \geq \sigma^{j,3}$ , and  $\|\mathbf{U}^{j,i}\| = \|\mathbf{V}^{j,i}\| = 1$ ,  $i = 1, 2, 3$ . Furthermore, the  $T$ -dimensional vectors  $\mathbf{U}^{j,1}$ ,  $\mathbf{U}^{j,2}$  and  $\mathbf{U}^{j,3}$  are orthogonal, and the 3-dimensional vectors  $\mathbf{V}^{j,1}$ ,  $\mathbf{V}^{j,2}$  and  $\mathbf{V}^{j,3}$  are also orthogonal. The vector  $\mathbf{V}^{j,1}$  is known as the polarization vector in the seismic literature. At each scale  $j$ , we only retain  $\mathbf{V}^{j,1}$  (and we discard all the remaining vectors), and we denote it by  $\mathbf{v}^j$ ,

$$\mathbf{v}^j = \mathbf{V}^{j,1} \quad (4)$$

We map the  $T \times 3$  network  $\mathbf{X}$  to the  $3 \times 1$  multiscale polarization lattice given by,

$$\mathbf{X} \mapsto [\mathbf{v}^1 \quad \dots \quad \mathbf{v}^l] \quad (5)$$

This map obtains the orientation of maximal polarization over various frequency groups. Basically, it is a sifting of the polarization vectors over various regions in the recurrence range. The wavelet channel utilized is significant and based on the channel the nature of the polarization sifting will change. Wavelet filters that are orthogonal, and have linear phase, are assumed to perform better than non-orthogonal wavelets. Figure 2 shows feature extraction results. Left: stationary wavelet transforms for a few  $\mathbf{X}_i$ ,  $i = 1, \dots$  superimposed on each other; top two lines: scaling capacity ( $W^1$ ), center two columns: coarsest (biggest scale) wavelet ( $W^2$ ), bottom two lines: next better scale wavelet ( $W^3$ ). Right: the vectors  $\mathbf{v}_i^j$  are spoken to as focuses on the sphere; each column compares to a similar scale  $j$  as the two plots on the left ( $j = 1, 2, 3$  start to finish). The shading and state of  $\mathbf{v}^j$  encode the sort of wave: red star for P, blue cross for L, and green sphere for testing information.

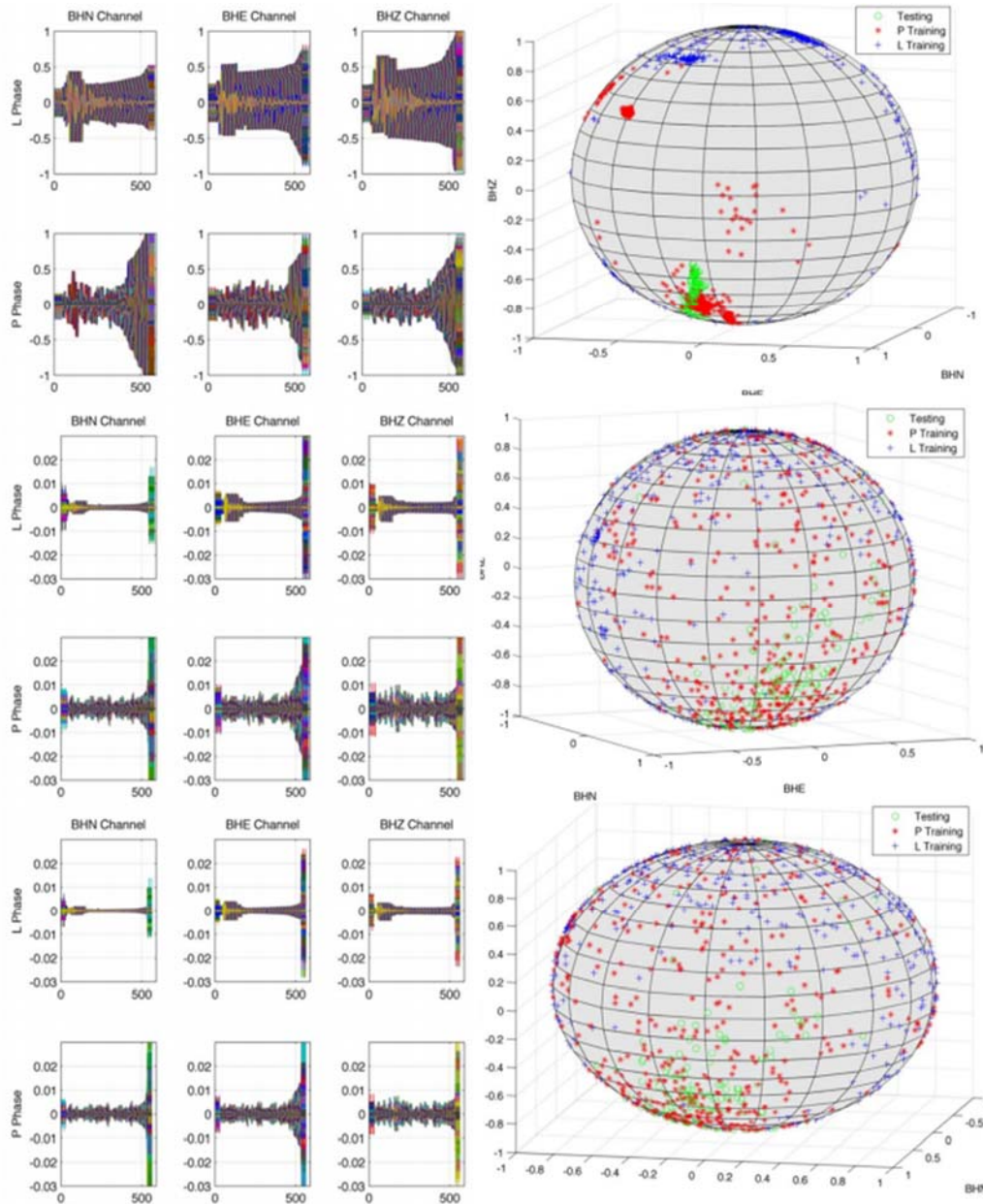


Figure 2 - Feature extraction results

## 2.2 Physical Interpretation

Each point of the sphere is tied to one-time window  $X$  over different levels of wavelet decomposition. This figure shows shading coded training and testing sets. The training set is made of marked stage classes, (blue-L stage and red-P stage), and the testing set is made of seismic windows of obscure stage.

Our first perception is that on every decomposition scale, points from comparable stage classes group close to one another and the concentration appears to decrease as the decay scale gets coarser. The grouping proposes possibility to characterize seismic waves as per a closeness to neighbors on the sphere. Figure 2 - Left shows a model where the plotted arrangement of testing focuses in green which are genuinely P-waves and watch they will in general bunch close to the P-wave preparing focuses.

The physical understanding of the polarization highlight vector is identified with the course of maximal signal energy and has been utilized to depict the orientation of predominant polarization in the seismology literature as mentioned before. For example, consider the following test. Assume you are blindfolded and need to find the position an individual who is conversing with you. At the point when that individual talks, you will in general turn toward the path where the sound is most grounded to give you a

thought of where the individual is standing. Comparably, the polarization highlight vector gives comparable data about the seismic wave. The bearings implanted on the sphere  $S^2$  comprise our element space.

### 3 Seismic Phase Learning

#### 3.1 Supervised Learning

“Learning is defined as acquiring new or modifying existing knowledge, behaviors, skills, values, or preferences and may involve synthesizing different types of information” [26]. Deep learning endeavors to learn examples and regularities in data. A well known model is that of proposal matrices, which endeavor to give recommendations dependent on past encounters. For example, when clients give criticism, regarding different preferences of a tune, on web radio destinations, proposal matrices can take these contributions to recommend new melodies that might be engaging the client. The way toward obtaining data from the information gave and performing activities the data discovered sums up the premise of deep learning.

All the more explicitly, there is a class of deep learning undertakings that gains from information that has been completely marked. This is known as supervised deep learning, or managed learning. This approach will be the focal point of this part and will be introduced from the viewpoint of learning the period of an unlabeled seismic wave.

#### 3.2 Classifiers

We expect that our preparation set is made out of  $N$  time windows  $\mathbf{X}_i$ ,  $i = 1, \dots, N$ . From every  $\mathbf{X}_i$ , we figure the  $l$  solitary vectors  $v^1_i, \dots, v^l_i$ . We will develop at each scale  $j$  a capacity  $f^j$  that maps a testing point  $v^j$ , with an obscure name, from the sphere  $S^2$  to the span  $[-1, 1]$ ,

$$f^j: v^j \in S^2 \mapsto f^j(v^j) \in [-1, 1]$$

We assessed three distinctive arrangement strategies (three unique sorts of  $f_j$ ) utilizing the part depicted in segment 3.3.1: kernel ridge regression, kernel support vector machines, and  $k$ -nearest neighbors. We utilize a similar sort of characterization strategy  $f_j$  for all the scales  $j = 1, \dots, l$ . Just the parameters of each capacity  $f_j$  differ over the scales. Further on, we depict the three distinct methodologies. Extra insights concerning the executions of these methods can be found in ([12,13] and references in that).

#### 3.3 Learning on the Sphere

Figure 2-left shows the yield of the stationary wavelet transform (plotted on each other) for a few time windows  $\mathbf{X}_i$  removed from various seismograms [14-18]. The main two columns show the scaling capacity coefficients ( $\mathbf{W}^1$ ) for the L and P waves, separately. The subsequent two lines show the coarsest (biggest scale) wavelet coefficients ( $\mathbf{W}^2$ ), and the third two columns show the following smaller scale wavelet coefficients ( $\mathbf{W}^3$ ). Figure 2-right shows the area of each  $v^j_i$  related with the time window  $\mathbf{X}_i$ . The shading and state of the spot speaking to  $v^j_i$  on the sphere encodes the kind of wave: red star for P, blue cross for L. The green spheres demonstrate the area of testing information for which we don't have the foggiest idea about the kind of wave [19-23]. The green spheres should be named red stars (P waves) or blue crosses (L waves). Notwithstanding the way that the  $\mathbf{X}_i$  are obtained from various seismograms estimated at various stations, the  $v^j_i$  normally bunch together (see for example  $v^1_j$  in the top line of Figure 2-right). We likewise see that the homogeneity of the conveyance of the  $v^j_i$  shifts as an element of the scale  $j$ , demonstrating that a few scales will be more helpful than others to arrange the time windows  $\mathbf{X}_i$ .

The genuine trouble here is that the standard Euclidean separation between two vectors  $v^j_1$  and  $v^j_2$ , beginning from two distinctive time windows  $\mathbf{X}_1$  and  $\mathbf{X}_2$ , is insubstantial in this specific circumstance. For the situation where focuses are inspected from a surface, or all the more for the most part a complex, we have to gauge separations utilizing the geodesic separation characterized on the complex. Then again, we can build an implanting of the complex into  $R^m$  that ideally saves separations (for example bi-Lipschitz), and measure separates in  $R^m$ .

For our situation, we approach a shut structure articulation for the geodesic separation and are in this way ready to represent the nonlinear structure of the element space to arrange the vectors  $v^j_i$ . We note that when the genuine geodesic separation isn't open, an estimate to the geodesic separation is generally near ideal. For example, Turaga et al. (2008) indicated that the Procrustes approximations to the geodesic separation on the Stiefel and Grassmann manifolds yield results that are near ideal for different issues

including the estimation of model parameters in dynamic matrix, movement acknowledgment, and video-based face acknowledgment [25]. In any case, the calculation of the geodesic separation may end up being over the top expensive. Sommer et al. (2010) indicated that the addition in precision accomplished with the genuine geodesic didn't exceed the calculation cost when the specific geodesic was contrasted with a direct estimation with regards to Principal Geodesic Analysis [24, 27-31].

### 3.4 Phase Classification on the Sphere

The grouping of a period window  $\mathbf{X}$  depends on a preparation set of named information to in part populate the spheres, at all scales  $j = 1, \dots, l$ , with data about the kind of waves at the relating areas on the spheres (see figure 2-right). We join the data given by the preparation marks with the information about the geometry of the sphere to become familiar with a capacity that depicts the limit between P-waves and L-waves. In this work, we assessed three distinctive regulated learning strategies to group the vectors  $v_i^j$ : portion edge relapse, part bolster vector machines, and k-closest neighbors. The key part is the meaning of a measurement and its related portion to evaluate vicinity on the sphere to in the end consolidate the characterization results at all scales to produce a mark.

In the accompanying sub-segments, we depict the order of the vector  $v_j$  at a given scale  $j$ . We at that point propose a data hypothetical measure to consolidate mono-scale characterization scores into a last grouping outcome.

## 4. Results

### 4.1 Evaluation Strategy

As a seismic tremor arrives at the RMSN, it is recorded put away for future examination. Over the system, a seismic tremor might be able to be detected at one station yet not at another. For example, a sensor at a chronicle station might be down for fixes which in this way, comprises a botched account chance.

In the regulated learning worldview, one must ensure a particular classifier isn't prepared with information that will be utilized for testing the given classifier. At the point when information is constrained, one must think about elective methodologies in surveying the viability of a calculation. In these cases, we utilize the strategies for cross-approval to assess the exhibition of the learning methods. These procedures ordinarily save a part of the general information for preparing and afterward utilize the rest of the information for testing. The segments held for testing and preparing are rotated, bringing about a  $n$ -overlap cross-approval, where  $n$  is the occasions the classifier is tried and prepared.

For the arrangement of seismic stages, we utilized a cross-approval technique that utilizes seismic information gathered over the full system. In an arrangement run, we are utilizing a subset of the informational collection, where each recording station in the system was utilized in gathering the information. The main imperative for a given run is that information from a seismic tremor isn't gathered at various chronicle stations in the system. In a characterization run, we work with 10 unmistakable seismic tremors estimated some place in the system. During cross-approval, we forget about one tremor seismogram and utilize the staying 9 seismograms to prepare our classifier. For instance, if we somehow managed to have a similar tremor show up twice at various detecting stations in our system, this grouping would be considered as cheating in light of the fact that a quake radiating from some source estimated at two unique stations will vary just by a direct change. The straight change would be incited on the signal because of the nearby geography of the account station. An elective methodology for cross-approval is use information just gathered at a given station. In spite of the fact that this would be a legitimate methodology, it isn't attainable in our examination because of information amount restrictions

### 4.2 Comparison of approaches

Table 1 shows test results under the methodology proposed by Jackson et al. (1991). It shows the level of time windows for which the speculation  $H_0$ , "X contains a P wave" was acknowledged as a component of the test edge  $\eta$ . These outcomes relate to our execution of the calculation of [15] on our dataset. At the point when the edge  $\eta = 0.30$ , the Lg are mistakenly characterized 33.25% of the time and thusly are effectively ordered 66.75% of the time. This is a sensible location level of the Lg waves. Sadly, a similar estimation of the edge for the invalid hypothesis was just acknowledged 23.62% and 44.56% of the ideal opportunity for the Pg and Pn waves, individually. By dismissing the invalid speculation, the classifier misses the P waves practically constantly, in this way yielding poor order of Pn and Pg waves.

Diminishing  $\eta$  surely helps the recognition of the P waves, yet comes at the cost of noteworthy misclassifications of the Lg waves in that the invalid speculation is acknowledged when it ought not be. In any case, our methodology had the option to identify with a similar precision both P and L waves.

Detection of the P and L waves using a hypothesis test.

$\eta$	$\Pr(H0 Lg) > \eta$	$\Pr(H0 Pg) > \eta$	$\Pr(H0 Pn) > \eta$
0.50	0.00	0.00	0.00
0.40	12.29	8.11	20.46
0.30	33.25	23.62	44.56
0.20	54.24	44.43	64.89
0.10	70.09	66.51	86.88
0.05	76.09	75.98	93.31

## 5 Conclusion

Our objective in this work was to investigate the utilization of AI as it applies to seismology. Specifically, we found a change that takes windows of three-channel seismic information and implants them on the unit circle over various recurrence scales with the end goal that the area of the installing portrays the seismic stage content in the wave. The utility of this change is focused on the stage grouping in the component space. Basically, windows of seismic information relating to the equivalent seismic stage class will in general be genuinely situated close to one another in the element space. Having this kind of structure over the component space at various degrees of sign goals gives a solid establishment to the use of regulated learning methods for stage grouping. Furthermore, the basic geometric structure of the component space takes into account administered learning strategies to be made an interpretation of straightforwardly to the element space. Fundamentally, we have told the best way to "lift" learning strategies to a non-straight complex.

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## МАШИНАЛЫҚ ОҚЫТУ НЕГІЗІНДЕГІ СЕЙСМИКАЛЫҚ ФАЗАЛАР КЛАССИФИКАЦИЯСЫ

**Аннотация.** Соңғы бірнеше жыл ішінде сенсорлық технологияның дамуы сейсмологиялық сигналдарды талдауға арналған компьютерлік әдістерге сұранысты арттырды. Сейсмикалық сигналдарды тудыратын механизмдерді түсінудің негізі – сейсмикалық толқындар фазасын жан-жақты білу мәнін білдіреді. Толқын түрін анықтау мүмкіндігі сейсмикалық ертерек ескерту жүйесінің жақсы жұмыс істеуіне септігін тигізеді.

Бұл жұмыста ұшарналы сейсмограммамен өлшенген сейсмикалық толқындарды жіктеудің жаңа әдісін ұсынамыз. Сейсмограммалар бір-бірімен қиылысатын уақытша терезелерге бөлінген. Мұнда уақытша әр терезе бірнеше физикалық масштабтағы терезеде орналасқан сейсмикалық толқын бағытын сипаттайтын көп масштабты, үшөлшемді унитарлық векторлар жиынтығымен көрсетіледі. Сейсмикалық толқындарды жіктеу міндеті екіөлшемді бірлік сфералары бойынша жіктеу нүктелерінің біріне айналады. Біз бұл есепті сфера геометриясына бейімделген машиналық оқыту әдістерін қолдану арқылы шешеміз. Сейсмикалық толқынның жіктелуі түрлі сейсмикалық толқындарға байланысты сфералардағы нүктелер жиынтығы арасындағы шекараны зерттеу қабілетіне негізделген. Әр сигнал шкаласында біз классификацияға қолданылатын белгісіздік ұғымын анықтаймыз, ол сфера бойынша үлгінің таралу геометриясын ескереді. Соңында әр шкала бойынша алынған жіктеу нәтижелерін ерекше белгіге біріктіреміз.

Сейсмология – уақыт өте келе жердегі ішкі өзгерістерді зерттеу арқылы жердің ішкі құрылымы туралы түсініктерді кеңейтуге бағытталған зерттеу саласы. Сейсмология мен дыбыстық толқындарды талдау арасында тығыз байланыс бар. Сейсмикалық толқын шығу көзінде пайда болады да, орта арқылы өтіп, оны

жазба құралдарымен басқаруға болады. Тиісінше дыбыстық толқын шығу көзі арқылы жасалады (мысалы, автомобиль сигналы), ауа арқылы қозғалады да, адам құлағына естіледі. Мұндай сигналдарды зерттеу көздің ауданы мен толқын өткен орта туралы мәліметтер бере алады.

Соңғы жылдары сейсмикалық сигналдарды зерттеу планетамыздың дамуын тереңірек түсінуге әкелді. Көптеген елдерге жерасты тұрақты активтері бар аймақтарды зерттеуге мүмкіндік берді және сейсмикалық жер сілкіністерінің адам санына әсерін азайту үшін құнды ақпарат берді. Бұл дамудың басты қызығушылығы – жер сілкінісінің қоғамға әсерін азайту мен құнды мәліметтер алу үшін сейсмикалық сигналдарды зерттеу әдістері болып саналады. Бұл әдістер ерте сейсмикалық ескерту жүйесінің (ESWS) инновациясындағы прогресті ынталандырады және сейсмикалық толқындарды зерттеудің балама перспективасын ұсынады. Атап айтқанда, AI әдістерін қолдану арқыл сейсмикалық фазаларды жіктеудің әдістемесін ұсынамыз.

Жұмыстағы мақсатымыз – AI әдістерін сейсмологияда қолдануды зерттеу. Атап айтқанда, ұшарналы сейсмикалық ақпарат терезелерін алатын және түпкілікті мақсаты бар түрлі қайталану ауқымында оларды бір шеңберге орналастыратын өзгерісті таптық, осылайша орнату аймағы сейсмикалық кезеңнің мазмұнын толқынмен көрсетеді. Бұл өзгерістің тиімділігі компоненттер кеңістігіндегі кезеңдерді топтауға бағытталған. Жалпы алғанда, сейсмикалық кезеңдердің эквиваленттік класына қатысты сейсмикалық ақпарат терезелері, жалпы жағдайда, элементтер кеңістігінде бір-біріне жақын орналасады. Өртүрлі деңгейдегі символдық мақсаты бар компоненттер кеңістігінде осындай құрылымның болуы топтық кезең үшін оқытудың реттелетін әдістерін қолдануға сенімді негіз береді. Сонымен қатар, компоненттік кеңістіктің негізгі геометриялық құрылымы элементтер кеңістігіне тікелей түсіндірілуі тиіс басшылыққа алынған оқыту стратегияларын ескереді. Шындығында біз оқыту стратегиясын кешенді түрде қалай «көтеру» керектігі туралы баяндадық.

**Түйін сөздер:** сейсмология, машиналық оқыту, AI әдісі, сейсмикалық толқындар, уақытша терезе, ұшарналы сейсмограммалар, поляризация мазмұны.

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### **КЛАССИФИКАЦИЯ СЕЙСМИЧЕСКИХ ФАЗ НА ОСНОВЕ МАШИННОГО ОБУЧЕНИЯ**

**Аннотация.** За последние несколько лет достижения в области сенсорных технологий привели к увеличению спроса на компьютеризированные методы анализа сейсмологических сигналов. Центральным в понимании механизмов, генерирующих сейсмические сигналы, является знание фаз сейсмических волн. Возможность указать тип волны приводит к лучшему функционированию сейсмических систем раннего предупреждения.

В данной работе мы предлагаем новый метод классификации сейсмических волн, измеренных по трехканальным сейсмограммам. Сейсмограммы разделены на перекрывающиеся временные окна, где каждое временное окно отображается на набор многомасштабных трехмерных унитарных векторов, которые описывают ориентацию сейсмической волны, присутствующей в окне, в нескольких физических масштабах. Задача классификации сейсмических волн становится одной из точек классификации на нескольких двумерных единичных сферах. Мы решаем эту проблему, используя методы машинного обучения, которые адаптированы к геометрии сферы. Классификация сейсмических волн основана на нашей способности изучать границы между наборами точек на сферах, связанных с различными типами сейсмических волн. На каждой шкале сигнала мы определяем понятие неопределенности, приложенное к классификации, которая учитывает геометрию распределения выборок на сфере. Наконец, мы объединяем результаты классификации, полученные в каждой шкале, в уникальный ярлык.

Сейсмология – эта область исследований, сосредоточенная на расширении понимания внутренней структуры Земли путем исследования внутренних изменений Земли во времени. Существует тесная связь между сейсмологией и анализом звуковых волн. Сейсмическая волна генерируется у источника, проходит через среду и может контролироваться записывающими инструментами. Соответственно, звуковая волна создается источником (например, автомобильным сигналом), движется по воздуху и может быть услышана человеческим ухом. Исследование таких сигналов может дать данные о площади источника и среды, через которую прошла волна.



В течение последних лет изучение сейсмических сигналов привело к более глубокому пониманию развития нашей планеты, позволило странам исследовать области для подземных регулярных активов и предоставило информацию, ценную для смягчения воздействия сейсмических землетрясений на человеческое население. Основным интересом этой разработки являются методы изучения сейсмических сигналов для получения данных, ценных для уменьшения воздействия землетрясений на общество. Эти методы могут стимулировать прогресс в инновациях систем раннего сейсмического оповещения (ESWS) и давать альтернативную точку зрения на исследование сейсмических волн. В частности, мы представляем методологию классификации сейсмических фаз с использованием методов AI.

Наша цель в этой работе состояла в том, чтобы исследовать использование метода AI применительно к сейсмологии. В частности, мы нашли изменение, которое берет окна трехканальной сейсмической информации и имплантирует их в единичный круг по различным шкалам повторения с конечной целью, чтобы область установки отображала содержание сейсмической ступени в волне. Полезность этого изменения сфокусирована на группировке этапов в пространстве компонентов. В основном, окна сейсмической информации, относящиеся к эквивалентному классу сейсмических стадий, в общем случае будут действительно расположены близко друг к другу в пространстве элементов. Наличие такого рода структуры в пространстве компонентов с различными уровнями знаковых целей дает прочное основание для использования регулируемых методов обучения для групповой стадии. Кроме того, базовая геометрическая структура пространства компонентов учитывает управляемые стратегии обучения, которые должны быть прямо интерпретированы для пространства элементов. По сути, мы рассказали, как лучше всего «поднять» стратегии обучения в сложный комплекс.

**Ключевые слова:** сейсмология, машинное обучение, метод AI, сейсмические волны, временное окно, трехканальные сейсмограммы, поляризационное содержание.

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## ANALYSIS OF THE EQUATIONS OF STATE FOR NEUTRON STARS

**Abstract.** In this work we consider various equations of state of neutron star matter, which include from the point of neutron drops formation to supra nuclear densities. Particular attention is paid to the nucleon – nucleon interaction since, in addition to the kinetic energies of the particles, the interactions among nucleons play a key role. Moreover, we investigate the properties of super-dense matter with diverse sets of particles such as electrons, protons, and the contribution of various particles-carriers of interaction. In order to achieve these goals, different potentials were considered, which are in a good agreement with experimental data. Furthermore, we find the energy of the system by using a variety of multi-particle methods, including the interaction of nucleons. Thanks to this information, thermodynamic parameters such as pressure, energy density and the speed of sound in the star are calculated. We compared similar equations of state of matter so that we could demonstrate the difference from each other. The Tolman-Oppenheimer-Volkoff system of equations has been solved numerically to construct mass-central density, radius-central density and mass-radius relations using different equations of state. In conclusion, the latest observational constraints on the equation of state are taken into account and we show that the observational data require that the equation of state be stiff, despite the fact that all stiff equations of state violate the principle of causality at high central densities, unlike soft ones.

**Keywords:** Equation of state, neutron stars, nuclear interactions, observational constraints.

**1. Introduction.** First theoretical calculations on the properties of neutron stars were carried out at the beginning of the 20th century by Tolman and Oppenheimer. It was shown that with increasing mass of a star, the electron pressure is no longer able to oppose the gravitational compression. Whereas the electrostatic corrections do not make a large contribution to the pressure of the star in the descriptions of white dwarf stars and were often neglected, the interactions between nucleons become crucial as the matter density becomes close to the nuclear density. The attraction between the nucleons reduces the pressure, but the repulsion caused by vector carriers increases the pressure, thereby preventing the collapse of the star [1]. If the Fermi energy is large, then in addition to neutrons other particles are formed in the neutron star. The Baym equation of state (EoS), which includes neutrons, protons and electrons, describes well neutron star matter at densities below the formation of neutron drops  $\rho_{drip} = 4.3 \times 10^{11} \text{ g/cm}^3$  then it goes quite smoothly into the Baym-Bem-Petrik EoS [2], which in turn describes the state of matter within  $\rho_{drip} < \rho < \rho_{nuc}$ , where  $\rho_{nuc} = 2.8 \times 10^{14} \text{ g/cm}^3$  – nuclear density. Further, the results vary widely within the densities above the density of nuclear matter. This is the range we will investigate in this work. The structure of the paper is organized as follows: in sections 2 and 3, a neutron star is considered at zero temperatures for the EoS of a degenerate neutron gas. In sections 4, 5, and 6, different equations of state (EoS) are compared, taking into account the interactions between particles. In section 7, the main results are discussed and the corresponding conclusions are summarized.

**2. Pure neutron gas.** The pressure of a degenerate neutron gas is calculated in the so-called phase spaces. With an increase in the density of a neutron star, the uncertainty principle greatly increases the momentum phase space and the radius of the star decreases. Further, due to the gravitational instability, it will decrease to the gravitational radius  $r_g = 2GM/c^2$  where the inner structure of the star is destroyed and a black hole is formed. Here  $G$  is the gravitational constant,  $c$  is the speed of light in vacuum and  $M$  is the total mass of the star. We write the relativistic hydrostatic equilibrium equation, Tolman-Oppenheimer-Volkoff (TOV) equation, for a perfect fluid as:

$$\left\{ \begin{array}{l} \frac{dm(r)}{dr} = \frac{4\pi r^2}{c^2} \varepsilon(r) \\ \frac{dp(r)}{dr} = -\varepsilon(r) \frac{Gm(r)}{c^2 r^2} \left[ 1 + \frac{p(r)}{\varepsilon(r)} \right] \left[ 1 + \frac{4\pi r^3 p(r)}{c^2 m(r)} \right] \left[ 1 - \frac{2Gm(r)}{c^2 r} \right]^{-1} \end{array} \right. \quad (1)$$

where  $\varepsilon(r) = c^2 \rho(r)$  is the energy density of hadron matter,  $p(r)$  is the pressure,  $\rho(r)$  is the density,  $m(r)$  is the mass of matter inside a sphere enclosed within radius  $r$ . This is a system of first-order ordinary differential equations, the solutions of which should represent equilibrium configurations of a perfect fluid with density  $\varepsilon(r)$  and pressure  $p(r)$ . The TOV equation cannot be solved in its present form because it is an open system of differential equations. To close it, we must add an equation, which is usually given in the form of an EoS (EoS). Clearly, not every EoS can be used to generate physically reasonable solutions of the TOV equation. In fact, not every EoS can lead to an equilibrium configuration. In the case of a neutron star an appropriate EoS can be written in a parametric form as:

$$\left\{ \begin{array}{l} \varepsilon(r) = \frac{\varepsilon_0}{8} \left[ (2y(r)^3 + y(r))\sqrt{1+y(r)^2} - \ln(y(r) + \sqrt{1+y(r)^2}) \right] \\ p(r) = \frac{\varepsilon_0}{24} \left[ (2y(r)^3 - 3y(r))\sqrt{1+y(r)^2} + 3\ln(y(r) + \sqrt{1+y(r)^2}) \right] \end{array} \right. \quad (2)$$

where,  $y(r) = k_n(r)/m_n c$ ,  $k_n(r)$  is the dimensionless Fermi momentum of a neutron,  $\varepsilon_0 = m_n^4 c^5 / \pi^2 \hbar^3$  constant having the dimension of energy density [3]. We introduce the dimensionless energy density and pressure in the following form  $\varepsilon = \bar{\varepsilon} c^4 / Gb^2$ ,  $p = \bar{p} c^4 / Gb^2$ ,  $\rho = \bar{\rho} c^2 / Gb^2$ , where  $\bar{\rho}$  is the dimensionless density,  $\bar{p}$  is the dimensionless pressure,  $\bar{\varepsilon}$  is the dimensionless energy density. To make systems of equations (1) and (2) dimensionless we introduce the quantities  $r = bx$ ,  $m = \bar{m} c^2 b / G$ , where  $b = \pi \sqrt{\hbar^3 / Gc m_n^4}$  is a parameter with dimension of length, satisfying the equality  $\varepsilon_0 = c^4 / Gb^2$  [3],  $x$  is dimensionless radial coordinate, and  $\bar{m}$  is the dimensionless mass. Then, the EoS (2) reduces to

$$\left\{ \begin{array}{l} \bar{\varepsilon}(x) = \frac{1}{8} \left[ (2y(x)^3 + y(x))\sqrt{1+y(x)^2} - \ln(y(x) + \sqrt{1+y(x)^2}) \right] \\ \bar{p}(x) = \frac{1}{24} \left[ (2y(x)^3 - 3y(x))\sqrt{1+y(x)^2} + 3\ln(y(x) + \sqrt{1+y(x)^2}) \right] \end{array} \right. \quad (3)$$

and the structure equations (1) become

$$\left\{ \begin{array}{l} \frac{d\bar{m}(x)}{dx} = 4\pi x^2 \bar{\varepsilon}(x) \\ \frac{d\bar{p}(x)}{dx} = -\bar{\varepsilon}(x) \frac{\bar{m}(x)}{x^2} \left[ 1 + \frac{\bar{p}(x)}{\bar{\varepsilon}(x)} \right] \left[ 1 + \frac{4\pi x^3 \bar{p}(x)}{\bar{m}(x)} \right] \left[ 1 - \frac{2\bar{m}(x)}{x} \right]^{-1} \end{array} \right. \quad (4)$$

equations (3)-(4) describe the behavior of matter at densities below the formation of neutron droplets and above nuclear matter.

**3. Neutron stars with protons, electrons.** The pressure caused by protons and electrons in a neutron star is small, but it is still present and softens the EoS by slightly reducing the maximum mass. In order to

achieve equilibrium the electro-neutrality condition  $n_e = n_p$ , where  $n$  is the particle number density [4], along with a balance between reactions  $n \rightarrow p + e + \bar{\nu}_e$  and  $p + e \rightarrow n + \nu_e$  must be fulfilled within the star. Hence, the EoS is constructed in the same way as for degenerate noninteracting neutrons and has the following form [5]:

$$\begin{cases} \varepsilon(r) = \pi \sum_{i=n,p,e} \frac{m_i c^2}{\Lambda_i^3} \left[ (2y_i(r)^3 + y_i(r)) \sqrt{1 + y_i(r)^2} - \ln(y_i(r) + \sqrt{1 + y_i(r)^2}) \right] \\ p(r) = \frac{\pi}{3} \sum_{i=n,p,e} \frac{m_i c^2}{\Lambda_i^3} \left[ (2y_i(r)^3 - 3y_i(r)) \sqrt{1 + y_i(r)^2} + 3 \ln(y_i(r) + \sqrt{1 + y_i(r)^2}) \right] \end{cases} \quad (5)$$

where, the Compton wavelength of particles are  $\Lambda_n = 1.319 fm$ ,  $\Lambda_p = 1.321 fm$ ,  $\Lambda_e = 2.42 \times 10^3 fm$  and the rest masses are  $m_e c^2 = 0.511 Mev$ ,  $m_p c^2 = 938.272 Mev$ , and the dimensionless Fermi momenta for protons and electrons are given by

$$y_p(x) = \frac{\sqrt{m_e^4 + (m_p^2 - m_n^2(1 + y_n(x)))^2 - 2m_e^2(m_p^2 + m_n^2(1 + y_n(x)))}}{2m_n m_p \sqrt{1 + y_n(x)^2}}, \quad y_e = y_p \quad (6)$$

where for convenience we have denoted  $m_i c^2 = m_i$ . The Fermi momentum can be calculated from the condition that the lower threshold for the neutron formation at  $y_n(x) = 0$  is equal to  $y_p(x) = 0.001265$ ,  $y_e(x) = y_p(x)$ , and central density is  $\rho = 1.186 \times 10^7 g/cm^3$ .

By solving the TOV equations using (5) we get the mass-radius relation for a neutron star depicted in Fig. 1. Note that in the  $M - \rho$  diagram (right panel) the configurations to the left of  $M_{Max}$  are unstable and collapse into a black hole. A similar situation also occurs in the  $M - \rho$  diagram (left panel), only here unstable configurations are to the right of  $M_{Max}$ . It can be seen that the contribution of protons and electrons to the pressure or density and correspondingly to the mass is not significant.

Another feature of the EoS with non-interacting protons, neutrons and electrons is that with increasing Fermi energies, electrons can decay into muons [6], and muons in turn decay into electrons with the emission of neutrinos or anti-neutrinos. This is one of the few ways for cooling of a neutron star at high temperatures [1]. The minimum density at which muons are formed is  $\rho = 8.21 \times 10^{14} g/cm^3$ . At densities  $\rho = 1.36 \times 10^{15} g/cm^3$ ,  $T < T_c = 2.2 \times 10^{11} K$  pion condensate forms [7]. If we take into account that during the formation of a neutron star, the temperature still reaches a value  $T > T_c$ , but due to the neutrino cooling the temperature decreases from  $10^{11}$  until  $10^9 K$  in a month, then the formation of pion condensation in the core of stars is inevitable. It means that, in realistic models and in all EsoS, the curves should strongly move down after  $\rho \approx 10^{15} g/cm^3$ , since pion condensations soften the EoS by decreasing the maximum mass and are highly dependent on the model, though they have no contribution to pressure. As for other pions, for example neutral pions  $\pi^0$  in 98% cases decay into two gamma quanta  $\pi^0 \rightarrow 2\gamma$ , that is, there is an equilibrium state between  $\pi^0 \Leftrightarrow 2\gamma$  but the equation for the chemical potential gives  $\mu_{\pi^0} = 0$ , and for  $\mu_{\pi^+} = -\mu_e < 0$ , therefore, in both cases, the distribution function for  $T = 0K$  tends to  $f = \frac{1}{e^{(E-\mu)/kT} - 1} \rightarrow 0$ . The formation of such particles is not expected in superdense matter, at least not at low temperatures, for the same reason positrons, anti-baryons and other mesons must be absent in an ideal gas [4].

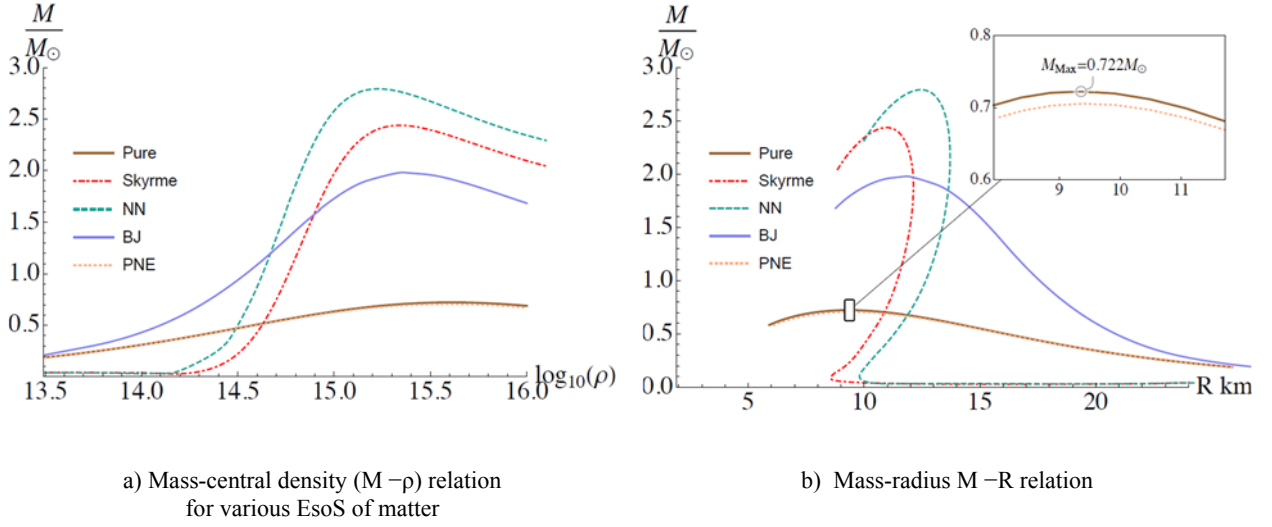


Figure 1 - The mass-central density (in  $\text{g}/\text{cm}^3$ ) and mass-radius relations, NN is the empirical EoS, Skyrme is for the skyrme potential, Pure is the degenerate neutron gas and BJ is the Bethe-Jones EoS. It can be seen that the maximum mass is decreased and is equal to  $M = 0.7057M_{sun}$  at  $R = 9.394 \text{ km}$  if we include electrons and protons into degenerate neutron gas

**4. Bethe-Johnson nuclear interaction.** Almost all EsoS of neutron stars are based on finding the energies of nucleon-nucleon interactions [8]. Nuclear scientists and theoreticians have spent plenty of time and effort studying nucleon-nucleon interactions. From interaction theories it is known that the attraction between nucleons is due to the exchange of scalar fields of pions, and repulsion by vector particles, in particular  $\rho$ ,  $\omega$ ,  $\varphi$  (where  $\rho$ ,  $\omega$ ,  $\varphi$  particle carriers of nuclear matter). Bethe and Johnson only considered repulsion from  $\omega$  ( $783 \text{ MeV}$ ), since it is this particle with nucleons that possesses the strongest coupling constant and is approximately estimated in  $g_{\omega}^2 = /\hbar c = 12.868$  [9], and most importantly, it describes well the experimental data on the binding energy of nucleons and the quadrupole moment of the deuteron. So using the Bethe and Jones potential, taking into account the variational method and considering the agreement with experiments, we obtain an EoS that relates density and pressure through the parametric equations [1]:

$$\begin{cases} \varepsilon = m_n n(x) + \frac{3(3\pi^2)^{2/3}(\hbar c)^2}{10m_n} n(x)^{5/3} + 236n(x)^{a+1} \\ p = n(x)^2 \frac{d(\varepsilon/n)}{dn} = \frac{(3\pi^2)^{2/3}(\hbar c)^2}{5m_n} n(x)^{5/3} + 363n(x)^{a+1} \end{cases} \quad (7)$$

where  $a = 1.54$ ,  $m_n = m_n c^2 \text{ MeV}$  and  $\hbar c = 197.327 \text{ MeV} \times \text{fm}$ . To make equation (7) dimensionless we multiplied by  $G b^2 / c^4 = 1.007 \times 10^{-4} \text{ fm}^3 / \text{MeV}$ . The speed of sound on the surface of a star is equal to

$$\left(\frac{c_s}{c}\right)^2 = \frac{dp}{d\varepsilon} = \frac{0.143n^{2/3} + 0.649n^a}{0.214n^{2/3} + n^a + 1.01} \quad (8)$$

where, the speed of sound is always  $c_s < c$  for any densities in the Bethe-Jones EoS, as well as in the case of a degenerate neutron gas or mixed with protons and electrons. Numerical solutions of the TOV equations are given in figures 1 and 2.

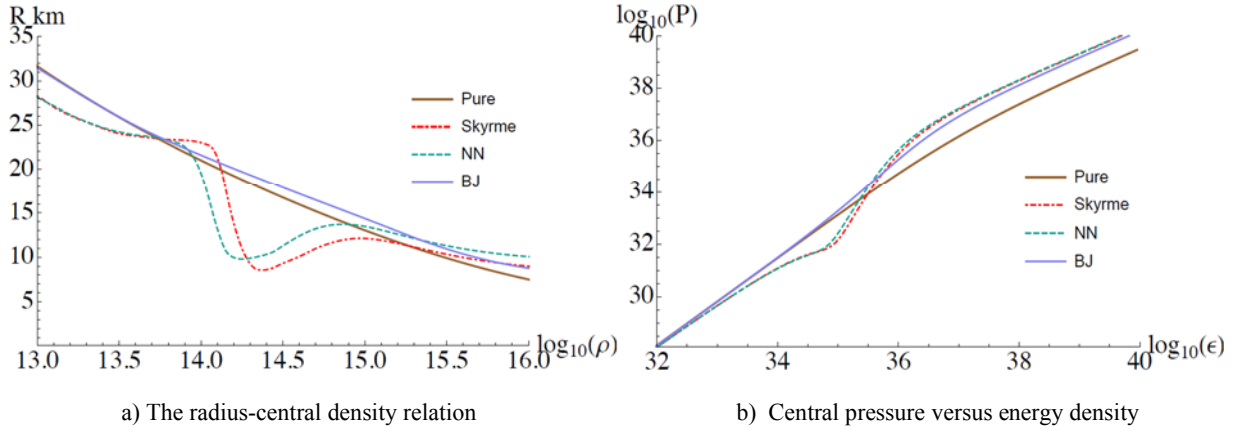


Figure 2 - Dependence of various parameters for the neutron star EoS

**5. Empirical nuclear interaction.** No matter how we choose the interaction potential between nucleons, it must be in agreement with the experimental data, that is, it must lead to the saturation of nuclear matter having satisfied 3 conditions: 1) the number density at which the saturation is achieved  $n_0 = 0.16 \text{ fm}^{-3}$ , 2) the binding energy at saturation  $BE = \frac{E}{A} \Big|_{n=n_0} = -16 \text{ MeV}$ , where  $BE$  is the binding energy, 3) the compressibility of the nuclear matter  $K(n_0) = 9 \frac{dp}{dn} = 400 \text{ MeV}$ , it is known from the experiment that the compressibility value changes depending on the potentials. In our case, a test function for the EoS of symmetric matter  $n_p = n_n$  has the following form:

$$\begin{cases} \varepsilon/n = m_n + \langle E_0 \rangle u^{2/3} + \frac{A}{2} u + \frac{B}{\sigma+1} u^\sigma \\ p/n_0 = \frac{2}{3} \langle E_0 \rangle u^{5/3} + \frac{A}{2} u^2 + \frac{B\sigma}{\sigma+1} u^{\sigma+1} \end{cases} \quad (9)$$

after substituting in the conditions for saturation of the nuclear matter, the constants are equal to  $A = -118.2 \text{ MeV}$ ,  $B = 65.39 \text{ MeV}$ ,  $\sigma = 2.112$ ,  $\langle E_0 \rangle = 22.1 \text{ MeV}$ . A model describing only neutron matter  $n = n_n$  is given by

$$\begin{cases} \varepsilon/n = m_n + 2^{2/3} \langle E_0 \rangle u^{2/3} + \left( \frac{A}{2} - (2^{2/3} - 1) \langle E_0 \rangle + S_0 \right) u + \frac{B}{\sigma+1} u^\sigma, \\ p/n_0 = \frac{2^{5/3}}{3} \langle E_0 \rangle u^{5/3} + \left( \frac{A}{2} - (2^{2/3} - 1) \langle E_0 \rangle + S_0 \right) u^2 + \frac{B\sigma}{\sigma+1} u^{\sigma+1} \end{cases} \quad (10)$$

where  $S_0$  is the volume symmetry coefficient (degree of deviation from symmetry),  $u = n/n_0$ , the range of applicability of the equation is from the order of the nuclear density to  $\rho = 1.135 \times 10^{15} \text{ g/cm}^3$  since beyond this value it will violate the principle of causality.

**6. Skyrme interaction.** In this section, we select a potential that gives a repulsive effect at high concentrations, since at high densities, which is equivalent to the reduction of the distance between nucleons, the repulsive force due to the exchange of vector particles dominates. Based on these arguments we choose the potential in the form of  $V(x-y) = \delta(x-y) \left( \frac{1}{6} t_3 n - t_0 \right)$ , where  $t_0$  is the parameter that

characterizes repulsion owing to the exchange of scalar particles between two nucleons,  $t_3$  is the parameter describing the repulsion at high densities. Next, we find the energy with help of the Hartree-Fock method, if the spin of particles is not considered (the calculation of the same potential using the

method of Hartree) we find that the energy increases twice. This is due to the fact that taking into account the spin of the particles reduces the total energy practical twice because in the first approximations only those pairs interact which have antiparallel spins. The EoS with spin reads as follows:

$$\begin{cases} \varepsilon = m_n n(x) + \frac{3(3\pi^2)^{2/3}(\hbar c)^2}{10m_n} n(x)^{5/3} - \frac{t_3}{24} n(x)^3 - \frac{t_4}{4} n(x)^2, \\ p = \frac{(3\pi^2)^{2/3}(\hbar c)^2}{5m_n} n(x)^{5/3} - \frac{t_3}{12} n(x)^3 - \frac{t_4}{4} n(x)^2, \end{cases} \quad (11)$$

where the constants  $t_3, t_0$  are found from the saturation condition for nuclear matter as well as for nucleon-nucleon interactions. After simple calculations one finds  $t_3 = 14600.8 \text{ MeV}$ ,  $t_0 = 1024.1 \text{ MeV}$ . The domain of applicability of the EoS is  $2.707 \times 10^{14} \text{ g/cm}^3 < \rho < 1.55 \times 10^{15} \text{ g/cm}^3$  [3].

**7. Observational constraint.** The physical constraints imposed on the EoS are known. This is primarily a restriction on the speed of sound, since the speed of sound in the core and on the surface of a star must fulfill the condition  $c_s/c < 1$ , in our cases, these conditions are not met for the skyrme EoS when  $\rho > 1.55 \times 10^{15} \text{ g/cm}^3$  and for the empirical nucleon- nucleon interaction at

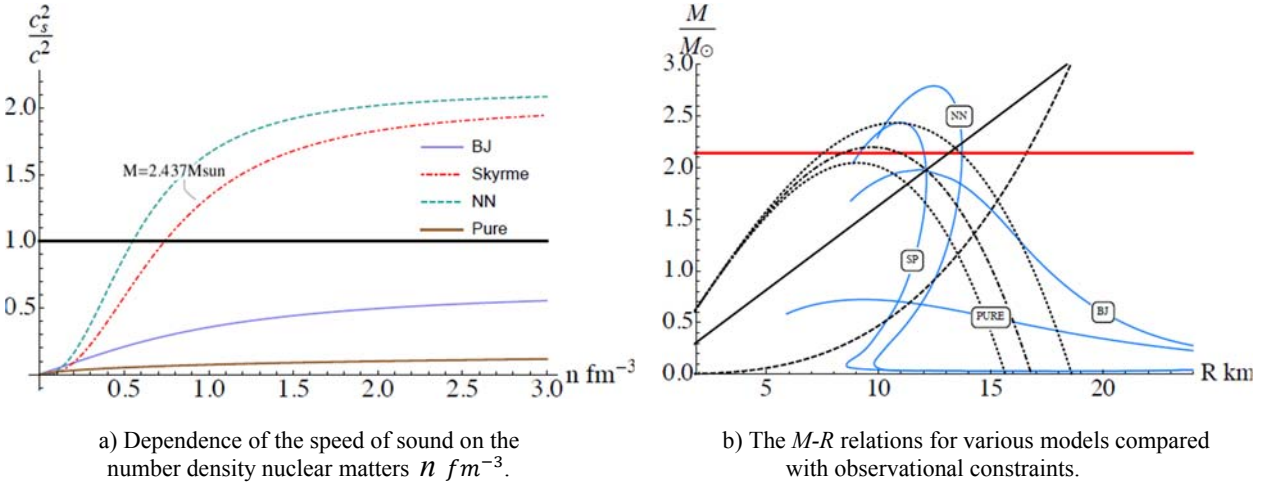


Figure 3 - Various EoS with theoretical and observational constraints

$\rho > 1.135 \times 10^{15} \text{ g/cm}^3$ , in Fig. 3a it can be seen that the maximum mass for the skyrme potential and for the empirical EoS must not exist from the physical point of view, since it lies above the plane  $c_s/c = 1$ . In figure 2 (b) the matter pressure is shown as a function of the energy density. As one can see the behavior of each EoS is different.

As for observational constraints, we learned from collecting data on neutron stars for 2019-2020 year [10] that with 95% confidence the mass of the most massive pulsar PSR J0740+6620, located at a distance of 1500 light years from Earth, lies in the range  $M = 2.14^{+0.20}_{-0.18} M_{\text{Sun}}$  with a rotation period in 2.89 ms (see figures 4c), it is marked with a thick red line. The lowest radius has been observed in the RX J1856-3754 [11]; for an observer at infinity the radius is  $R_{\infty} = R \sqrt{1 - \frac{2GM}{c^2 R}}$  so that for  $R_{\text{min}} = 16.8 \text{ km}$  [12], the

conditions read  $2GM/c^2 R > R - R^3/(R_{\infty}^{\text{min}})^2$  [13]. In figure 3b, this is indicated by a dotted curve and a dotted line. Furthermore, the largest surface gravity of a neutron star is for  $M = 1.4 M_{\text{Sun}}$ .

On the other hand, the observed radius with 90% confidence is  $15.64 \text{ km} < R_{\infty} < 18.86 \text{ km}$ ; in figure 3b, we mark the lower limit of the surface gravity with two dotted curves of the form



$2GM/c^2R > R - R^3/(R_\infty)^2$  [13]. The most famous and fastest rotating neutron star PSR J1748-2246 [14] has the highest rotation frequency 716 Hz and the equation for the frequency is determined as  $v_{\max} = 1045(M/M_{Sun})^{1/2}(10km/R)^{3/2}$ . Therefore, the constraint on the rotation looks like this  $M \geq 0.47(R/10km)^3 M_{Sun}$  and the figure it is shown as a dashed curve. Finally, the upper limit on the surface gravity of a neutron star is determined by the pulsar XTE J1814-338 and the constraint is  $M/M_{Sun} < 2.4 \times 10^5 c^2 R / GM_{Sun}$  shown in the figure as a solid black line [15]. Realistic mass-radius relations must pass through the region surrounded by the observational constraints. Therefore, as one can see, the pure degenerate neutron gas EoS is not realistic.

**8. Discussion and conclusion.** In this paper, we obtained the dependence of mass on radius and on the central density for different EoSs of Neutron Stars. It can be seen in figure 1 that if we do not take into account the nucleon-nucleon interaction, the EoS will be extremely soft to describe the dependence of the mass on the radius, which does not correspond to the observations. In section III, we discussed the appearance of pion condensation, but in realistic models, the formation of condensation must be prevented by many factors such as the pion-nucleon interaction [7]. In Ref. [16] the EoS with protons, neutrons and electrons was studied, but instead of the local electroneutrality condition  $n_p - n_e = 0$  the global electroneutrality condition was used in the form  $\int \rho_{ch} d^3r = \int e[n_p(r) - n_e(r)] = 0$ . At the same time, the Lagrangian density  $L$  takes into account the repulsive force between nucleons from vector bosons  $\rho_\mu$  and  $\omega_\mu$  also the electromagnetic 4-potential  $A_\mu$  or any Lagrangian types [17].

As can be seen from figures 1 and 3, the empirical EoS has the most rigid dependence of mass on radius, but the plots quickly fall down with the inclusion of protons. The equation of Bethe - Jones is in good agreement with the experimental data, but at high densities, it is not enough to describe the repulsive force with a single  $\omega$  meson. Nevertheless, the EoS does not contradict the latest observational and theoretical constraints, which is certainly a big advantage. The EoS for the skyrme potential is just a good description of the matter at high densities up to  $\rho < 5.5\rho_{nuc}$  which corresponds to a star with mass  $M = 2.37M_{Sun}$ .

The final results are shown in figure 3, as expected, the lower limit of observational constraints lies in the interval  $1.3M_{Sun} < M < 1.4M_{Sun}$  that is close to the Chandrasekhar limit, which in turn lies in the range of  $1.38 - 1.44 M_{sun}$  for white dwarf stars. According to some estimates the maximum observed mass of a neutron star PSR J0740+6620 is  $2.17M_{Sun}$ . The existence of such an object is explained by the fact that a rapidly rotating neutron star increases the maximum mass by almost 15%. Apparently, in the past, PSR J0740+6620 absorbed a significant part of the substance of its companion - most likely, when it was still at the stage of the red giant [18].

According to 2015 data, about 2500 neutron stars are known. Out of them, only 10% have companions. Many massive neutron stars also have an inner core. The radius of the inner core can reach up to several kilometers, and the density in the center of the nucleus can exceed the density of atomic nuclei by 7-8 times. The composition and EoS of the substance of the inner core are not known for certain. At such densities, neutrons can give way to hyperons, three-quark particles that include at least one strange quark, or even consist of free quarks and gluons [19].

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## НЕЙТРОНДЫҚ ЖҰЛДЫЗДАРДЫҢ КҮЙ ТЕНДЕУЛЕРІН ТАЛДАУ

**Аннотация.** Жұмыста нейтрондық жұлдыздардың, нейтрондық тамшылардың пайда болуынан бастап, асқын ядролық тығыздыққа дейінгі әртүрлі күй теңдеулері: айныған нейтрондық газ, Бета және Джонс күй теңдеуі, эмпирикалық күй теңдеуі және Skyrme потенциалды күй теңдеуі қарастырылған. Алғашқы қадам ретінде Толман-Оппенгеймер-Волковтың релятивтік гидростатикалық тепе-теңдік теңдеуі масса-орталық тығыздық, масса-радиус және радиус-орталық тығыздық қатынасын тұрғызу үшін қолданылды. Себебі нейтрондық жұлдыз таза релятивтік объект. Күй теңдеуін таңдау барысында нейтрондар және кейбір жағдайлар үшін протондар арасындағы нуклон-нуклондық әсерлесуге аса мән берілді. Өйткені, кинетикалық энергиямен қатар нуклондар арасындағы әсерлесу де маңызды рөл атқарады.

Сонымен қатар, нейтроннан басқа электрон және протон сияқты әртүрлі бөлшектер жиынтығынан тұратын аса тығыз материяның қасиеттері және әсерлесуді тасымалдаушы бөлшектердің үлесі зерттелді. Осы мақсатқа жету барысында тәжірибемен жақсы үйлесетін Бета мен Джонс потенциалы және Skyrme потенциалы қарастырылды. Нуклондардың әсерлесуімен қоса, әртүрлі көпбөлшектік әдістері, Хартри және Хартри-Фок әдістері қолданылып, жүйенің энергиясы анықталды. Осы ақпаратқа сүйене отырып қысым, энергия тығыздығы және нейтронды жұлдыздағы дыбыс жылдамдығы сияқты термодинамикалық параметрлер тығыздық функциясы ретінде есептелді. Бір-бірінен айырмашылықтарын көрсету үшін қасиеттері ұқсас келетін заттың күй теңдеулері салыстырылды.

Жалпы алғанда, фундаменталды заңдарды тексеру барысында маңызды астрофизикалық объект болып саналатын миллисекундтық пульсарлар бақыланады. Бақылаудың соңғы нәтижелері күй теңдеулерінің, жұмсақ күй теңдеулеріне қарағанда, жоғарғы тығыздықта себеп-салдар принципін бұзатынына қарамастан, қатаң болуды талап етті. Бұл нәтижелер, айналатын пульсарды бақылау барысында Шапиро эффекті арқылы пульсар массасын анықтауға мүмкіндік беретін физикалық параметрлерді анықтауға болатынын көрсеткен. Солтүстік Америка обсерваториясында гравитациялық толқындар (NANOGrav) үшін наногерц бойынша мәліметтерді біріктіру барысында 12,5 жыл бойы орбиталды фазаны бақылауда Green Bank телескобын қолдана отырып, PSR J0740 + 6620 – нейтрондық жұлдыздың массасы  $2,14_{-0,09}^{+0,10}$  күн массасына тең болатыны анықталды. Біздің есептеуімізде PSR J1748-2246 нейтронды жұлдыздың айналуы стандартты бақылаудан белгілі шектеу болып саналады. Аталған пульсар секундына 1122 айналым жасайтын ХТЕ J1739-285 пульсардан кейін жылдамдығы бойынша екінші болып саналады. Дегенмен басқа астрономдар ХТЕ J1739-285 пульсардың айналу периодын әлі толық растамаған.

Қорытындылай келе, жұмыста әсерлесуді ескермеген есептеу нәтижелері, бір тасымалдаушының әсері ескерілгендегі әсерлесу (Бета және Джонс күй теңдеулері жағдайында) нәтижелерінен едәуір айырмашылық көрсететіні баяндалған. Сонымен қатар, эмпирикалық жолмен тұрғызылған, жұлдыздың толық радиусының орталық тығыздыққа тәуелділігі айныған нейтрондық газдан ерекшеленетіні көрсетілді. Жұмыстың жаңалығы, бақылаудан алынған мәліметтермен қоса, нейтрондық жұлдыздарды сипаттау үшін әртүрлі күй теңдеулерін қолдануда және олардың қайсысы жеткілікті түрде физикалық шынайылықты сипаттайтынын көрсетуге.

**Түйін сөздер:** күй теңдеуі, нейтронды жұлдыз, масса-радиус арақатынасы, ядролық әсерлесу, бақылаудан алынған шектеулер.

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## АНАЛИЗ УРАВНЕНИЙ СОСТОЯНИЯ НЕЙТРОННЫХ ЗВЕЗД

**Аннотация.** В данной работе рассмотрены различные уравнения состояния нейтронных звёзд, такие как уравнение состояния вырожденного нейтронного газа, уравнение состояния Бета и Джонса, эмпирическое

уравнение состояния и уравнение состояния с потенциалом Skyrme, от точки образования нейтронных капель до сверхядерных плотностей. Поскольку нейтронная звезда является чисто релятивистским объектом, было использовано релятивистское уравнение гидростатического равновесия Толмана-Оппенгеймера-Волкова в качестве отправной точки для получения соотношений масса-центральная плотность, масса-радиус и радиус-центральная плотность. При выборе уравнений состояния было уделено особое внимание тем уравнениям, в которых учитывается нуклон-нуклонные взаимодействия между нейтронами, в некоторых случаях с протонами, поскольку кроме кинетической энергии частицы, большую роль играют взаимодействия между нуклонами.

Были исследованы свойства сверхплотной материи с различным набором частиц, такими как нейтроны, а также электроны и протоны. Также был исследован вклад различных частиц-переносчиков взаимодействия. Для достижения этих целей были рассмотрены различные потенциалы: потенциал Бета и Джонса, потенциал Skyrme, которые хорошо согласуются с экспериментальными данными. Кроме этого, вычисляется энергия системы, используя различные многочастичные методы: методы Хартри и Хартри-Фока, включая взаимодействие нуклонов. Благодаря этой информации рассчитываются термодинамические параметры, такие как давление, плотность энергии и скорость звука как функция плотности нейтронной звезды. Уравнения состояния вещества нейтронной звезды были сравнены между собой попарно, а затем были продемонстрированы тонкие отличия друг от друга.

Благодаря наблюдениям за миллисекундными пульсарами, которые являются одним из самых полезных астрофизических объектов для проверки фундаментальных законов физики были учтены последние наблюдательные ограничения на уравнение состояния. Показано, что наблюдательные данные требуют, чтобы уравнения состояния были жесткими, несмотря на то, что все жесткие уравнения состояния, в отличие от мягких, нарушают принцип причинности при больших плотностях. Наблюдательные ограничения были получены путем длительных наблюдений за пульсарами. При наблюдении за вращающимися пульсарами было обнаружено, что можно измерить физические параметры, которые позволяют вычислять массу самих пульсаров с помощью эффекта Шапиро. Путем объединения набора данных Североамериканской наногерцовой обсерватории гравитационных волн (NANOGrav) за 12,5 лет с недавними наблюдениями за орбитальной фазой, используя телескоп GreenBank, была измерена масса источника PSR J0740+6620 (нейтронная звезда), которая равна  $2,14^{+0,10}_{-0,09}$  солнечных масс. В качестве типичного наблюдательного ограничения на вращение нейтронных звезд, используются параметры источника PSR J1748-2246, считающегося вторым по скорости вращения пульсаром, а первым в настоящий момент считается XTEJ1739-285, совершающий 1122 оборота в секунду, хотя другие астрономы этого не подтвердили.

В заключение показано, что результаты вычисления без взаимодействия сильно разнятся с результатами уравнения состояния Бета и Джонса, где учитывался, как минимум, один переносчик взаимодействия. Также было определено, что зависимость полного радиуса звезды от центральной плотности, построенной на эмпирических соображениях, сильно отличается от результатов, полученных из уравнения состояния вырожденных нейтронных газов, начиная со значений плотности порядка ядерной. Новизна статьи заключается в исследовании нейтронных звезд путем сравнения современных наблюдательных данных и результатов, полученных с помощью применения различных уравнений состояния и на основе этого проведен анализ того какие из этих данных более подробно описывают физическую реальность.

**Ключевые слова:** уравнение состояния, нейтронная звезда, отношение масса-радиус, ядерные взаимодействия, наблюдательные ограничения.

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**DEVELOPMENT OF A METHOD FOR DESIGNING  
INFORMATION SYSTEMS FOR ENERGY COMPANIES**

**Abstract.** The creation of information models requires the use of known methods and the development of new methods of formalizing the pre-design research process. The modeling process consists of four stages: data collection on the object of management - pre-project research; creation of a graphical model of business processes taking place in the enterprise; development of a formal model of business processes; business research by optimizing the formal model.

To support the creation of workflow management services and systems, the complex offers methodologies, standards and specialized software that make up the developer's tools.

This can be ensured only by modern automated methods based on information systems. It is important that the information collected is structured to meet the needs of potential users and stored in a form that allows the use of modern access technologies.

Before discussing the effectiveness of FIM, it should be noted that the basic concept of information itself is still not the same. In a pragmatic way, it is a set of messages in the form of an important document for the system. Information can be evaluated not only by volume, but also by various parameters, the most important of which are: timeliness, relevance, value, aging, accuracy, etc. in addition, the information may be clear, probable and accurate. The methods of its reception and processing are different in each case.

**Keywords:** information systems, management information systems (MIS), integrated databases, sadt methodology.

Information support is the basis for all management activities. Here the set of information should be considered as various messages, data, relevant objects, phenomena, processes, relationships about the data. A necessary condition for the successful operation of any enterprise is the normal operation of the following processes:

- targeted collection and primary processing of information;
- organization of channels for users to access the collected information;
- timely use of collected information for decision making.

The main task of gathering the necessary information:

- completeness, adequacy and integrity of information;
- reducing the technological delay between the moment of information generation and the moment of access to information.

This can be ensured only by modern automated methods based on information systems. It is important that the information collected is structured to meet the needs of potential users and stored in a form that allows the use of modern access technologies.

Modern management information systems.

Development of management information systems (DMIS) is a very complex process that requires significant time and resources [1-4]. Modern large MIS projects are usually characterized by the following features:

- the complexity of the characteristics that require careful modeling and analysis of data and processes (there are many functions, processes, data elements and complex relationships between them);

- its local tasks and objectives (for example, the presence of a set of closely interacting components (subsystems) with traditional applications related to transaction processing, regulatory tasks and analytical processing (decision support) applications using unregulated requests for large amounts of data);

- lack of direct analogues, which limits the possibility of using any standard design solutions and application systems;

- Significant time duration of the project, based on the limited capabilities of the development team, on the one hand, and the scale of the customer organization and the different levels of readiness of its individual units to implement the MIS, on the other hand.

For the successful implementation of the project, the design object (DO) must first be adequately described, complete and functionally non-contradictory MIS and information models must be built. In addition, the information needs of users may change or be clarified during the creation and operation of the fee, which makes it difficult to develop and maintain such systems. Currently, one of the most complex and important stages in the development of MIS, the stage of creating an information model, remains largely informal. The initial phases of the project have a decisive impact on the achieved result, as they make key decisions that determine the quality of the information system. The share of the final result of the conceptual phase reaches 30%.

The creation of information models requires the use of known methods and the development of new methods of formalizing the pre-design research process. The modeling process consists of four stages: data collection on the object of management - pre-project research; creation of a graphical model of business processes taking place in the enterprise; development of a formal model of business processes; business research by optimizing the formal model. In addition, it is the most promising method among the known methods of pre-design research in terms of its automation. The method involves the collection and initial processing of information. As a result of the initial processing, we obtain a database containing data on the enterprise, which is suitable for further automated analysis, rather than a mass of unsystematic primary information. We will use the same presentation of research results in future work.

We set the task to create an information model that allows not only to show the relationship between the structural units of the enterprise and their weight, but also to assess the nature of the processes taking place in the organization. In this case - what operations (functions) are performed on the information (documents) within the organization. Hereinafter, such an information model is called functionally oriented. Obtaining such a model allows to set the task of optimizing the organizational structure of the enterprise on new criteria, for example, on the loading of individual functions, uniform loading, etc. b. on.

The obtained functional-oriented model allows to implement the technique of creating an organizational model of the enterprise "from below": in the first stage to determine the full list of functions that must be implemented in the enterprise for effective management and achievement of goals; identify internal and external relationships between functions; estimate the amount of information passed on these contacts; reorganization of departments and services by redistribution of these functions on a similar basis. At this stage, communication between departments is carried out automatically - through communication between newly created departments and services. New features, document management can be included.

A wide class of mathematical theories (third and fourth stages) can be used to formally describe and analyze the graphical model of business processes, which we will discuss in detail in this paper.

Analysis of existing approaches to building an information model of the enterprise.

At present, the automation of enterprise (organization) management is still an important and topical issue [4, 16, 19, 23], the intuition and personal experience of the leader is often insufficient to make effective and timely management decisions. Therefore, a modern approach to management involves not only a large investment in the purchase of expensive equipment, but also the creation and implementation of automated management decision support systems (SS). Creating an SS has always been a complex system process and still remains:

- a modern enterprise is a complex system of interacting elements (divisions);

- each enterprise requires unique and standard design solutions that require complex adaptation;

- unique and flows (information and material) that connect the subsystems of the enterprise, as well as the enterprise with the environment. Therefore, it is necessary to carefully study the information flows during the development of the SS. To do this, it is necessary to create an adequate information model of the enterprise [13]. This process is not simple.

Currently, there is a set of tools that facilitate the process of creating an information model, such as CASE-tools [9]. They can significantly simplify the modeling process. However, the preliminary stages related to the description of the subject area are beyond the competence of CASE-tools and are performed informally at the verbal level [13]. In addition, the adequacy of the information model depends on the quality of their implementation. We note that the information model is very important for them (in terms of functionality), as it significantly determines the efficiency of all SS. For them, only the document flow (traffic or traffic) is important and can be optimized only after a careful study of this traffic, that is, it should be organized so that the documents arrive on time and without queuing. In other words, it would be possible to make timely management decisions at all SS levels. Of course, a lot depends on the technical implementation of the IS, but this is only a necessary but insufficient condition for the effective operation of the IS.

One type of MIS is called corporate information systems (IS) [20]. Recently, great results have been achieved in the practice of creating such IS [20]. This was largely due to the fact that all information available to the corporation was entered into a common integrated database (IDB) and that all divisions of the corporation were included in this database in accordance with its competence. IDB is a multidisciplinary database. IDB is the most important, but not the only, component of IS. Another important part of it is the telephone, telegraph, etc. is a communication network that includes typical communication channels. That is, the communication network is gradually called an integrated service network. The information component is present in any MIS, thus defining it as an information subsystem and having a significant impact on the structure and effectiveness of the duty. Thus, it makes sense to study and optimize individual IS. You can do this by building an IS model. At the same time, it is always necessary to take into account in what subject area and to what extent the information is adequately collected in the IP.

Thus, when studying the IP, it is necessary to take into account that the information contained in it is a model of some areas of the real world. The main requirement for any IS is to ensure the adequacy of this model. The main tools to improve the efficiency of complex information systems are: operational analysis of the situation, development of operational and calendar plans, modeling of management processes. Modeling is the study of the properties of the original by replacing one object (the original) with another object, called the model, and studying the properties of the model. The need to use models arises when it is expensive, difficult or impossible to make a decision on a particular object. The model simplifies, reduces and speeds up the process of studying the original.

Nowadays, the sadt method is widely used. (Structured Analysis and Design Tecnique) — Structural analysis and design methodology that provides a number of advantages in control systems:

- formalize the description of the workflow;
- tolerance: process models created within one system can work under the control of another system;
- universality: the use of a single mechanism for describing the management of workflows in different areas of activity.

Currently, a number of standards have been developed to describe specific workflows that can be divided into two categories:

- Graphic models depicting the tree-like structure of the process.
- Block models closest to the block structure of programming languages.

**Conclusion.** Thus, the above models are suitable for use in a number of cases to effectively describe the system environment and its operation. In addition, information about the flows that serve the system is a determinant of any system. Therefore, it is important to study not only the model of the system, but also its information model to the level of functions in detail and its filling with information flows, not blocks of the system. Such a model can be called functionally oriented (FIM). The use of FIM is also important for modeling the functional structure of the system. The use of FIM allows you to set and solve new tasks at the level of organizational and functional structure, for example, to determine the load of functions, their full load, etc. b. redistribution (optimization) of document flow between individual functions in order to ensure

Before discussing the effectiveness of FIM, it should be noted that the basic concept of information itself is still not the same. In a pragmatic way, it is a set of messages in the form of an important document for the system. Information can be evaluated not only by volume, but also by various parameters, the most

important of which are: timeliness, relevance, value, aging, accuracy, etc. in addition, the information may be clear, probable and accurate. The methods of its reception and processing are different in each case.

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### **ЭНЕРГЕТИКАЛЫҚ КӘСІПОРЫННЫҢ АҚПАРАТТЫҚ ЖҮЙЕЛЕРІН ЖОБАЛАУ ӘДІСІН ӘЗІРЛЕУ**

**Аннотация.** Ақпараттық модельдерді құру белгілі әдістерді қолдануды және жобалау алдындағы зерттеу үдерісін формальдаудың жаңа әдістерін әзірлеуді талап етеді. Модельдеу үдерісі төрт кезеңнен тұрады: басқару объектісі туралы деректерді жинау – жоба алдындағы зерттеу; кәсіпорында болып жатқан бизнес үдерістерінің графикалық моделін құру; бизнес үдерістердің формальды моделін әзірлеу; формальды модельді оңтайландыру жолымен бизнесті зерттеу.

Жұмыс ағынын басқару қызметтері мен жүйелерін құруды қолдау үшін кешенде әзірлеушінің құрал-сайман құралдарын құрайтын әдіснамалар, стандарттар және мамандандырылған бағдарламалық жасақтама ұсынылады.

Ақпараттық қамтамасыз ету барлық басқару қызметінде құрылатын база болып саналады. Бұл жерде ақпаратты жиынтығы әртүрлі хабар, мәлімет, деректер туралы тиісті зат, құбылыс, үдеріс, қатынастар ретінде қарастырған жөн.

Мұны тек ақпараттық жүйелер негізінде заманауи автоматтандырылған әдістермен ғана қамтамасыз етуге болады. Жиналған ақпараттың әлеуетті пайдаланушылардың қажеттіліктерін ескере отырып құрылымдалуы және қазіргі заманғы қол жеткізу технологияларын пайдалануға мүмкіндік беретін нысанда сақталуы өте маңызды.

Сондықтан басқаруға заманауи көзқарас қымбат құрал-жабдықтарды сатып алуға үлкен қаражат салуды ғана емес, басқарушылық шешім қабылдауды қолдаудың автоматтандырылған жүйелерін (АЖ) құру мен енгізуді көздейді.

Кез келген кәсіпорындардың табысты жұмыс істеуі үшін қажетті шарт келесі үдерістердің қалыпты жұмысы болып саналады:

- ақпаратты мақсатты жинау және алғашқы өңдеу;
- пайдаланушылардың жиналған ақпаратқа қол жеткізу арналарын ұйымдастыру;
- шешім қабылдау үшін жиналған ақпаратты уақытында пайдалану.

Қажетті ақпаратты жинаудың негізгі мәселесі:

- ақпараттың толықтығы, барабарлығы және тұтастығы;
- ақпараттың пайда болу уақыты мен ақпаратқа қол жеткізу басталған сәттің арасындағы технологиялық

кешігуді азайту.

АЖ әзірлеу кезінде ақпараттық ағынды мұқият алдын ала зерттеу қажет. Ол үшін кәсіпорынның адекватты ақпараттық моделін құру кеорек. Бұл үдеріс қарапайым емес.

**Түйін сөздер:** ақпараттық жүйелер, басқарудың ақпараттық жүйелері (БАЖ), интегралдық деректер базасы, sadt әдістемесі.

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### **РАЗРАБОТКА МЕТОДИКИ ПРОЕКТИРОВАНИЯ ИНФОРМАЦИОННЫХ СИСТЕМ ДЛЯ ЭНЕРГЕТИЧЕСКИХ КОМПАНИЙ**

**Аннотация.** Создание информационных моделей требует использования известных методов и разработки новых методов формализации процесса предпроектных исследований. Процесс моделирования состоит из четырех этапов: сбор данных по объекту управления – предпроектное исследование; создание графической модели бизнес-процессов, происходящих на предприятии; разработка формальной модели бизнес-процессов; бизнес-исследования путем оптимизации формальной модели.

Для поддержки создания сервисов и систем управления рабочими процессами комплекс предлагает методологии, стандарты и специализированное программное обеспечение, составляющие инструменты разработчика.



Информационная поддержка – основа всей управленческой деятельности. Здесь набор информации следует рассматривать как множество сообщений, данных, соответствующих элементов, явлений, процессов, взаимосвязей между данными.

Обеспечить это можно только современными автоматизированными методами на базе информационных систем. Важно, чтобы собранная информация была структурирована для удовлетворения потребностей потенциальных пользователей и хранилась в форме, позволяющей использовать современные технологии доступа.

Поэтому современный подход к управлению предполагает не только крупные вложения в закупку дорогостоящего оборудования, но и создание и внедрение автоматизированных систем поддержки принятия управленческих решений (ИС).

Необходимым условием успешной работы любого предприятия является нормальная работа следующих процессов:

- целевой сбор и первичная обработка информации;
- организация каналов доступа пользователей к собранной информации;
- своевременное использование собранной информации для принятия решений.

Основная задача сбора необходимой информации:

- полнота, адекватность и целостность информации;
- сокращение технологической задержки между моментом формирования информации и моментом доступа к информации.

При разработке ИС необходимо заранее внимательно изучить информационные потоки. Для этого необходимо создать адекватную информационную модель предприятия. Это непростой процесс.

**Ключевые слова:** информационные системы, информационные системы управления (ИСУ), интегрированные базы данных, sadt-методология.

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**FALSIFICATIONS OF POISSON ADIABATE,  
HUGONIO ADIABATE, LAPLACE SOUND SPEEDS.  
MODERNIZATION OF FOUNDATIONS OF THERMODYNAMICS**

**Abstract.** The inequality of the universal gas constant of the difference in the heat capacity of a gas at constant pressure with the heat capacity of a gas at a constant volume is proved. The falsifications of using the heat capacity of a gas at constant pressure, false enthalpy, Poisson adiabat, Laplace sound speed, Hugoniot adiabat, based on the use of the false equality of the universal gas constant difference in the heat capacity of a gas at constant pressure with the heat capacity of a gas at a constant volume, have been established. The dependence of pressure on temperature in an adiabatic gas with heat capacity at constant volume has been established. On the basis of the heat capacity of a gas at a constant volume, new formulas are derived: the adiabats of an ideal gas, the speed of sound, and the adiabats on a shock wave. The variability of pressure in the field of gravity is proved and it is indicated that the use of the specific coefficient of ideal gas at constant pressure in gas-dynamic formulas is pointless. It is shown that the false “basic formula of thermodynamics” implies the falseness of the equation with the specific heat capacity at constant pressure. New formulas are given for the adiabat of an ideal gas, adiabat on a shock wave, and the speed of sound, which, in principle, do not contain the coefficient of the specific heat capacity of a gas at constant pressure. It is shown that the well-known equation of heat conductivity with the gas heat capacity coefficient at constant pressure contradicts the basic energy balance equation with the gas heat capacity coefficient at constant volume.

**Key words:** speed of sound, heat capacity, adiabat, isobaric, isothermal.

**1. Falsifications of the difference in heat capacities  $C_p - C_v = R$**

The difference in heat capacities of gas  $C_p - C_v = R$  is considered the basic formula of thermodynamics and is derived from the first law of thermodynamics [2, 3]:

$$d'Q = dU_{\text{km}} + p dV_{\text{km}} \quad (1)$$

In (1)  $d'Q$  the amount of heat,  $dU_{\text{km}}$  the differential of the internal energy of a kilomole of gas  $U_{\text{km}} = C_v T$ , according to the equation of state of Clapeyron  $V_{\text{km}} = RT/p$ :

$$d'Q = C_v dT + p R d \frac{T}{p} \quad (2)$$

Only by the hypothesis of constant pressure, putting in (2)  $d'Q = C_p dT$ , we get formula  $C_p - C_v = R$ , where  $C_p$  is the heat capacity of gas at constant pressure  $p = \text{const}$ . The heat capacity

at a constant volume is  $C_v = \frac{iR}{2}$ , where  $i = n_{\text{translat}} + n_{\text{rotat}} + 2n_{\text{vibrat}}$  is the sum of the translational, rotational and doubled number of vibrational degrees of freedom of the molecule, the universal gas constant  $R = 8,31 \cdot 10^3 \frac{\text{joule}}{\text{deg} \cdot \text{kilomole}}$ .

**Theorem 1.** In an adiabatic gas, pressure is a variable and depends on temperature  $\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{i}{2}+1}$ .

The constant pressure hypothesis in adiabatic and non-isothermal gas leads to false equality  $C_v = -R$ , because  $C_v = \frac{iR}{2}$ .

*Evidence.* Let the pressure in adiabatic gas  $d'Q = 0$  be constant:

$$0 = C_v dT + pRd \frac{T}{p} \quad (3)$$

In nonisometric gas  $T \neq \text{const}$ ,  $dT \neq 0$  in (3), the pressure decreases:

$$0 = C_v dT + RdT, \quad C_v + R = 0 \quad (4)$$

The second equality of (4) implies the negativity of the specific heat  $C_v = -R$ , which contradicts formula  $C_v = \frac{iR}{2} > 0$ . The heat capacity of gas  $C_v = \frac{iR}{2}$  in equation (3) establishes the dependence of pressure on temperature:

$$0 = \frac{i}{2} RdT + p d \frac{RT}{p}, \quad \left(\frac{i}{2} + 1\right) \frac{dT}{T} = \frac{dp}{p}, \quad \frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{i}{2}+1} \quad (5)$$

the Theorem is proved.

According to theorem 1, the constancy of pressure in adiabatic and non-isothermal gas led to a false equality of  $C_v = -R$ . In Exactly the same way in [2,3] et al .the hypothesis of pressure constancy in the 1st law of thermodynamics (2) was applied to derive the false formula  $C_p - C_v = R$ .

**Theorem 2.** In an isothermal and Isobaric gas, the difference in heat capacity is not equal to the universal gas constant  $C_p - C_v \neq R$ .

*Evidence.* Let a kilomole of gas be supplied with heat at a constant step  $d'Q = C_p dT$ :

$$C_p dT = C_v dT + pRd \frac{T}{p}$$

After reducing the pressure  $p = \text{const}$  is obtained

$$C_p dT = C_v dT + RdT, \quad (C_p - C_v - R)dT = 0 \quad (6)$$

Equality to zero in (6) will take place if one of the cofactors is equal to zero. By the condition of theorem  $dT = 0$  in an isothermal gas, equality to zero in (6) holds, so the second factor is not equal to zero:

$$(C_p - C_v - R) \neq 0, \quad (7)$$

The conclusion follows from inequality (7): in an isothermal and Isobaric gas, the difference in heat capacity is not equal to the universal gas constant  $C_p - C_v \neq R$ . The theorem is proved.

**Theorem 3.** The difference between an arbitrary heat capacity gas heat capacity of gas at constant volume is not equal to universal gas constant:  $C - C_v \neq R, \forall C$ .

*Evidence.* Let the gas be supplied with heat  $d'Q = CdT$  :  $CdT = C_v dT + pRd \frac{T}{p}$ .

Transform the differential of the quotient in the right part:  $CdT = C_v dT + pR \frac{pdT - Tdp}{p^2}$ .

After abbreviations and bringing similar results

$$(C - C_v - R)dT = -RT \frac{dp}{p} \quad (8)$$

From (8) follows the inequality to zero  $C - C_v - R \neq 0$  for all heat capacities of the gas. The hypothesis of constancy of pressure in the right part (8)  $dp = 0$ ,  $p = const$  means equality to zero  $dT = 0$  in the left part, that is, the isothermicity of the gas and the validity of the inequality  $C - C_v \neq R, \forall C$ . The theorem is proved.

## 2. Adiabatic of ideal gas with heat capacity $C_v = \frac{iR}{2}$

Let heat  $d'Q = CdT$  be applied to the kilomole of gas. The 1-st law of thermodynamics (2) is transformed based on the equation of state of Clapeyron  $pV = mRT / \mu$ , written for volume  $V$ , which contains a gas mass of  $m$  with a density of  $\rho = m / V$ . Equation of state of Clapeyron by equalities  $V_{km} = \frac{\mu}{m} V$  takes the form of  $p = \frac{\rho RT}{\mu}$ ,  $\mu = const$  -the mass of a kilomol. Equation (1) uses differentials  $d'Q = CdT$ ,  $dU_{km} = C_v dT$ .

Then the necessary transformations are performed:

$$(C - C_v)dT = pdV_{km}, \quad (C - C_v)dT = pd\left(\frac{\mu}{m}V\right), \quad \rho = \frac{m}{V},$$

$$\frac{(C - C_v)}{R} \frac{dp}{p} = \left(\frac{C - C_v}{R} - 1\right) \frac{d\rho}{\rho}, \quad \frac{dp}{p} = \left(1 - \frac{R}{C - C_v}\right) \frac{d\rho}{\rho}$$

From the last equality follows the barotropy of the gas at  $C \neq C_v$  :

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{1 - \frac{R}{C - C_v}} \quad (9)$$

Apply the formula (9) in adiabatic gas  $d'Q = 0$ ,  $C = 0$ ,  $C_v = \frac{i}{2}R$ ,  $\frac{R}{C_v} = \frac{2}{i}$  :

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{1 + \frac{R}{C_v}}, \quad \frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{1 + \frac{2}{i}}, \quad i = n_{translat} + n_{rotat} + 2n_{vibrat} \quad (10)$$

The Poisson adiabat  $\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\frac{c_p}{c_v}}$  is derived in [2,3] by a false formula  $c_p - c_v = \frac{R}{\mu}$ ,

adiabata Jakupov (10) is based on  $C_v = \frac{iR}{2}$ .

### 3. The speed of sound. Falsifications of Poisson, Laplace, Hugonio. Alibata on the shock wave

The speed of sound is calculated using the Jakupov  $\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{1+\frac{2}{i}}$  adiabat :  $a = \sqrt{\frac{\partial p}{\partial \rho}}$ ,

$$a = \sqrt{\left(1 + \frac{2}{i}\right) \frac{p}{\rho}}, \quad a = \sqrt{\left(1 + \frac{2}{i}\right) \frac{R}{\mu} T}, \quad i = n_{\text{translat}} + n_{\text{rotat}} + 2n_{\text{vibrat}}$$

Falsifications [2] of Poisson's adiabata  $\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\frac{c_p}{c_v}}$  and the speed of sound Laplace's

$$a = \sqrt{\frac{c_p}{c_v} RT} \quad \text{are consequences of the false equality } C_p - C_v = R.$$

Adiabata Hugonio [2] on a shock wave is obtained by repeated using a false formula  $C_p - C_v = R$  :

$$\frac{p_2}{p_1} = \frac{(k+1)\rho_2 / \rho_1 - (k-1)}{(k+1) - (k-1)\rho_2 / \rho_1}, \quad (11)$$

$$k = \frac{c_p}{c_v}, \quad \text{specific heat capacity ratio } c_p = \frac{C_p}{\mu}, \quad c_v = \frac{C_v}{\mu}, \quad c_p - c_v = \frac{R}{\mu}.$$

The Hugonio relations [2] relate the parameters of a gas on a shock wave

$$\rho_2 V_2^2 + p_2 = \rho_1 V_1^2 + p_1, \quad \rho_2 V_2 = \rho_1 V_1, \quad (12)$$

$$c_v T_2 + \frac{V_2^2}{2} + \frac{p_2}{\rho_2} = c_v T_1 + \frac{V_1^2}{2} + \frac{p_1}{\rho_1} \quad (13)$$

In gas dynamics [2] in order to apply the specific heat capacity at constant pressure, the enthalpy

$$h = c_p T \quad \text{is widely used, which is converted to the form of a false formula } c_p - c_v = \frac{R}{\mu} :$$

$$h = c_p T = \frac{c_p}{R} \frac{R}{\mu} T = \frac{c_p}{c_p - c_v} \frac{R}{\mu} T \quad (14)$$

Because of the false formula  $c_p - c_v = \frac{R}{\mu}$  is obtained counterfeit enthalpy

$$h = \frac{c_p}{c_p - c_v} \frac{R}{\mu} T = \frac{c_p}{c_p - c_v} \frac{p}{\rho} = \frac{k}{k-1} \frac{p}{\rho}, \quad k = \frac{c_p}{c_v}, \quad h = \frac{k}{k-1} \frac{p}{\rho}$$

Further, according to the Clapeyron equation  $p = \frac{\rho RT}{\mu}$  and formula  $c_p - c_v = \frac{R}{\mu} = R_*$  fake

made conversion [2]:

$$c_v T = \frac{c_v}{R_*} R_* T = \frac{c_v}{c_p - c_v} \frac{R}{\mu} T = \frac{c_v / c_v}{c_p / c_v - c_v / c_v} \frac{p}{\rho} = \frac{1}{k-1} \frac{p}{\rho} \quad (15)$$

The correct ratio shows that Hugoniot at the shock wave (13) is reduced to a fake form:

$$\frac{k}{k-1} \frac{p_2}{\rho_2} + \frac{V_2^2}{2} = \frac{k}{k-1} \frac{p_1}{\rho_1} + \frac{V_1^2}{2} \quad (16)$$

Conservation of the total momentum on the shock wave (12) is to mind

$$p_1 - p_2 = \rho_2 V_2^2 - \rho_1 V_1^2 = \rho_1 V_1 (V_2 - V_1)$$

Both parts of this expression are multiplied by the ratio

$$\frac{V_2 + V_1}{\rho_1 V_1} = \frac{V_2}{\rho_1 V_1} + \frac{V_1}{\rho_1 V_1} = \frac{V_2}{\rho_2 V_2} + \frac{V_1}{\rho_1 V_1} = \frac{1}{\rho_2} + \frac{1}{\rho_1},$$

and from the left to  $\frac{1}{\rho_2} + \frac{1}{\rho_1}$ , and from the right to  $\frac{V_2 + V_1}{\rho_1 V_1}$ .

The result is

$$(p_1 - p_2) \left( \frac{1}{\rho_2} + \frac{1}{\rho_1} \right) = V_2^2 - V_1^2 \quad (17)$$

Multiplying (13) by 2, we find

$$V_2^2 - V_1^2 = 2 \left( \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} \right) + 2c_v (T_1 - T_2) \quad (18)$$

According to equation of state  $T = \frac{\mu}{R} \frac{p}{\rho}$ , expression (18) takes the form

$$V_2^2 - V_1^2 = 2 \left( 1 + \frac{c_v \mu}{R} \right) \left( \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} \right) \quad (19)$$

Substituting (17) in the right part (19), we find

$$(p_1 - p_2) \left( \frac{1}{\rho_2} + \frac{1}{\rho_1} \right) = 2 \left( 1 + \frac{c_v \mu}{R} \right) \left( \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} \right) \quad (20)$$

Multiplying both parts (20) by  $\rho_2 / p_1$ , we get the shock adiabat

$$\frac{p_2}{p_1} = \frac{2 \left( 1 + \frac{c_v \mu}{R} \right) \frac{\rho_2}{\rho_1} - \left( 1 + \frac{\rho_2}{\rho_1} \right)}{2 \left( 1 + \frac{c_v \mu}{R} \right) - \left( 1 + \frac{\rho_2}{\rho_1} \right)} = \frac{\left( 1 + 2 \frac{c_v \mu}{R} \right) \frac{\rho_2}{\rho_1} - 1}{\left( 1 + 2 \frac{c_v \mu}{R} \right) - \frac{\rho_2}{\rho_1}} \quad (21)$$

Here the ratio is  $\frac{c_v \mu}{R} = \frac{C_v}{R} = \frac{iR}{2} / R = \frac{i}{2}$ , so the adiabat on the shock wave will be like this:

$$\frac{p_2}{p_1} = \frac{(1+i) \frac{\rho_2}{\rho_1} - 1}{(1+i) - \frac{\rho_2}{\rho_1}}, \quad i = n_{\text{translat}} + n_{\text{rotat}} + 2n_{\text{vibrat}} \quad (22)$$

#### 4. Justification of pressure variability

The false formula of thermodynamics  $C_p - C_v = R$  is a direct consequence of the first law of thermodynamics, in which the gas pressure is considered constant  $p = const$ . We prove the variability of pressure in statics and in gas dynamics. Theorem 1 proved the non-physical constancy of pressure in an adiabatic gas.

**Static  $\mathbf{v} = \mathbf{0}$ .** The coordinate system of Oxyz will be connected to the earth's surface. The ORT of the  $\mathbf{k}$ -axis Oz is parallel and opposite to the  $\mathbf{g}$ -axis acceleration of gravity:

$$\mathbf{k} \uparrow \downarrow \mathbf{g}, \mathbf{g} = 0\mathbf{i} + 0\mathbf{j} - g\mathbf{k}, g = 9,81 \frac{m}{c^2}.$$

Consider a stationary gas  $\mathbf{v} \equiv \mathbf{0}$  in volume  $\Delta V = (z_2 - z_1)S$ , where  $S$  the area of the cylinder base,  $z_2 - z_1$  – the height of the cylinder, equal to the distance between parallel planes  $z_1 = const$ ,  $z_2 = const$ ,  $z_1 < z_2$ . Denote  $\mathbf{n}_1 = -\mathbf{k}$  the external normal of the surface  $z_1 = const$ ,  $\mathbf{n}_2 = \mathbf{k}$ , the external normal of the surface  $z_2 = const$ . In an ideal gas, the Euler stress is  $\mathbf{p} = -p\mathbf{n}$ ,  $p$  is the pressure in the gas  $S$ , located in the plane  $z_1 = const$ , the force  $\mathbf{f}_1 = -p_1\mathbf{n}_1S$ ,  $S$  in a plane  $z_2 = const$  the force  $\mathbf{f}_2 = -p_2\mathbf{n}_2S$ , the whole mass of gas in the cylinder is the force of gravity  $m\mathbf{g}$ ,  $\mathbf{g} = -g\mathbf{k}$ , the mass of gas equal to  $m = \rho\Delta V$ . According to Newton's second law we have the equation

$$m \frac{d\mathbf{v}}{dt} = \mathbf{f}_1 + \mathbf{f}_2 + m\mathbf{g} \quad (23)$$

In statics  $\mathbf{v} = \mathbf{0}$ ,  $\frac{d\mathbf{v}}{dt} = \mathbf{0}$ , Newton's 2 law (23) takes the form

$$0 = \mathbf{f}_1 + \mathbf{f}_2 + m\mathbf{g}, -p_1S\mathbf{n}_1 - p_2S\mathbf{n}_2 + \rho(z_2 - z_1)S\mathbf{g} = 0 \quad (24)$$

In (24), we use the ORT equalities  $\mathbf{n}_1 = (-\mathbf{k})$ ,  $\mathbf{n}_2 = \mathbf{k}$ ,  $\mathbf{g} = -g\mathbf{k}$ :

$$-p_1S(-\mathbf{k}) - p_2S\mathbf{k} + \rho(z_2 - z_1)S(-g\mathbf{k}) = 0, p_1 - p_2 - \rho(z_2 - z_1)g = 0,$$

$$p_1 - p_2 = \rho(z_2 - z_1)g, \frac{p_2 - p_1}{z_2 - z_1} = -\rho g, \lim_{z_2 - z_1 \rightarrow 0} \frac{p_2 - p_1}{z_2 - z_1} = -\rho g$$

In the limit, the equation is obtained  $\frac{dp}{dz} = -\rho g$ , whose integral for an incompressible  $\rho g \neq 0$ ,

liquid confirms pressure variability  $p \neq const$ . In a compressible gas  $\rho = \frac{\mu p}{RT}$ , the integral of

equation is called the barometric formula  $p = p_0 e^{-\frac{\mu g}{RT}(z - z_0)}$  [3], here also  $p \neq const$ .

**Dynamics  $\mathbf{v} \neq \mathbf{0}$ .** On the physics of Euler's equations of an ideal gas

$$\nabla p = \rho\mathbf{g} - \rho \frac{d\mathbf{v}}{dt}, \frac{\partial \rho}{\partial t} + \nabla \cdot \rho\mathbf{v} = 0, \rho c_v \frac{dT}{dt} = -p\nabla \cdot \mathbf{v}$$

the pressure cannot be constant:  $\nabla p \neq 0$ ,  $p \neq const$ .

**Note.** For fake connection  $C_p - C_v = R$ , fake heat capacity at constant pressure is calculated

$$C_p = R + C_v = R + \frac{i}{2}R = (1 + \frac{i}{2})R$$

Thus, in both static and dynamic gas  $p \neq const$ , the pressure cannot be constant in the field of gravitational force, for example, under Earth conditions. Therefore, the definition of heat capacity  $C_p$  and specific heat of gas  $c_p = \frac{C_p}{\mu}$  at constant pressure lose their physical meaning. Loses the physical meaning of the heat function of enthalpy  $h = c_p T$ .

### 5. Paradoxes of the equation of thermal conductivity with the coefficient of heat capacity at constant pressure

The thermal conductivity equation is derived from the law of conservation of energy with a coefficient of heat capacity at a constant volume [2]:

$$\rho c_v \frac{dT}{dt} = \text{div}(\lambda \text{grad}T) + \mu \sum_{i=1}^3 \sum_{j=1}^3 \left( \frac{\partial v_i}{\partial x_j} \right)^2 - p \text{div} \mathbf{v}$$

In [4], a false equation of thermal conductivity with a coefficient of heat capacity at constant pressure is given:

$$\rho c_p \frac{dT}{dt} - \frac{dp}{dt} = \nabla \cdot (\lambda \nabla T) + \frac{\mu}{2} \sum_{i=1}^3 \sum_{j=1}^3 \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)^2 - \frac{2}{3} \mu (\nabla \cdot \mathbf{v})^2$$

The falsity consists in applying the Stokes stress tensor, formula  $C_p - C_v = R$ , as indicated by the specific heat capacity coefficient at constant pressure  $c_p$ , which is located on the left side of the equation. Similarly the thermal conductivity equation of an ideal gas is derived with a specific coefficient of heat capacity at a constant volume of  $c_v$ :

$$\rho c_v \frac{dT}{dt} = -p \nabla \cdot \mathbf{v} \tag{25}$$

In [2,4] is replaced in the left part (25) by a false formula  $c_p - c_v = \frac{R}{\mu}$  using the Clapeyron equation of state:

$$c_v T = \left( c_p - \frac{R}{\mu} \right) T = c_p T - \frac{RT}{\mu} = c_p T - \frac{p}{\rho}$$

As a result, equation (25) is converted to the form

$$\rho c_p \frac{dT}{dt} - \frac{dp}{dt} + \frac{p}{\rho} \frac{d\rho}{dt} = -p \nabla \cdot \mathbf{v} \tag{26}$$

In (26), the continuity equation  $d\rho/dt + \rho \nabla \cdot \mathbf{v} = 0$ . Gives a false equation with a heat capacity at constant pressure

$$\rho c_p \frac{dT}{dt} - \frac{dp}{dt} = 0 \tag{27}$$



By definition, the specific coefficient of heat capacity  $c_p$  takes place at a constant pressure  $p = \frac{\rho RT}{\mu} = const$ . At constant pressure, the false equation (27) implies equality to zero:

$$\rho c_p \frac{dT}{dt} = 0, \quad \frac{dT}{dt} = 0,$$

which contradicts the original equation (31) of the dynamic gas  $\mathbf{v} \neq \mathbf{0}$  :

$$\rho c_v \frac{dT}{dt} = -p \nabla \cdot \mathbf{v} \neq 0, \quad \frac{d\rho}{dt} = -\rho \nabla \cdot \mathbf{v}, \quad \rho c_v \frac{dT}{dt} = \frac{p}{\rho} \frac{d\rho}{dt} \neq 0$$

According to equation (25), there is an inequality to zero  $\frac{dT}{dt} \neq 0$ . It turns out to be absurd, which confirms the falsity of the "basic formula of thermodynamics"  $C_p - C_v = R$ ,  $c_p - c_v = \frac{R}{\mu}$ . Hence,  $C_p - C_v \neq R$ .

### 6. False Poisson adiabat of ideal gas

The Poisson adiabat [3] is derived from the false equation (27). The reduction of  $dt$  in equation (27) gives a connection of  $\frac{dp}{\rho} = c_p dT$ , the Clapeyron equation is Applied, the false formula

$c_p - c_v = \frac{R}{\mu}$  is used again, and the transformations are made:

$$\frac{dp}{\rho} = c_p dT = \frac{c_p \mu}{R} d\left(\frac{RT}{\mu}\right) = \frac{c_p \mu}{R} d\left(\frac{p}{\rho}\right) = \frac{c_p}{c_p - c_v} \frac{\rho dp - p d\rho}{\rho^2},$$

$$(c_p - c_v) dp = c_p \left(dp - \frac{p}{\rho} d\rho\right), \quad \frac{dp}{p} = \frac{c_p}{c_v} \frac{d\rho}{\rho}, \quad d \ln p = d \ln \rho^{\frac{c_p}{c_v}}$$

From the last equality we get the Poisson adiabat  $\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\frac{c_p}{c_v}}$  with a false degree indicator due to a coefficient of  $C_p$ .

### Summary

**Proved:** falsity of the capacity of gas  $C_p$ , falsity of the "basic formula of thermodynamics"

$C_p - C_v = R$ , falsity of the of the Poisson adiabat  $\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\frac{c_p}{c_v}}$ , falsity of the of the speed of sound

Laplace  $a = \sqrt{\frac{c_p}{c_v} RT}$ , falsity of the adiabat Hugonio  $\frac{p_2}{p_1} = \frac{(1 + \frac{c_p}{c_v}) \frac{\rho_2}{\rho_1} - (\frac{c_p}{c_v} - 1)}{(1 + \frac{c_p}{c_v}) - (\frac{c_p}{c_v} - 1) \frac{\rho_2}{\rho_1}}$ .

**Justified:** adiabat of ideal gas  $\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{1+\frac{2}{i}}$ ,  $i = n_{\text{translat}} + n_{\text{rotat}} + 2n_{\text{vibrat}}$ , speed of

sound  $a = \sqrt{\left(1 + \frac{2}{i}\right) \frac{RT}{\mu}}$ ,  $i = n_{\text{translat}} + n_{\text{rotat}} + 2n_{\text{vibrat}}$  adiabat percussion wave  $\frac{p_2}{p_1} = \frac{(1+i)\frac{\rho_2}{\rho_1} - 1}{(1+i) - \frac{\rho_2}{\rho_1}}$ ,

$$i = n_{\text{translat}} + n_{\text{rotat}} + 2n_{\text{vibrat}}$$

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**ПУАССОН, ГЮГОНИО АДИАБАТЫ,  
ЛАПЛАС ДЫБЫС ЖЫЛДАМДЫҒЫНЫҢ ЖАСАНДЫЛЫҒЫ.  
ТЕРМОДИНАМИКА НЕГІЗДЕРІН ЖАҢҒЫРТУ**

**Аннотация.** Термодинамиканың бірінші заңынан шығарылған термодинамиканың негізгі формуласы болып есептелетін тұжырымдама жасандылығы сипатталған. Тұрақты көлемдегі газдың тұрақты қысымы мен жылу сыйымдылығы бар газдың жылу сыйымдылығының әмбебап газ тұрақты айырмашылығының теңсіздігі дәлелденді.

Тұрақты көлемдегі жылу сыйымдылығы бар адиабатты газдағы температураға қысымның тәуелділік формуласы қорытылды. Тұрақты қысым, энтальпия, Пуассон адиабаты, Лаплас дыбыс жылдамдығы, гюгонио адиабаты кезінде газдың жылу сыйымдылығын термодинамикада қолданудың жасандылығы мен физикалық еместігі дәлелденді, онда тұрақты көлемде газдың жылу сыйымдылығымен тұрақты қысым кезінде газдың жылу сыйымдылығының әмбебап газды тұрақты айырмашылығының жасанды теңдігі қолданылды.

Тартылыс күші өрісінде қысымның айнымалдылығы дәлелденді. Газ-динамикалық формулаларда тұрақты қысым кезінде идеалды газдың нақты коэффициентін қолдану мағынасыз екендігі көрсетілген. Жасанды «термодинамиканың негізгі формуласы» тұрақты қысым кезінде нақты жылу сыйымдылығы коэффициентімен жасанды жылу теңдеуіне әкелетіні көрсетілген. Тұрақты көлемдегі мінсіз газдың жылу сыйымдылығына сүйене отырып, идеалды газ адиабаты үшін жаңа формула, соққы толқынындағы адекватты адиабат, дыбыс жылдамдығының жаңартылған формуласы анықталды, олар, негізінен, тұрақты қысым кезінде газдың нақты жылу сыйымдылығы коэффициентін қамтымайды. Тұрақты қысымдағы газдың жылу сыйымдылығы коэффициенті бар белгілі жылу өткізгіштік теңдеуі тұрақты көлемдегі газдың жылу сыйымдылығы коэффициентімен энергия балансының негізгі теңдеуіне қайшы келетіні көрсетілген.

**Түйін сөздер:** дыбыс жылдамдығы, жылу сыйымдылық, адиабата, изобарлық, изотермалық.

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**ФАЛЬСИФИКАЦИИ АДИАБАТЫ ПУАССОНА, АДИАБАТЫ ГЮГОНИО,  
СКОРОСТИ ЗВУКА ЛАПЛАСА. МОДЕРНИЗАЦИЯ ОСНОВ ТЕРМОДИНАМИКИ**

**Аннотация.** Излагаются фальсификации вывода из первого закона термодинамики основной формулы термодинамики. Доказано неравенство универсальной газовой постоянной разности теплоемкости газа при постоянном давлении с теплоемкостью газа при постоянном объеме. Установлена формула зависимости давления от температуры в адиабатическом газе с теплоемкостью при постоянном объеме. Доказаны фальсификации и нефизичность применения в термодинамике теплоемкости газа при постоянном давлении, энтальпии, адиабаты Пуассона, скорости звука Лапласа, адиабаты Гюгонио, в которых применено

фальшивое равенство универсальной газовой постоянной разности теплоемкости газа при постоянном давлении с теплоемкостью газа при постоянном объеме.

Доказана переменность давления в поле силы тяжести. Указано, что использование удельного коэффициента идеального газа при постоянном давлении в газодинамических формулах бессмысленно. Показано, что ложная «основная формула термодинамики» приводит к фальшивому уравнению теплопроводности с коэффициентом удельной теплоемкости при постоянном давлении. Основываясь на теплоемкости совершенного газа при постоянном объеме, установлены новая формула для адиабаты идеального газа, адекватная адиабата на ударной волне, модернизированная формула скорости звука, которые, в принципе, не содержат коэффициента удельной теплоемкости газа при постоянном давлении. Показано, что известное уравнение теплопроводности с коэффициентом теплоемкости газа при постоянном давлении противоречит основному уравнению баланса энергии с коэффициентом теплоемкости газа при постоянном объеме.

**Ключевые слова:** скорость звука, теплоемкость, адиабата, изобарический, изотермический.

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**SEMANTIC ANALYSIS OF THE KAZAKH LANGUAGE BASED  
ON THE APPROACH OF NEURAL NETWORKS**

**Abstract.** This paper provides an overview of existing modern methods and software approaches for semantic analysis. Based on the research done, it was revealed that, for the semantic analysis of text resources, an approach based on machine learning is most used. This article presents the developed algorithm for the semantic analysis of the text in the Kazakh language. The paper also presents a software solution to this approach implemented in the Python programming language. The vector representation of words was obtained by machine learning based on the corpus, which is 1 million sentences in the Kazakh language. In the software implementation, well-known libraries such as gensim, matplotlib, sklearn, numpy, etc. were used. Based on a set of semantically related pairs of words, an ontology for a specific document is built, which is formed during the operation of a neural network. The paper presents the results of the experiments in the graphical form of a set of words. The novelty of the proposed approach lies in the identification of semantic close words in meaning in texts in the Kazakh language. This work contributes to solving problems in machine translation systems, information retrieval, as well as in analysis and processing systems in the Kazakh language.

**Keywords:** word2vec, model, vector, word, representation, semantic, analysis, Kazakh, language.

**1 Introduction**

Computer semantic analysis is closely related to the problem of text understanding by a machine. There are many interpretations of the concept "meaning of the text" and the tasks of understanding it. For example, according to D.A. Pospelov [1], the system understands the text entered into it if from the point of view of a person (or a group of experts) it correctly answers questions related to the information contained in the text. Here we can talk not about simply obtaining facts that are clearly present in the text, but about revealing the hidden meanings that the author introduces. D.A. Pospelov identifies several levels of text comprehension, from the point of view of the complexity of the questions that the intellectual system should be able to answer. Guided by the definition from [1], the meaning of the text can be considered as a description of the knowledge contained in it, in the formal language of knowledge representation, which allows solving a fairly wide range of problems related to text analysis, and the problem of semantic analysis - as a translation of a natural language - expressions into the language of knowledge representation. For example, the language of first-order predicates, semantic networks, frames, as well as ontologies and thesauri can act as a language for representing knowledge of a text in a natural language.

In the 60s and 70s, the main approach to representing the semantics of a language was the component approach, within which the meaning of each word in a natural language had to be represented as a combination of semantic universals. By the mid-1980s, it became clear that a generally accepted set of such universals had never been compiled. Relational semantics has become an alternative to the component approach in semantics. In this approach, the meanings of the words of the language are described by setting connections with the meanings of other words, and the entire conceptual system of the language is represented as a semantic network [2].

Review of methods and software approaches for semantic analysis. Of course, no software can replace the analysis that humans can think of. However, the programs that are currently being developed can reduce the time spent studying large databases. In this regard, the work of the following programs for

solving problems of semantic text analysis is considered. Software offered by various manufacturers, such as “Semantic LLC”, “Tomita-parser (Yandex)”, Semantic Analyst “JHON”, “SummarizeBot API”, “TextAnalyst 2.0”, “Galaktika-ZOOM”, “NLP ISA”, “Natasha” and etc. is used in various subject areas and for different languages [3-10]. A complete overview of existing modern systems of semantic analysis and their description are presented in table 1.

Table 1 – Review of modern software systems for semantic text analysis

System name	Description
“Semantic LLC”	is a program for editing unstructured text. The semiconductor line is graphically oriented, each node is a semantic element, and the walls represent the elements of the elements. Each node attribute has a great value, the set of attributes depends on the element type.
“Tomita-parser (Yandex)”	a program that allows you to extract facts from structured text. Separation of facts is based on context-independent grammar rules. And the program requires a dictionary of keywords. The parser will write its own grammar.
“JHON”	The semantic analyst "JHON" receives the meanings of a natural language in Russian and solves the following tasks: lexical analysis, morphological analysis, syntactic analysis, semantic analysis - involving the triad of subject-object relations, creating a semantic network of text, fact of events.
“SummarizeBot API”	The web service offers a RESTful API to handle all text and image processing tasks. It uses over 100 languages including Russian, English, Chinese, Japanese and uses machine learning technology. The current version uses the following parameters: 1) automatically link to text; 2) Selection of keywords and conceptual documents; 3) Analysis of a sample of documents and selection of material objects and attributes; 4) Automatically detect the language of the document; 5) Obtaining unpublished data: the main text of articles, forums, forums, etc.; 6) Image processing: identification and recognition of objects in images.
“TextAnalyst 2.0”	the program was developed by the research and production innovation center MicroSystems as a tool for text analysis. Text links allow you to create a semantic web of comments, expressed in processed text. The request has the ability to semantic search for text fragments, taking into account the semantic links hidden in the text. Allows you to parse text by composing a hierarchical tree / heading topics containing text.
“Galaktika-ZOOM”	an automated information search and analysis system manufactured by the Galaktika Corporation. It is a powerful editing and processing tool that allows you to get the information you need in large quantities. It is offered as a commercial system with consumers in advertising, government, and media. This program allows you to build semantic networks, but its program codes are not shared with the system.
“NLP ISA”	For the text, a tree of analyzed analysis was built, the semantic role and connections were established. Allows you to select serialized syntax and semantic analysis mode. Alternatively, you can also select a mode that has a syntax-semantic mode combination.
“Natasha”	it is a set of rules for getting a Tomita parser for Python and a set of ready-to-execute rules, addresses, terms, sums and other objects.

Scientific works [11-13] describe the basic ideas of information retrieval. Various options for finding text statistics are presented, which include counting the number of occurrences of words in documents and the frequency of word contiguity, and new model architectures for computing continuous vector representations of words from very large datasets. The quality of vector representations of words obtained by various models was studied using a set of syntactic and semantic language problems. In [14], the application of language models of a neural network to the problem of calculating semantic similarity for the Russian language is shown. The tools and bodies used, and the results achieved are described.

The above software products are designed for multi-resource languages such as English, Spanish, Russian, etc. Unfortunately, for the Kazakh language now there is no software implementation in the open access. This is since the Kazakh language differs in its semantic and linguistic properties from others, and also does not have large linguistic resources for conducting applied research.

## 2 Algorithm for semantic analysis of text in the Kazakh language

During digital technologies, given the constant growth of the volume of digital data, an important role is played by improving the quality of information retrieval using new semantic approaches and methods.

To work with big data, various algorithms and methods are being developed for the machine solution of this problem, since the amount of data does not allow for manual analysis. Any natural language is

complex, unique, and multifaceted in its own way, therefore, extracting data from documents and text resources is a large and time-consuming work that requires preliminary processing.

Based on the research done from the developed models used most for the semantic analysis of text resources, there is an approach based on machine learning. Below will be presented the developed algorithm for semantic analysis of text in the Kazakh language and implementation based on this approach. When developing an algorithm to map certain information to a certain attribute, we opted for a neural network (NN) with a hidden layer (100). Neural network training consists of the following parts:

- Text preprocessing. Text preprocessing consists of three stages: tokenization, removal of stop words, normalization of words.
- Construction of the feature vector. The feature vector is a sign of the characteristic we are interested in. For one descriptor, the features were taken as follows: a window of two words after, five before was taken in the text of the article at the place of occurrence of the element. Moreover, a dictionary is formed for each descriptor, which is responsible for the presence of the specified word in the dictionary. All features of each descriptor are collected into one and a feature vector is constructed.
- Training the neural network. The network is trained by presenting each input dataset and then propagating the error.

At the second stage, the neural network was trained. For text preprocessing, the developed natural language processing modules were used. After applying these modules, we extracted the features of our descriptor. A feature vector was then constructed using the extracted data. The constructed feature vector was compared with certain keywords, determined by the modified TF-IDF method for the Kazakh language.

### **3 Software solution and algorithm implementation**

This is one of the most difficult and demanded tasks facing artificial intelligence is NLP (Natural Language Processing). To solve and implement NLP tasks currently, there are several software systems and libraries, which include the tasks of speech recognition, language formation and information acquisition, etc.

Python is currently one of the most promising programs for solving NLP problems. Libraries written in Python are designed to solve NLP problems and allow you to simulate various languages and processing functions.

There are also many types of libraries, consider the most famous and applicable for word processing tasks:

Spacy, NLTK, CoreNLP, StanfordNER, etc. Table 2 below shows a comparison of the functional capabilities for solving the NLP problem.

Table 2 – Comparison of the capabilities of libraries aimed at solving NLP problems

Function	Spacy	NLTK	CoreNLP
Programming language	Python	Python	Java/Python
Neural network models	+	-	+
Vector of integrated words	+	-	-
Multilingual model	+	+	+
Tokenization	+	+	+
POS tagging	+	+	+
Segmentation	+	+	+
Parsing	+	-	+
Highlighting named objects	+	+	+
Communication between objects	-	-	-

Having studied the technical possibilities for the implementation of the semantic analysis algorithm and training the neural network, the authors will use the Spacy and StanfordNER libraries. The StanfordNER and Spacy libraries allow us to model our own model. It also allows you to make the necessary configurations, depending on the specifics of the (Kazakh) language in question.

It is necessary to define the StanfordCoreNLP settings [15]: token- tokenize; ssplit - distribution of offers; pos - speech definition; lemma - find the original form of each word; ner - highlighting named objects; - regexner - work with regular expressions; parse - semantic analysis of each word; depparse - definition of syntax between words and sentences.

Further, figure 1 shows the developed algorithm for the implementation of semantic analysis taking into account keywords and describes the work of the modules.

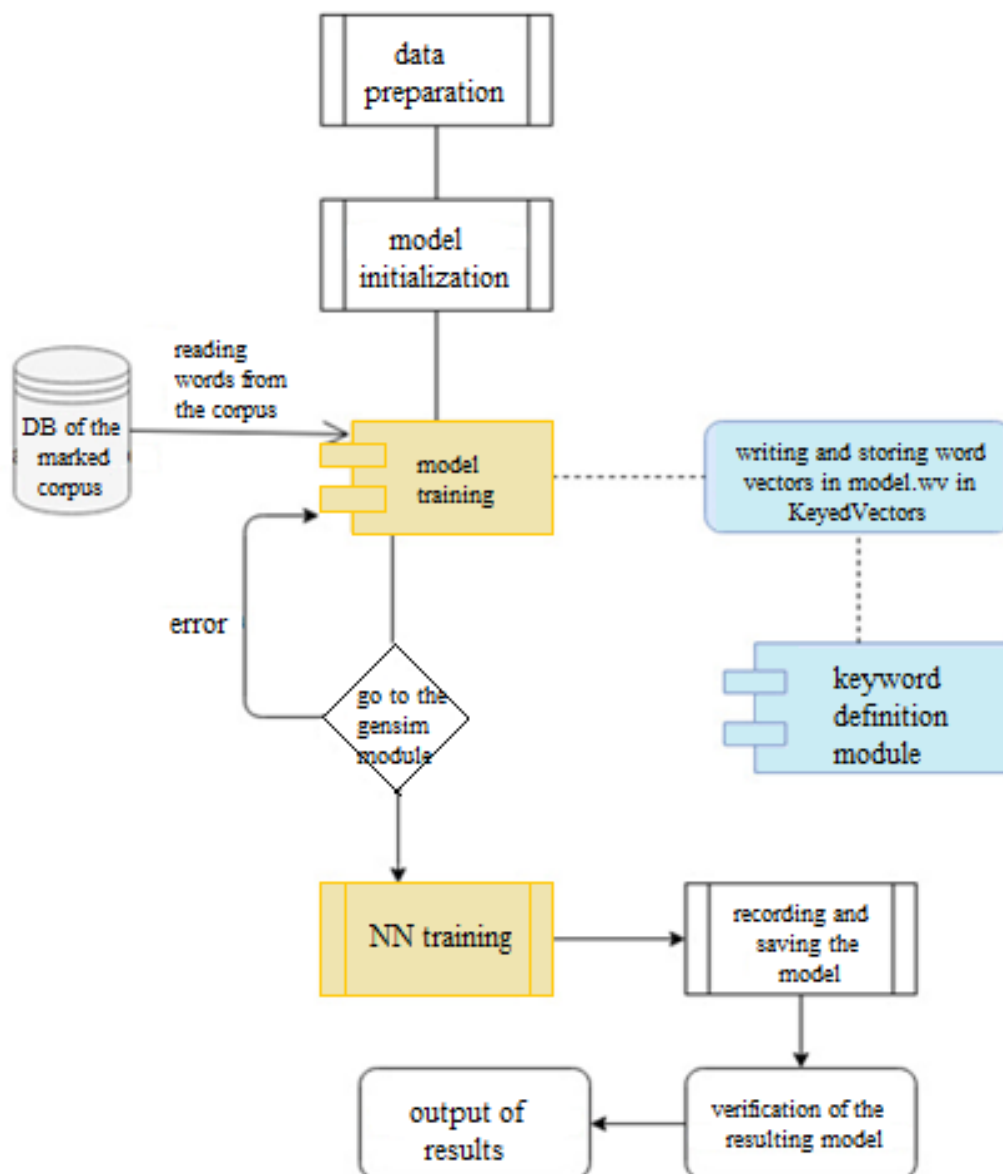


Figure 1 – Algorithm for the implementation of semantic analysis taking into account keywords

The input is text data. To train the model, set the following parameters: The dimension of the feature vectors is 100; The maximum distance between the current and predicted word in a sentence is 5; The minimum education level is 1; The cutoff frequency is 4 words.

```
>>> model = WordVec(sentences, size=100, window=5, min_count=5,
workers=4)
```

Record initialized model

```
>>> model.save(fname)
```

```
>>> model = WordVec.load(fname) #
```

Now you can train with the resulting model. For training the model, a monolingual Kazakh corpus was prepared, which is in the SQL database. When the text is processed by the model, vectors of words are identified, which are stored in the model.wv module in KeyedVectors. The resulting vectors of words are also compared with keywords (phrases) from the text corpus for the purpose of further use as possible values of semantic attributes of entities. Once the model finishes training, you can go to `gensim.models.KeyedVectors` in `wv`:

```
>>> word_vectors = model.wv
```

```
>>> del model
```

The `gensim.models.phrases` module automatically detects a long chain of words. This module allows us to define phrases through learning.

```
>>> bigram_transformer = gensim.models.Phrases(sentences)
```

```
>>> model = Word2Vec(bigram_transformer[sentences], size=100, ...)
```

```
class gensim.models.wordvec.Corpora(dirname)
```

```
class
```

```
gensim.models.wordvec.LineSentence(source,max_sentence_length=10000,
limit=None)
```

After completing the `gensim` module, you can then start training the neural network.

```
sentences = LineSentence('myfile.txt')
```

```
from gensim.models import Word2Vec # define training data
```

```
sentences = [['ұл (ul)', 'балалар (balalar)', 'қыздарға (qyzdarga)',
'қарағанда (qaraganda)', 'мықты (myqtu)', 'болады (bolady)'],
['Ал (Al)', 'қыз (qyz)', 'балалар (balalar)', 'ұлдарға (uldarga)',
'қарағанда (qaraganda )', 'нәзік (nazik)'], ['Қыз (Qyz)', 'әлемнің
(alemning)', 'көркі (korki)'],
['Гүл (Gul)', 'жердің (zherding)', 'көркі (korki)'],
['Қазақстан (Kazakhstan)', 'республикасы (respublikasy)', 'тәуелсіз
(tauelsiz)', 'мемлекет (memleket)']]...
```

As a result of the obtained trained model, it is necessary to check the obtained data. You can also create a graphical interpretation of the results (Figure 2).

The NER Stanford software package was used to train the model. The following is a listing of working with the Stanford NER library and software implementation

```
>>> trainFile = train/dummy-kazakh-corpus.tsv serializeTo = dummy-
ner-kazakh-french.ser.gz map = word=0,answer=1
useClassFeature=true useWord=true useNGrams=true noMidNGrams=true
maxNGramLeng=6 usePrev=true useNext=true
useSequences=true usePrevSequences=true maxLeft=1 useTypeSeqs=true
useTypeSeqs2=true useTypeySequences=true wordShape=chris2useLC
useDisjunctive= true
```





Figure 2 - Graphical representation of the vector space of practical results of the semantic analysis of the text in the Kazakh language

```
>>> cd stanford-ner-tagger/
java -cp "stanford-ner.jar:lib/*" -mx4g edu.stanford.nlp.ie.crf.CRFClassifier -prop train/prop.txt
```

```
1 # coding: utf-8
2
3 import nltk
4 from nltk.tag.stanford import StanfordNERTagger
5
6 # Optional
7 import os
8 java_path = "C:\Program Files (x86)\Java\jdk1.8.0_201"
9 os.environ['JAVA_HOME'] = java_path
10
11 sentence = u"Қазақстанда алма өседі. Алматы қаласында ҚазНУ жоғары оқу орны орналасқан"
12
13 jar = './stanford-ner-tagger/stanford-ner.jar'
14 model = './stanford-ner-tagger/my-ner-model-french.ser.gz'
15
16 ner_tagger = StanfordNERTagger(model, jar, encoding='utf8')
17
18 words = nltk.word_tokenize(sentence)
19 print(ner_tagger.tag(words))
```

Figure 3 – An example of input data of text for the program .NER

The problem was successfully solved by using a morphological parser for marking up parts of speech in texts with the subsequent application of the machine learning method of semantically related keywords (phrases). A trained neural network with a hidden layer is applied to the set of these phrases in order to assign a specific phrase to a specific attribute of the entity described in the text. Thus, based on a set of semantically related pairs of words, an ontology is built for a specific document, which is formed during the operation of a neural network.

### Conclusion

To solve the problem, semantic analysis in the Kazakh language is based on machine learning. The program is implemented in the python programming language, using the libraries gensim, matplotlib, sklearn, numpy, etc. A set of vectors of words in the Kazakh language was obtained, which was trained on the corpus, which is 1 million sentences. The corpus is fed to the program input in a normalized form. Further, to improve the result, the corpus will be supplemented with proposals on various topics.

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## НЕЙРОНДЫҚ ЖЕЛІЛЕРГЕ НЕГІЗДЕЛГЕН ҚАЗАҚ ТІЛІНІҢ СЕМАНТИКАЛЫҚ ТАЛДАУЫ

**Аннотация.** Ақпараттық және смарт технологиялардың, жасанды интеллект жүйелерінің дамуына табиғи тілдерді өңдеу ғылыми зерттеу саласы үлкен ықпалын тигізіп жатыр. Мақалада семантикалық талдауға арналған қазіргі уақыттағы әдістер мен бағдарламалық тәсілдеріне жасалған жалпы шолу қамтылған. Жүргізілген зерттеулер негізінде мәтіндік ресурстарды семантикалық талдау үшін машиналық оқытуға негізделген әдіс көп қолданылатыны анықталды. Мақалада кілт сөздерді ескеру арқылы қазақ тіліндегі мәтінге семантикалық талдау жасаудың алгоритмі ұсынылған және модульдер жұмысы сипатталған. Белгілі бір ақпаратты белгілі бір атрибутқа сәйкестендіру алгоритмін жасағанда таңдау жасырын қабаты бар (100) нейрондық желіге тоқтады (NN). Бастапқы кезеңде мәтінді алдын ала өңделеді. Мәтінді алдын ала өңдеу үшін табиғи тілді өңдеуге арналған өзіміз жасаған модульдер қолданылды. Осы модульдерді қолданғаннан кейін дескриптор белгілері алынды. Содан кейін алынған мәліметтерді қолдана отырып, белгілер векторы құрылды. Құрылған белгілер векторы қазақ тілі үшін өзгертілген TF-IDF әдісімен анықталған белгілі бір кілт сөздермен салыстырылды. Екінші кезеңде нейрондық желі оқытылды. Сондай-ақ, берілген әдістің Python бағдарламалау тілінде орындалған бағдарламалық шешімі ұсынылған. Бағдарламалық жасақтаманы іске асыруда gensim, matplotlib, sklearn, numpy және т.б. сияқты кең тараған кітапханалар қолданылды. Модельді оқыту үшін келесі параметрлер орнатылды: мүмкіндік векторының өлшемі 100; сөйлемдегі ағымдағы және болжанатын сөз арасындағы аса үлкен арақашықтық 5; білімнің минималды деңгейі - 1; кесу жиілігі 4 сөзден тұрады. Сонымен қатар, модельді оқыту үшін қазақ тілінде 1 миллион сөйлемнен тұратын және SQL мәліметтер базасында орналасқан біртұтас қазақ корпусы дайындалды. Мәтінді модельмен өңдеген кезде KeyedVector-да model.wv модулінде сақталған сөз векторлары анықталады. Алынған сөз векторлары мәтін корпусындағы кілт сөздермен (сөз тіркестерімен) салыстырылады, бұл семантикалық атрибуттардың ықтимал мәнін әрі қарай пайдалану мақсатында жасалады. Нақты бір құжатқа арналған онтология нейрондық желінің жұмысы барысында пайда болатын семантикалық байланысты жұп жиынтығын қолдану арқылы жасалған. Жұмысымызда жүргізілген тәжірибе нәтижесі сөз жиынтығының графикалық түрінде көрсетілген. Ұсынылған тәсілдің жаңалығы – қазақ тіліндегі мәтін мағынасы жағынан жақын сөздерді анықтау. Бұл жұмыс машиналық аударма жүйесіндегі, ақпаратты іздеудегі, сонымен қатар талдау және өңдеу жүйесіндегі мәселелерді шешуге ықпал етеді.

**Түйін сөздер:** word2vec, модель, сөз, векторлық, көрініс, семантикалық, талдау, қазақ, тілі.

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## СЕМАНТИЧЕСКИЙ АНАЛИЗ КАЗАХСКОГО ЯЗЫКА НА ОСНОВЕ ПОДХОДА НЕЙРОННЫХ СЕТЕЙ

**Аннотация.** В данной работе представлен обзор существующих современных методов и программных подходов семантического анализа. На основе проделанных исследований выявлено, что для семантического анализа текстовых ресурсов наиболее применяется подход, основанный на машинном обучении. В данной статье представлен

разработанный алгоритм семантического анализа текста на казахском языке с учетом ключевых слов и описаны работы модулей. При разработке алгоритма для сопоставления определенной информации определенному атрибуту, выбор был остановлен на нейронной сети (НС) со скрытым слоем (100). Для начала выполняется предобработка текста. Для предобработки текста были использованы разработанные модули обработки естественного языка. После применения данных модулей были извлечены признаки нашего дескриптора. Затем с помощью извлеченных данных был построен вектор признаков. Построенный вектор признаков сопоставлялся с определенными ключевыми словами, определенный модифицированным методом TF-IDF для казахского языка. На втором этапе происходило обучение нейронной сети. В работе также представлено программное решение данного подхода, реализованного на языке программирования Python. В программной реализации были использованы известные библиотеки, такие как gensim, matplotlib, sklearn, numpy и т.д. Для обучения модели были заданы следующие параметры: Размерность векторов признаков составляет 100; Максимальное расстояние между текущим и предсказанным словом в предложении составляет 5; Минимальный уровень обучения 1; Пороговая частота среза 4 слов. Также для обучения модели был подготовлен одноязычный казахский корпус, который составляет 1 млн предложений на казахском языке и который находится в БД SQL. При обработке текста моделью выявляются вектора слов, которые хранятся в модуле model.wv в KeyedVectors. Полученные вектора слов также сопоставляются с ключевыми словами (словосочетаниями) из корпуса текстов с целью дальнейшего использования в качестве возможных значений семантических атрибутов сущностей. По набору семантически связанных пар слов строится онтология для конкретного документа, формирующаяся при работе нейронной сети. В работе представлены результаты проведенных экспериментов в графическом виде набора слов. Новизна предлагаемого подхода заключается во выявлении семантически близких слов по смыслу в текстах на казахском языке. Эта работа несет свой вклад в решение задач в системах машинного перевода, информационного поиска, а также в системах анализа и обработки на казахском языке.

**Ключевые слова:** модель, word2vec, векторное, представление, слов, семантический, анализ, казахский, язык.

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**SAFE THRESHOLDS FOR THE PARAMETERS OF CRYPTOGRAPHIC  
ALGORITHMS AND STANDARD ST RK 1073-2007**

**Abstract.** This article considers the problems of finding modern secure thresholds for the parameters of cryptographic algorithms. We conducted a comparative analysis of the obtained secure thresholds against the thresholds of the state standard of the Republic of Kazakhstan ST RK 1073-2007 "Means of cryptographic protection of information. General technical requirements." Based on the results of the analysis and taking into account the experience of certification tests, we worked out specific proposals for amendments and additions to this Standard.

**Key words:** information security, cryptography, cryptographic algorithm parameters, state standard, security level.

**Introduction.** The state standard of the Republic of Kazakhstan ST RK 1073-2007 "Means of cryptographic protection of information. General technical requirements" (hereinafter "the Standard") was adopted 12 years ago and became the main Kazakhstan standard for assessing the quality of Means of Cryptographic Protection of Information (hereinafter – MCPIs) [1, 2].

Over the past time, theoretical cryptography has received new development, as well as the qualifications and computational capabilities of potential adversaries using distributed (network based) and quantum computing have also increased. At the same time, the use of global communication networks has significantly increased, including in banking information and payment systems that need cryptographic protection of information. All of this, makes it relevant, to define modern safe thresholds for parameters of cryptographic algorithms. Moreover, the built models of cryptographic information protection adversaries of also prove that some provisions of the Standard, especially those related to the first and second level of security, are outdated, and the Standard itself needs to be updated[3].

When updating the Standard, it is advisable to be guided by the following conceptual principles that had been substantiated and, to a large extent, verified by previous editions of the Standard [1, 4-6]:

1. Ensuring consistency with the previous editions of the Standard. This, on the one hand, will make it easier for developers and researchers of MCPIs to make a smooth transition to new requirements, and on the other hand, it will allow government bodies and other users of MCPIs to be guided by previously adopted regulatory legal acts in the field of MCPIs, presumably with minor changes.

2. Defining in the Standard all used cryptographic terms will facilitate an unambiguous understanding and application of the Standard, eliminate the need to constantly refer to the scientific and technical literature for the interpretation of terms.

3. Defining in the Standard four security levels of MCPIs associated with possible damage from disclosure, imposition, or uncontrolled changes in the protected information, the budget of a potential adversary, as well as the computational and spatial complexity of the known cryptographic-protection breaking algorithms. This will allow even unskilled users of MCPIs to build adequate protection. Besides, for the sake of consistency, it is advisable not only to maintain the four security levels but if the

requirements for them are tightened, it is highly desirable to do so in such a way that the requirements of each security level of a new version of the Standard do not exceed the requirements of the higher security level of the previous version. This will prevent or limit at one level a decrease in the security level of MCPIs when switching to a new edition of the Standard, which is extremely important for already deployed information and telecommunication systems.

4. Defining in the Standard general requirements, which are elementary in theoretical and applied cryptography and, therefore, are imposed on all MCPIs regardless of the level of security. This will help to counter threats exploiting the lack of deep cryptographic knowledge among individual developers and owners of MCPIs.

5. Defining in the Standard the main parameters of cryptographic algorithms and their secure threshold values, as a rule, with a 20% margin of strength for their absolute values, which provide resistance to known universal algorithms (attacks) for breaking cryptographic protection of the appropriate computational complexity. The need to introduce a margin of strength is confirmed by the history of the development of cryptography, which demonstrates the emergence of cryptographic attacks even on admittedly strong cryptographic algorithms that are more effective than universal attacks. These requirements will help the designers of MCPIs in the development and selection of cryptographic algorithms and protocols of the required strength, as well as, in certification and other tests, effectively identify cryptographic algorithms that are insecure against both universal algorithms for breaking cryptographic protection and most special attacks.

6. Defining in the Standard additional organizational and technical requirements for MCPIs depending on the level of security. This will allow MCPIs to withstand additional threats that substantially depend on the scientific, technical, operational and financial capabilities of the adversary.

7. Quitting in the Standard any definition of specific cryptographic algorithms and protocols. This makes it possible to conduct certification tests of MCPIs of different types and purposes, various domestic and foreign manufacturers.

**Analysis of general requirements.** In paragraphs 4 "General provisions", 5.1 "General requirements for MCPIs" and 5.2 "Requirements for technical documentation for MCPIs" of the Standard [1] set forth the common requirements for all MCPIs.

1.1. Subparagraphs 4.3.1, 4.3.2, 4.3.3, 4.3.4 and paragraph 4.4 of the Standard define four security levels, linked to the cost of the information to be protected with no more than 100, 10,000, 1,000,000 and 100,000,000 MCI for 1, 2, 3 and 4 security levels, as well as with the computational complexity of known algorithms for breaking cryptographic protection at  $2^{50}$ ,  $2^{80}$ ,  $2^{120}$  and  $2^{160}$ , respectively. The thresholds for the computational complexity of breaking algorithms for levels 1 and 2 are no longer secure [3]. Besides, the development of bank information and payment systems requires the processing of higher-priced information. Therefore, taking into account the models of cryptographic information protection adversaries, it is expedient to amend the wording in subparagraphs 4.3.1, 4.3.2, 4.3.3, 4.3.4 and paragraph 4.4 of the Standard, as follows:

"4.3.\* MCPIs of the first (second, third, fourth) security level are designed to protect the information, the damage from disclosure, imposition or unauthorized modification of which in the amount protected using the same key (the same keys) does not exceed 100 (50 thousand, 25 million, 10 billion) monthly calculation indices, from potential adversaries with a budget of no more than 1,000 (1 million, 1 billion and 1 trillion) monthly calculation indices.

4.4 MCPIs cannot be recognized as appropriate to the first, second, third or fourth security levels if an algorithm for breaking cryptographic protection provided by them is known, the computational complexity of which is less than  $2^{64}$ ,  $2^{96}$ ,  $2^{128}$  and  $2^{160}$  operations, respectively, with due consideration of the inverse multiplicative correction for the probability of its successful application. If an algorithm for breaking cryptographic protection has a space complexity of at least  $2^{60}$ ,  $2^{80}$ ,  $2^{100}$  and  $2^{120}$  bits, respectively, then this algorithm is considered inapplicable."

1.2. Paragraph 4.2 of the Standard states that MCPIs are considered as technologically complete (workable) means. This wording does not allow us to unambiguously interpret this paragraph as a requirement, although the workability of the MCPIs is of fundamental importance. To correct this, it is advisable to transfer paragraph 4.2 of the Standard from Section 4 "General Provisions" to Section 5 as follows:

"5.1.1 MCPIs shall be technologically complete (workable) hardware, software, or firmware."

1.3. Subparagraphs 5.1.1, 5.3.6, 5.4.6, 5.5.6 and 5.6.6 of the Standard impose requirements on the randomness of generated and formed keys. However, the requirements are largely duplicated, not localized in one paragraph for each level of security, which also complicates the presentation and understanding of the results of certification tests in scientific reports and protocols. To correct this, it is advisable to exclude subparagraph 5.1.1 of the Standard, amend the wording of subparagraphs 5.3.6, 5.4.6, 5.5.6 and 5.6.6, and supplement the list of terms with the notions "random sequence of bits", "pseudo-random sequence of bits" and "non-deterministic pseudo-random sequence of bits".

1.4. Subparagraph 5.2.1 of the Standard requires a complete description of the implemented cryptographic transformation algorithms, and subparagraph 5.2.2 allows replacing the complete description with references to standards defining these algorithms. To correct the formal contradiction between these subparagraphs, it is reasonable to combine them into one subparagraph with the requirement for a complete description of the implemented algorithms or the presence of references to the standards defining these algorithms:

"5.2.1 The technical documentation (design, technological and software documentation, depending on the type of MCPI) for all cryptographic transformation, key generation, formation, distribution, and management algorithms implemented in a MCPI shall contain their full description or references to the defining them state and interstate standards or other regulatory documents on standardization, effective or applicable in the Republic of Kazakhstan in the prescribed manner."

**Analysis of the requirements for the parameters of cryptographic algorithms.** Paragraphs 5.3 "Requirements for MCPIs of the first security level", 5.4 "Requirements for MCPIs of the second security level", 5.5 "Requirements for MCPIs of the third security level" and 5.6 "Requirements for MCPIs of the fourth security level" of the Standard [1] set forth the requirements for the parameters of cryptographic algorithms that directly depend on the computational complexity of the well-known universal algorithms for breaking cryptographic protection. These requirements only partially comply with the above secure thresholds of  $2^{64}$ ,  $2^{96}$ ,  $2^{128}$  and  $2^{160}$  operations for security levels 1, 2, 3 and 4, respectively.

2.1. Subparagraphs 5.3.1, 5.4.1, 5.5.1 and 5.6.1 of the Standard specify that the key length of symmetric cryptographic transformation algorithms implemented by MCPIs shall be at least 60, 100, 150 and 200 bits for security levels 1, 2, 3 and 4, respectively. In the case of using these threshold key lengths, the all-key brute-force algorithm, which is universal and the only one applicable to all symmetric algorithms, will be able to break cryptographic protection in  $2^{60}$ ,  $2^{100}$ ,  $2^{150}$  and  $2^{200}$  encryption operations [7-11]. The value of  $2^{60}$  is below the secure threshold of computational complexity for security level 1, as  $2^{60} < 2^{64}$ . The values of  $2^{100}$  and  $2^{150}$  correspond to secure thresholds of computational complexity for 2nd and 3rd security levels, but do not provide additional security margins of 20%, since  $2^{100*80\%} = 2^{80} < 2^{96}$  and  $2^{150*80\%} = 2^{120} < 2^{128}$ . Therefore, to ensure the secure key length of symmetric cryptographic transformation algorithms, it is necessary to increase their threshold values in subparagraphs 5.3.1, 5.4.1 and 5.5.1 of the Standard to 80, 120 and 160 bits, respectively, while the requirements of subparagraph 5.6.1 of the Standard can be left unchanged. Then the brute-force algorithm will be able to break cryptographic protection in only  $2^{80}$ ,  $2^{120}$ ,  $2^{160}$  and  $2^{200}$  encryption operations with an additional security margin of 20%, since  $2^{80*80\%} = 2^{64}$ ,  $2^{120*80\%} = 2^{96}$ ,  $2^{160*80\%} = 2^{128}$  and  $2^{200*80\%} = 2^{160}$ .

2.2. Subparagraphs 5.3.2, 5.4.2, 5.5.2 and 5.6.2 of the Standard indicate that the key length of asymmetric cryptographic transformation algorithms implemented by MCPIs shall be at least 120, 160, 250 and 400 bits for security levels 1, 2, 3 and 4, respectively. In the case of using these threshold key lengths, the all-key brute-force algorithm will be able to break cryptographic protection in  $2^{120}$ ,  $2^{160}$ ,  $2^{250}$  and  $2^{400}$  encryption operations, respectively. However, in all modern MCPIs, the implemented asymmetric cryptographic transformation algorithms use, as a one-way function, exponentiation in some finite multiplicative group  $G$ , usually cyclic and with order  $ord(G) \approx 2^k$ , where  $k$  is the length of the secret and/or public key. Thus the cryptographic strength of these algorithms is based on the computational complexity of the discrete logarithm problem in an arbitrary or cyclic finite multiplicative group. The computational complexity of known effective discrete logarithm algorithms in these groups (the Gelfond-Shanks algorithm, also called the baby-step giant-step algorithm, Pollard's kangaroo ( $\lambda$ ) algorithm, Pollard's rho ( $\rho$ ) algorithm, etc.), which do not impose significant additional restrictions on the group properties, is  $O(\sqrt{ord(G)}) \approx \sqrt{ord(G)} \approx \sqrt{2^k} = 2^{k/2}$  and, respectively,  $2^{60}$ ,  $2^{80}$ ,  $2^{125}$ , and  $2^{200}$  operations for security levels 1, 2, 3 and 4 [7, 8, 12, 13]. The values of  $2^{60}$ ,  $2^{80}$ , and  $2^{125}$  do not attain the secure

thresholds of computational complexity for security levels 1, 2, and 3, since,  $2^{60} < 2^{64}$ ,  $2^{80} < 2^{96}$  and  $2^{125} < 2^{128}$ . Therefore, to ensure the secure key length of asymmetric cryptographic transformation algorithms, it is necessary to increase their threshold values in subparagraphs 5.3.2, 5.4.2 and 5.5.2 of the Standard to 160, 240 and 320 bits, respectively, while the requirements of subparagraph 5.6.2 of the Standard can be left unchanged. Then, discrete logarithm algorithms in an arbitrary or cyclic finite multiplicative group will be able to break cryptographic protection in only  $2^{80}$ ,  $2^{120}$ ,  $2^{160}$  and  $2^{200}$  operations, respectively, with an additional security margin of 20%, since  $2^{80*80\%} = 2^{64}$ ,  $2^{120*80\%} = 2^{96}$ ,  $2^{160*80\%} = 2^{128}$  and  $2^{200*80\%} = 2^{160}$ .

2.3. Subparagraphs 5.3.3, 5.4.3, 5.5.3 and 5.6.3 of the Standard state that the key length of implemented by MCPIs asymmetric cryptographic transformation algorithms, the cryptographic strength of which is based on the computational complexity of the factorization problem or the discrete logarithm problem in a finite field, shall be at least 500, 1500, 4000 and 8000 bits for security levels 1, 2, 3 and 4, respectively. These subparagraphs reflect the fact that in many MCPIs, the implemented asymmetric cryptographic transformation algorithms use, as a mandatory one-way function, exponentiation in a finite field  $P$  whose order  $ord(P) = n \approx 2^k$ , where  $k$  is the length of the secret and/or public key. Notice that to provide a "loophole" for such a one-way function, the popular RSA algorithm uses the product of two secret primes:  $n = p \times q$  as an element of the public key, and the secret key  $d$  falls in the range from 1 to  $\varphi(n) = (p-1)(q-1)$ , i.e.  $n = p \times q \approx (p-1)(q-1) \approx 2^k$ , where  $k$  is the length of the private key  $d$ . Hence, the cryptographic strength of these algorithms is based on the computational complexity of composite factorization or the discrete logarithm problem in a finite field. The computational complexity of the known effective factorization and discrete logarithm algorithms in a finite field (sieve algorithms for a number field) that do not impose additional restrictions on the properties of a composite number and a finite field is subexponential and evaluated as  $L_n(1/3, (64/9)^{1/3})$ , where  $L_n(\alpha, c) = O(\exp((c+o(1))(\ln n)^\alpha (\ln \ln n)^{1-\alpha}))$  [7, 8, 12, 13]. Special algorithms for solving these problems (for example, special number field sieve algorithms) that impose significant additional restrictions on factorizable numbers and finite fields have a slightly lower computational complexity of  $L_n(1/3, (32/9)^{1/3})$ . In the case of using the existing threshold key lengths, the above universal algorithms will be able to break cryptographic protection in  $2^{63.3}$ ,  $2^{102.3}$ ,  $2^{155.0}$  and  $2^{206.5}$  operations, and special algorithms – in  $2^{50.2}$ ,  $2^{81.2}$ ,  $2^{123.0}$ , and  $2^{163.9}$  operations for security levels 1, 2, 3 and 4, respectively. The value  $2^{63.3}$  is below the secure threshold of computational complexity for security level 1, since  $2^{63.3} < 2^{64}$ . The value  $2^{102.3}$  exceeds the secure threshold of computational complexity for security level 2, but does not provide an additional security margin even at 7%, since  $2^{102.3*93\%} \approx 2^{95.1} < 2^{96}$ . Therefore, to ensure the secure key length of asymmetric cryptographic conversion algorithms, the cryptographic strength of which is based on the computational complexity of the problem of composite factorization or the discrete logarithm problem in a finite field, it is necessary to increase their threshold values in subparagraphs 5.3.3 and 5.4.3 of the Standard to 1000 and 2000 bits, respectively, while the requirements of subparagraphs 5.5.3 and 5.6.3 of the Standard can be left unchanged. Then, the above universal algorithms will be able to break cryptographic protection in only  $2^{85.9}$ ,  $2^{115.7}$ ,  $2^{155.0}$  and  $2^{206.5}$  operations with additional security margins of about 20%, since  $2^{85.9*74.5\%} \approx 2^{64}$ ,  $2^{115.7*83.0\%} \approx 2^{96}$ ,  $2^{155.0*82.6\%} \approx 2^{128}$  and  $2^{206.5*77.5\%} \approx 2^{160}$ .

2.4. Subparagraphs 5.3.4, 5.4.4, 5.5.4, and 5.6.4 of the Standard specify that the length of the hash code calculated by MCPIs shall be at least 120, 160, 250 and 400 bits for security levels 1, 2, 3 and 4, respectively. If these threshold hash code lengths are used, the universal pre-image search algorithm by exhaustive search of pre-images will be able to break cryptographic protection in  $2^{120}$ ,  $2^{160}$ ,  $2^{250}$  and  $2^{400}$  hash operations, but Yuval's algorithm (attack) for collision search, based on the birthday paradox and having computational complexity of  $\approx 2^{m/2}$ , where  $m$  is the length of the hash code, will be able to break cryptographic protection in  $2^{60}$ ,  $2^{80}$ ,  $2^{125}$  and  $2^{200}$  hashing operations, respectively [7, 8, 14]. The values  $2^{60}$ ,  $2^{80}$  and  $2^{125}$  are below the corresponding secure thresholds of computational complexity for security levels 1, 2, and 3, since  $2^{60} < 2^{64}$ ,  $2^{80} < 2^{96}$  and  $2^{125} < 2^{128}$ . Therefore, to ensure secure hash code lengths, it is necessary to increase their threshold values to 160, 240 and 320 bits, respectively, in subparagraphs 5.3.4, 5.4.4 and 5.5.4 of the Standard, while the requirements of subparagraph 5.6.4 of the Standard can be left unchanged. Then Yuval's algorithm will be able to break cryptographic protection in only  $2^{80}$ ,  $2^{120}$ ,  $2^{160}$

and  $2^{200}$  hashing operations with an additional security margin of 20%, since  $2^{80*80\%} = 2^{64}$ ,  $2^{120*80\%} = 2^{96}$ ,  $2^{160*80\%} = 2^{128}$  and  $2^{200*80\%} = 2^{160}$ .

2.5. The Standard does not impose any requirements on the length of Message Authentication Code (MAC) computed by MCPIs. However, the complexity of breaking cryptographic protection provided by a MAC substantially depends on its length  $m$ . Since message authentication codes are designed to control data integrity and provide protection against falsified data entry and unauthorized modification of messages, then in addition to the all-key brute-force algorithm accounted for in subparagraphs 5.3.1, 5.4.1, 5.5.1 and 5.6.1 of the Standard, for breaking cryptographic protection, universal search algorithms for MACs (guessing attempts) can be used. The exhaustive algorithm for MACs has a computational complexity of  $2^m$  operations for checking MACs [7, 8]. However, its characteristic feature is that the algorithm is not executed on the computing tools of an adversary, but by MCPIs, whose performance is significantly lower. Besides, MCPIs can limit the number of attempts to receive data with incorrect MACs. For these reasons, adversaries usually use for MACs a partial enumeration algorithm with the computational complexity  $r$  of operations for checking MACs or, taking into account the correction for the probability of its successful application,  $r / (r/2^m) = 2^m$ , where  $r$  is the number of attempts  $1 \leq r \ll 2^m$ . Therefore, to ensure a secure length of MACs, it is necessary to supplement paragraphs 5.3, 5.4, 5.5, and 5.6 of the Standard with the following subparagraphs:

"5.\*.5 The length of a MAC calculated by a MCPI shall be at least 80 (120, 160, 200) bits. For the sole purpose of protecting a MCPI against unauthorized modification, identifying corrupted keys, and garbled encrypted data, it is allowed to use shorter MACs, but not less than 15 (20, 30, 40) bits."

In the general case, search algorithms for MACs will be able to break cryptographic protection in only  $2^{80}$ ,  $2^{120}$ ,  $2^{160}$  and  $2^{200}$  operations for checking MACs with an additional security margin of 20%, since  $2^{80*80\%} = 2^{64}$ ,  $2^{120*80\%} = 2^{96}$ ,  $2^{160*80\%} = 2^{128}$  and  $2^{200*80\%} = 2^{160}$ . In the above-mentioned exceptional cases, the algorithms will be able to break the cryptographic protection provided by a MAC, in just  $2^{15}$ ,  $2^{20}$ ,  $2^{30}$  and  $2^{40}$  operations, as corrected. However, in these cases, a MAC is an additional line of defense and breaking by an adversary of the entire cryptographic protection of a MCPI will be complicated by the expected organizational and physical protection of the MCPI against unauthorized access and modifications, the complexity of the directed and secretive change of the MCPI operation; organizational, cryptographic (encryption) and other technical protection of keys at the stage of their distribution and loading, identification of mismatch of downloaded keys by synchronization protocols of several MCPIs; cryptographic (encryption) protection of encrypted data, as well as the execution of a search algorithm for MACs by a MCPI itself, and not on the high-performance computing facilities of an adversary. The need for introducing exceptions is dictated by the principles of maintaining consistency with the previous edition of the Standard and ensuring that the requirements of each security level of the new version of the Standard do not exceed the requirements of the higher security level of the previous edition and, in particular, the requirements of subparagraphs 5.4.7, 5.4.8, 5.5.7, 5.5.8, 5.6.7 and 5.6.8 of the current Standard.

2.6. Subparagraphs 5.3.5, 5.4.5, 5.5.5 and 5.6.5 of the Standard state that the length of an electronic digital signature (DS) generated by a MCPI shall be at least 120, 200, 300 and 400 bits for security levels 1, 2, 3 and 4, respectively. In the case of using these threshold lengths of DS, the universal exhaustive algorithm for signatures will be able to break cryptographic protection in  $2^{120}$ ,  $2^{200}$ ,  $2^{300}$  and  $2^{400}$  operations for DS checking. However, in many modern MCPIs, the implemented DS generation and verification algorithms use the El-Gamal scheme, in which the signature is a pair  $(r, s)$  of length  $m$  bits with the elements  $r$  and  $s$ , as a rule, of the same length  $m/2$  bits, and the signature verification reduces to comparing the value of an expression with the element  $s$ . Consequently, the exhaustive algorithm for elements  $s$  has the computational complexity of  $2^{m/2}$  public key signature verification operations and, if threshold DS lengths are used, it can break cryptographic protection in  $2^{60}$ ,  $2^{100}$ ,  $2^{150}$ , and  $2^{200}$  operations for security levels 1, 2, 3 and 4, respectively [7, 8, 12, 13]. The value of  $2^{60}$  does not attain the safe threshold of computational complexity for security level 1, since  $2^{60} < 2^{64}$ . The values of  $2^{100}$  and  $2^{150}$  are above the secure thresholds of computational complexity for 2nd and 3rd security levels, but do not provide an additional security margin of 20%, since  $2^{100*80\%} = 2^{80} < 2^{96}$  and  $2^{150*80\%} = 2^{120} < 2^{128}$ . Therefore, to ensure a secure DS length, it is necessary, in subparagraphs 5.3.5, 5.4.5 and 5.5.5 of the Standard, to increase their threshold values to 160, 240 and 320 bits, respectively, while the requirements of subparagraph 5.6.5



of the Standard can be left unchanged. Then the exhaustive algorithm of the signature element will be able to break cryptographic protection in, respectively, only  $2^{80}$ ,  $2^{120}$ ,  $2^{160}$  and  $2^{200}$  operations of signature verification with an additional security margin of 20%, since  $2^{80*80\%} = 2^{64}$ ,  $2^{120*80\%} = 2^{96}$ ,  $2^{160*80\%} = 2^{128}$  and  $2^{200*80\%} = 2^{160}$ .

2.7. Subparagraphs 5.3.6, 5.4.6, 5.5.6 and 5.6.6 of the Standard specify that the principle of generating and forming keys implemented by a MCPI shall ensure that each bit of the key takes on the value of one with a probability from the interval  $(0.5 \pm 0.03)$ ,  $(0.5 \pm 0.01)$ ,  $(0.5 \pm 0.003)$  and  $(0.500 \pm 0.001)$  for security levels 1, 2, 3 and 4, respectively, and, additionally, for security levels 3 and 4, keys shall be random-number sequences and generated using random-noise generators based on physical processes. These requirements, even with the increased specification for the computational complexity of cryptographic protection breaking algorithms, provide the necessary level of security [15]. So, a partial enumeration algorithm for the most probable keys of the same weight will have the computational complexity of  $C_n^w$  encryption operations or, subject to the correction for the probability of its successful application,  $C_n^w / (C_n^w (0,5+d)^w (0,5-d)^{n-w}) = (0,5+d)^{-w} (0,5-d)^{w-n} \geq (0,5+d)^{-w} (0,5+d)^{w-n} = (0,5+d)^{-n}$ , where  $n$  is the key length,  $w$  is the key weight (the number of 1-bits),  $0,5+d$  is the probability that a bit of the key takes on a value of one. In the case of threshold values of the specified intervals, that is, for  $d = 0.03$ ,  $0.01$ ,  $0.003$  and  $0.001$ , the partial enumeration algorithm for the most probable keys with the lengths of 80, 120, 160 and 200 bits of cryptographic transformation algorithms will be able to break cryptographic protection in only more than  $2^{73.2}$ ,  $2^{116.5}$ ,  $2^{158.6}$  and  $2^{199.4}$  operations, as corrected, and with a security margin of more than 12%, since  $2^{73.2*87.4\%} \approx 2^{64}$ ,  $2^{116.5*82.4\%} \approx 2^{96}$ ,  $2^{158.6*80.7\%} \approx 2^{128}$  and  $2^{199.4*80.2\%} \approx 2^{160}$ . Consequently, to ensure a secure interval of the probability that each bit of the key takes on the value of one, the threshold values in subparagraphs 5.3.6, 5.4.6, 5.5.6 and 5.6.6 of the Standard can be left unchanged, but the requirements themselves can be reworded as follows:

"5.3.6 The keys generated and formed by a MCPI shall be a random or non-deterministic pseudo-random sequence of bits, where each bit takes the value of one with a probability within the interval  $(0.500 \pm 0.03)$ .

5.4.6 The keys generated and formed by a MCPI shall be a random or non-deterministic pseudo-random sequence of bits, where each bit takes the value of one with a probability within the interval  $(0.500 \pm 0.01)$ .

5.5.6 The keys generated and formed by a MCPI shall be a random sequence of bits, where each bit takes the value of one with a probability within the interval  $(0.500 \pm 0.003)$ .

5.6.6 The keys generated and formed by a MCPI shall be a random sequence of bits, where each bit takes the value of one with a probability within the interval  $(0.500 \pm 0.001)$ ."

**Analysis of additional requirements.** In paragraphs 5.4 "Requirements for MCPIs of the second security level", 5.5 "Requirements for MCPIs of the third security level" and 5.6 "Requirements for MCPIs of the fourth security level" of the Standard [1] set forth additional security requirements.

3.1. Subparagraphs 5.4.7, 5.5.7 and 5.6.7 of the Standard state that MCPIs shall implement procedures for calculating and verifying key checking information to prevent the use of keys corrupted at the stage of distribution and loading with a probability of at least 0.9999, 0.999999 and 0.99999999 for levels 2, 3 and 4, respectively. In order to achieve compliance with the aforementioned lengths of MACs of 20, 30 and 40 bits, it is necessary to increase the threshold probability values to  $1-10^{-6}$ ,  $1-10^{-9}$  and  $1-10^{-12}$ , respectively.

3.2. Subparagraphs 5.4.8, 5.5.8 and 5.6.8 of the Standard specify that MCPIs shall implement procedures for calculating and verifying checking information about encrypted data to identify corrupted encrypted data with a probability of at least 0.9999, 0.999999 and 0.99999999 for security levels 2, 3 and 4, respectively, but for security levels 2 and 3 the requirement applies only to pre-encryption. This exception for online encryption was introduced for the possibility to certify for security levels 2 and 3 cryptographically strong encryptors for analog telephones and radio stations operating on a very narrowband voice data transfer channel and, therefore, excluding its use for transmitting additional checking information. However, the known definitions of preliminary encryption are more theoretical than practical, which leads to a subjective interpretation and circumvention of the requirements of the Standard when a MCPI is under development and certification tests. On the other hand, modern telecommunication

facilities and their data transfer protocols have built-in tools for detecting and/or correcting errors, which often makes it unnecessary for a MCPI to detect corrupted encrypted data. Given this and intending to comply with the aforementioned lengths of MACs of 20, 30 and 40 bits, it is necessary to increase the threshold probability values to  $1-10^{-6}$ ,  $1-10^{-9}$  and  $1-10^{-12}$ , respectively, and extend this requirement in the above subparagraphs to all types of encryption as follows:

"5.4.8 MCPIs shall implement procedures for calculating and verifying checking information about encrypting data to identify random errors in encrypted data with a probability of at least  $1-10^{-6}$  or the MCPI documentation shall contain organizational and technical measures to ensure protection against this threat.

5.5.8 MCPIs shall implement procedures for generating and verifying MACs or DS for encrypting data to identify randomly or deliberately corrupted encrypted data with a probability of at least  $1-10^{-9}$  or the MCPI documentation shall contain organizational and technical measures to ensure protection against this threat.

5.6.8 MCPIs shall implement procedures for generating and verifying MACs or DS for encrypting data for encrypted data to identify randomly or deliberately corrupted encrypted data with a probability of at least  $1-10^{-12}$ ."

3.3. Subparagraph 5.4.9 of the Standard states that MCPIs shall inform the operator of the establishment, reset, and also the impossibility of establishing an encryption mode. Subparagraph 5.5.9 of the Standard, additionally, requires reporting other irregularities in operation. Moreover, subparagraph 5.6.9 of the Standard adds the requirement to prevent the transfer of open data to the storage, distribution, and subsequent processing of encrypted data. That is, the current version of these subparagraphs is oriented towards encryption and takes little account of authentication and key generation issues. The requirement to prevent transfer is poorly combined with the requirements of reporting and is also more theoretical than practical, which leads to a subjective interpretation of data areas and circumvention of the requirements of the Standard when a MCPI is under development and certification tests. At the same time, the Standard does consider issues of logging, which in recent years have been given a significant place in the integrated security system. Based on the foregoing, it is proposed to amend subparagraphs 5.4.9, 5.5.9, and 5.6.9 of the Standard as follows:

"5.4.9 MCPIs shall inform the operator about the current mode of operation.

5.5.9 MCPIs shall inform the operator about the current mode of operation and irregularities in operation.

5.6.9 MCPIs shall inform the operator about the current mode of operation and irregularities in operation, and to automatically log these events."

3.4. Subparagraph 5.5.11 of the Standard states that the routine procedures for deletion (destruction) of keys by a MCPI shall ensure that they cannot be recovered. In addition to this, subparagraph 5.6.11 of the Standard requires the delivery of technical means completed with the MCPI that implement the specified procedures, if the MCPI itself does not implement them. However, the issues of recovering information deleted in the RAM or external memory of a computer on various electronic and optical media are often very knowledge-based and ambiguous, which leads to a subjective interpretation of the impossibility of recovering deleted keys when a MCPI is under development and certification tests. The inclusion of a paper shredder or an incinerator for burning paper and other key carriers in each set of MCPI is excessive, especially in the presence of several MCPIs of the same type in an enterprise. Hence, it is advisable to amend subparagraphs 5.5.11 and 5.6.11 of the Standard as follows:

"5.\*.11 MCPIs shall delete (destruct) keys on completing their distribution, management, and use, or the MCPI operational documentation shall contain organizational and technical measures for the deletion (destruction) of keys."

**Conclusion.** The secure thresholds for the parameters of cryptographic algorithms defined in the paper make it necessary to quickly revise the standard ST RK 1073-2007 to bring it into line with the current level of development of theoretical cryptography and the capabilities of potential adversaries. The developed proposals are specific and consistent with the current version of the Standard.

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### КРИПТОГРАФИЯЛЫҚ АЛГОРИТМ ПАРАМЕТРЛЕРІНЕ АРНАЛҒАН ҚАУІПСІЗ ШЕГІ ЖӘНЕ СТ РК 1073-2007 СТАНДАРТЫ

**Аннотация.** Қазақстан Республикасының ҚР СТ 1073-2007 «Ақпаратты криптографиялық қорғау құралдары. Жалпы техникалық талаптар» мемлекеттік стандарты 12 жыл бұрын қабылданды және ақпаратты криптографиялық қорғау құралдарының (АКҚК) сапасын бағалауда негізгі қазақстандық стандарт болып саналады. Осы күнге дейін теориялық криптография жаңаша дамуда, сондай-ақ әлуетті бұзушылардың біліктілігі мен есептеу мүмкіншіліктері де артты. Ақпараттың криптографиялық қорғанысын бұзушылардың құрастырған моделі, осы стандарттың бірнеше ережелерінің, әсіресе, бірінші және екінші деңгейдегі қауіпсіздікке қатыстылары ескірген, ал стандарттың өзін жаңалау қажеттілігін дәлелдейді.

Стандартты жаңалауда төмендегі тұжырымдымалық қағидаларға негізделген жөн:

1. Стандарттың алдыңғы редакциясымен сабақтастығын сақтау.

2. Стандартта қолданылатын барлық криптографиялық терминдерді анықтау.

3. Жариялаудан, мәжбүрлі немесе қорғаныстағы ақпаратты басқара алмайтындай етіп бұрмалаудан, әлуетті бұзушы бюджетінен, сондай-ақ криптографиялық қорғаныстың белгілі алгоритмдерін есептеудегі және кеңістігіндегі қиыншылықтан келетін зақымды ескеріп, стандарттағы АКҚК қауіпсіз төрт деңгейін анықтау.

- Бірінші, екінші, үшінші және төртінші деңгейдегі АКҚК бюджеті 1000, 1 млн., 1 млрд, және 1 трлн АЕК болған әлуеттік бұзушылардан бағасы сәйкес 100, 50 мың, 25 млн. және 10 млрд АЕК артпайтын ақпаратты қорғауға арналған;

- егер қолдану ықтималдылығы сәтті деп кері мультипликативті түзетуді ескере отырып алынған алгоритм күрделілігі  $2^{64}$ ,  $2^{96}$ ,  $2^{128}$  және  $2^{160}$  операциядан кем болса және криптографиялық қорғаныс алгоритмі белгілі болса, АКҚК қауіпсіздіктің сәйкес бірінші, екінші, үшінші және төртінші дейгейі бола алмайды. Егер криптографиялық қорғанысты ашу алгоритмінің күрделілігі  $2^{60}$ ,  $2^{80}$ ,  $2^{100}$  және  $2^{120}$  биттен аз болса, мұндай алгоритм қолдануға болмайды деп болжам жасаймыз.

4. Стандарттың жалпы талаптарында қорғаныс деңгейіне қарамастан барлық АКҚК қолданылатын теориялық және қолданбалы криптографияда белгілі нәрсені анықтап алу.

5. Стандартта криптографиялық алгоритмдердің негізгі параметрлерін және олардың есептеу күрделілігіне сәйкес криптографиялық қорғаныстарды ашудың (шабуыл) универсал алгоритмдерге төзімділігін қамтамасыз етететін, беріктігін ескере отырып қауіпсіздік шегінің шамасын анықтау. Нақты айтқанда:

- АКҚК криптографиялық түрлендірудегі симметриялық алгоритмде іске асырылатын кілт ұзындығы 1, 2, 3 және 4 қауіпсіздік деңгейі үшін сәйкес 80, 120, 160 және 200 биттен аз болмауы тиіс;

- АКҚК криптографиялық түрлендірудегі асимметриялық алгоритмде іске асырылатын кілт ұзындығы сәйкес 160, 240, 320 және 400 биттен аз болмауы тиіс;

- АКҚК криптографиялық түрлендірудегі асимметриялық алгоритмде іске асырылатын кілт ұзындығы криптографиялық төзімділігі құрама санның көбейткішке жіктеуге немесе ақырғы өрісте дискретті логорифмдеу мәселесіне негізделе отырып, есептің есептеу күрделілігіне сәйкес 1000, 2000, 4000 және 8000 биттен аз болмауы тиіс.

6. АКҚК қауіпсіздік деңгейіне сәйкес, стандарттағы ұйымдастырушы және техникалық қосымша талаптарды анықтау.

7. Стандарттағы нақты криптографиялық алгоритмдер мен протоколдарды анықтаудан бас тарту.

**Түйін сөздер:** ақпаратты қорғау, криптография, криптографиялық алгоритмдер параметрі, мемлекеттік стандарт, қорғаныс деңгейі.

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### БЕЗОПАСНЫЕ ПОРОГИ ДЛЯ ПАРАМЕТРОВ КРИПТОГРАФИЧЕСКИХ АЛГОРИТМОВ И СТАНДАРТ СТ РК 1073-2007

**Аннотация.** Государственный стандарт Республики Казахстан СТ РК 1073-2007 "Средства криптографической защиты информации. Общие технические требования" был принят 12 лет назад и стал

основным казахстанским стандартом для оценки качества средств криптографической защиты информации (СКЗИ). За прошедшее время теоретическая криптография получила новое развитие, а также выросли квалификация и вычислительные возможности потенциальных нарушителей. Построенные модели нарушителей криптографической защиты информации доказывают то, что ряд положений этого Стандарта, особенно касающиеся первого и второго уровня безопасности, устарели, а сам Стандарт необходимо обновить.

При обновлении Стандарта целесообразно руководствоваться следующими концептуальными принципами:

1. Сохранение преемственности с предыдущими редакциями Стандарта.

2. Определение в Стандарте всех используемых в нем криптографических терминов.

3. Определение в Стандарте четырех уровней безопасности СКЗИ, увязанных с возможным ущербом от разглашения, навязывания или неконтролируемого изменения защищаемой информации, с бюджетом потенциального нарушителя, а также с вычислительной и пространственной сложностью известных алгоритмов вскрытия криптографической защиты:

- СКЗИ первого, второго, третьего и четвертого уровней безопасности предназначены для защиты информации стоимостью не более 100, 50 тыс., 25 млн. и 10 млрд. МРП от потенциальных нарушителей с бюджетом не более 1000, 1 млн., 1 млрд. и 1 трлн. МРП соответственно;

- СКЗИ не могут быть признаны соответствующими первому, второму, третьему или четвертому уровню безопасности, если известен алгоритм вскрытия криптографической защиты, обеспечиваемой ими, вычислительная сложность которого составляет менее  $2^{64}$ ,  $2^{96}$ ,  $2^{128}$  и  $2^{160}$  операций соответственно, с учетом обратной мультипликативной поправки на вероятность его успешного применения. Если алгоритм вскрытия криптографической защиты имеет пространственную сложность не менее  $2^{60}$ ,  $2^{80}$ ,  $2^{100}$  и  $2^{120}$  бит соответственно, то этот алгоритм полагается неприменимым.

4. Определение в Стандарте общих требований, являющихся азбучными в теоретической и прикладной криптографии и, потому, предъявляемых ко всем СКЗИ независимо от уровня безопасности.

5. Определение в Стандарте основных параметров криптографических алгоритмов и их безопасных пороговых значений с запасом прочности, обеспечивающих стойкость к известным универсальным алгоритмам (атакам) вскрытия криптографической защиты соответствующей вычислительной сложности. В частности:

- длина ключа реализуемых СКЗИ симметричных алгоритмов криптографического преобразования должна быть не менее 80, 120, 160 и 200 бит для 1, 2, 3 и 4 уровней безопасности соответственно;

- длина ключа реализуемых СКЗИ асимметричных алгоритмов криптографического преобразования должна быть не менее 160, 240, 320 и 400 бит соответственно;

- длина ключа реализуемых СКЗИ асимметричных алгоритмов криптографического преобразования, криптографическая стойкость которых основана на вычислительной сложности задачи разложения составного числа на множители или задачи дискретного логарифмирования в конечном поле, должна быть не менее 1000, 2000, 4000 и 8000 бит соответственно.

6. Определение в Стандарте дополнительных организационных и технических требований, предъявляемых к СКЗИ в зависимости от уровня безопасности.

7. Отказ от определения в Стандарте конкретных криптографических алгоритмов и протоколов.

**Ключевые слова:** защита информации, криптография, параметры криптографических алгоритмов, государственный стандарт, уровень безопасности.

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## **OPTIMAL CONTROL OF POWER SYSTEMS**

**Abstract.** This article discusses the study of problems of optimal control for electric power systems. The numerical solution of optimal control problems for complex electric power systems using an iterative algorithm is shown. Also considered are issues of solving the optimal control of a nonlinear system of ordinary differential equations in two different cases. The proposed solution methods follow the principle of continuation of extremal problems based on sufficient conditions for optimality of V. F. Krotov. A special case of optimal control problems is considered. Numerical experiments showed sufficient efficiency of the implemented algorithms. The problem of optimal motion control of a two-system electric power system is graphically illustrated in the proposed numerical example.

**Keywords:** optimal control, electric power systems, an iterative algorithm.

### **1 Introduction**

The industrial growth of any country largely depends on the reliability of a large interconnected power system. The electric power system is an important form of modern energy source, since it is used in almost all spheres of human activity for socio-economic development. In an interconnected power system, the purpose of an electric power system's engine is to generate electrical energy in sufficient quantities at the most appropriate place of generation, transfer it in large quantities to load centers, and then distribute it to individual consumers in the proper order.

Mathematical model of modern electric power complex, consisting of turbo generators and complex multiply connected power units, is a system of nonlinear ordinary differential expressions. It is known [1-3] that this model serves as the basis of a broad and relevant class of control problem.

It should be noted that mathematical simulation of various processes and systems, including electric power system, are closely related with problem of making the best decisions. The optimization problems, as well as the creation of methods of building control on the principle of feedback for such systems, still attract attention of many researchers.

Optimal control theory is based on the maximum principle of L.S. Pontryagin and the method of dynamic programming of R. Bellman. It is known that the maximum principle reduces extreme challenge to the decision of a special system of ordinary differential equations, and dynamic programming methods to the solution of partial differential equations.

In many cases, the exact solution of these tasks is quite difficult. Why we developed numerical methods for solving extreme problems [5], based on the extension principle [6-11], which differ in the considerable variety of approaches and results.

These methods found a wide and effective application in solving some optimal control problems of large dimension and of various complexities [12-23]. Note that our works [24-26] were also devoted to the solution of optimal control problems. A study of global asymptotic stability was carried out in [27].

There are two widely spread areas in engineering practice among different research directions in optimal control theory based on the method of state space. One of them combines the methods of optimal control, which involve optimizing the system by minimizing the functional that characterizes, as a rule, the quality of regulation [21]. The second area contains the methods of modal control, i.e. methods of forming

feedback circuits, giving a closed-loop automatic control system (ACS), pre-selected root distribution of the characteristic equation [28].

The need to reconfigure the values of fudge factors of industrial controllers due to several factors associated with changes in the characteristics of complex energy facilities. Factors occur due to load changes, properties of energy resources, work of parallel regulation channels associated with controller via the object, equipment deterioration, impact of uncontrolled external disturbances, etc. For example, the load change of thermal power unit causes the change of position of regulating units. Tilt of operating characteristics of the regulating units of different types (slide, dampers, dampers, valves, etc.) may change in 2-3 times in different positions. The gain ratio of the object changes in 2-3 times accordingly that results in deteriorating the quality of transition processes. These degradations essentially influence the technical-and-economic performance of the equipment by decreasing its efficiency.

Testing parameters similarly change in other technical processes of complex objects of power-supply branch. In order to increase the effectiveness of the disturbance control and suppression processes, which are caused by the change of equipment operation load and other performance factors, it needs to use optimal digital control systems.

Thus, the study of modern principles of optimal control systems of complex objects is an actual scientific-technical problem.

In this paper, solving of optimal control problems for power system uses the principle of expansion extreme problems based on sufficient optimality conditions.

## 2 The optimal control formulation

It is required to minimize the functional

$$J(u) = 0.5 \sum_{i=1}^l \int_0^T (k_i y_i^2 + r_i u_i^2) dt + \Lambda(x(T), y(T)), \quad (1)$$

under the condition:

$$\begin{aligned} \frac{dx_i}{dt} &= y_i, & \frac{dy_i}{dt} &= -\lambda_i y_i + f_i(x) + b_i u_i, \\ x_i(0) &= x_{i0}, y_i(0) = y_{i0}, i = \overline{1, l}, t \in (0, T), \\ x(t), y(t) &: (0, T) \rightarrow R^l, \end{aligned} \quad (2)$$

where  $\{x_i, y_i\}_{i=1}^l$  – is system condition  $\{u_i\}_{i=1}^l$  – control;  $\{f_i(x)\}_{i=1}^l, \Lambda(x, y)$  – given continuously differentiable functions and functions

$f_i(x)$  satisfy the integrability conditions:

$$\frac{\partial f_i(x)}{\partial x_k} = \frac{\partial f_k(x)}{\partial x_i}, \forall i \neq k; \quad (3)$$

We consider point in time  $T$  and initial states  $\{x_{i0}, y_{i0}\}$  ordered;  $r_i, \lambda_i, k_i, b_i$  – positive constants; terminal values  $x(T), y(T)$  are unknown earlier.

We should note that if we appropriately set the  $f_i(x), i = 1, \dots, l$ , – function, non-linear problem of Cauchy (1)-(2) images the electric power system, for which the problem of synthesis is an important practical task of optimal control.

### Special case of the control problem (1)-(2).

Further, in the optimal control problem (1)-(2) we assume that there are following data for the control problem:

$$k_i = 2\lambda_i + \frac{b_i^2}{r_i}, \quad i \in \overline{1, l}.$$

In this case, we can solve the problem (1)-(2), following the Bellman-Krotov formalism [9,10]. At first, we show the correctness of the next Lemma.

**Lemma 1.** In order that the control  $u_i^0(y_i) = -\frac{b_i}{r_i} y_i, i = \overline{1, l}$  and the relevant solution of system (2)-(3)  $\{x(t), y(t)\}$  could be optimal, it is necessary and sufficient that

$$\varphi(x(T), y(T)) = -\Lambda(x(T), y(T)), k_i = 2\lambda_i + \frac{b_i^2}{r_i}, i \in \overline{1, l}. \quad (4^0)$$

$$\varphi(x, y) = 0.5 \sum_{i=1}^l y_i^2 - \int_{x_j=0, j>i}^l \int_{i=1}^{x_i} f_i(x_1, \dots, x_{i-1}, \xi_i, x_{i+1}, \dots, x_l) d\xi_i \quad (4)$$

$\varphi(x, y)$  – the Bellman-Krotov function, where

$$J(u^0) = \min_u J(u) = -\varphi(x(t_0), y(t_0)).$$

**Implementation of iterative algorithm for the problem (1)-(2).**

Let us describe the procedure of improving a given s-order approximation

$$\vartheta_s(t) = \{x_{1,s}(t), \dots, x_{l,s}(t), y_{1,s}(t), \dots, y_{l,s}(t), \dots, u_{1,s}(t), \dots, u_{l,s}(t)\}.$$

Step 1. Let us find a solution the next dual problem

$$\begin{cases} \frac{d\psi'_{i,s}(t)}{dt} = -\frac{\partial H(x_s(t), y_s(t), \nabla_{x,y}\varphi(x_s(t), y_s(t), t))}{\partial x_i}, \\ \frac{d\psi'_{l+i,s}(t)}{dt} = -\frac{\partial H(x_s(t), y_s(t), \nabla_{x,y}\varphi(x_s(t), y_s(t), t))}{\partial y_i}, i = 1, \dots, l, \\ \psi_{i,s}(T) = -\frac{\partial \Lambda(x(T), y(T))}{\partial x_i(T)}, \psi_{l+i,s}(T) = -\frac{\partial \Lambda(x(T), y(T))}{\partial y_i(T)}, i = 1, \dots, l, \end{cases}$$

where

$$\begin{aligned} H(x_s(t), y_s(t), \nabla_{x,y}\varphi(x_s(t), y_s(t), t)) &= \max_u H(x_s(t), y_s(t), \nabla_{x,y}\varphi(x_s(t), y_s(t), t), u, t), \\ &= -0.5[k_i y_i^2 + r_i u_i^2] + \sum_{i=1}^l \left[ \frac{\partial \varphi(x, y, t)}{\partial x^i} y^i + \frac{\partial \varphi(x, y, t)}{\partial y^i} [-\lambda_i y_i + f_i(x) + b_i u_i] \right] \\ &H(x, y, \nabla_{x,y}\varphi, u, t) \end{aligned}$$

$$\tilde{u}(x, y, \nabla_{x,y}\varphi, t) \in \text{Arg max}_u H(x, y, \nabla_{x,y}\varphi, u, t),$$

$$H(x, y, \nabla_{x,y}(\varphi, t), t) = H(x, y, \nabla_{x,y}\varphi, \tilde{u}(x, y, \nabla_{x,y}\varphi, t), t),$$

$$\psi_s(t) = \nabla_{x,y}\varphi(x, y, t)|_{x=x_s(t), y=y_s(t)}$$

Step 2. We solve Cauchy problem (2) at  $u = \tilde{u}(x, y, \nabla_{x,y}\varphi, t)$  and find function of state  $\{x_{s+1}(t), y_{s+1}(t)\}$  and control function

$$u_{s+1}(t) = \tilde{u}(x, y, \nabla_{x,y}\varphi_s(x, y, t), t)|_{x=x_s(t), y=y_s(t)}$$

Thus, we find new functional approximation of control state  $\{x_{s+1}(t), y_{s+1}(t), u_{s+1}(t)\}$ , for which the inequality is true:

$$J(x_{s+1}(t), y_{s+1}(t), u_{s+1}(t)) \leq J(x_s(t), y_s(t), u_s(t)).$$

**Application of iterative algorithm to solve the problem of optimal control of steam turbines' capacity.**

One of the models describing the transient processes in electrical system is the following system of differential equations [1, 2]:

$$\begin{aligned} \frac{d\delta_i}{dt} &= S_i, \quad H_i \frac{dS_i}{dt} = -D_i S_i - E_i^2 Y_{ii} \sin \alpha_{ii} - P_i \sin(\delta_i - \alpha_i) - \\ &- \sum_{j=1, j \neq i}^l P_{ij} \sin(\delta_{ij} - \alpha_{ij}) + u_i, \quad i \in \overline{1, l}, \quad t \in (0, T), \end{aligned} \quad (10)$$



$$\delta_{ij} = \delta_i - \delta_j, P_i = E_i U Y_{i,n+1}, P_{ij} = E_i E_j Y_{ij},$$

where  $\delta_i$  is an angle of rotor deflection of  $i$ -alternator towards some synchronous roll axis (roll axis of constant voltage bus, which makes rotation at a speed of 50 rpm/sec.;  $S_i$  – slip of  $i$ -alternator;  $H_i$  – an inertia constant of  $i$ -alternator;  $u_i = P_i$  – mechanical outputs, which fed to alternator;  $E_i$  – EMF of  $i$ -alternator;  $Y_{ij}$  – mutual conductance of system branches  $i$  – and  $j$ ;  $U = const$  is tension in constant voltage bus;  $Y_{i,n+1}$  characterizes connection (conductivity) of  $i$  – alternator with constant voltage bus;  $D_i = const \geq 0$  – mechanical dumping;  $a_{ij}, a_i$  – constant values with active resistance influence in armature alternator circuits.

The complexity of the model's analysis (10) is in taking account  $a_{ij}, a_{ij} = a_{ji}, i, j = \overline{1, l}$ . Because  $\delta_{ij} = -\delta_{ji}$ , then the model (10) is not a conservative; you cannot build a Lyapunov function for it in the form of the first integral. The system is called positional model.

Let the state variable and control variable in the established post-emergency mode are equal to:

$$S_i = 0, \delta_i = \delta_i^F, u_i = u_i^F, i = \overline{1, l}. \quad (11)$$

To obtain the system of perturbed motion let us pass on to equations in fluctuations, supposing that:

$$S_i = \Delta S_i, \delta_i = \delta_i^F + \Delta \delta_i, u_i = u_i^F + \Delta u_i, i = \overline{1, l}. \quad (12)$$

Next, for the convenience of the variables  $\Delta S_i, \Delta \delta_i, \Delta u_i$ , again symbolizing  $S_i, \delta_i, u_i$  from (11) we get:

$$\frac{d\delta_i}{dt} = S_i, \frac{dS_i}{dt} = \frac{1}{H_i} [-D_i S_i - f_i(\delta_i) - N_i(\delta) + M_i(\delta) + u_i],$$

$$i = \overline{1, l}, t \in (0, T), \quad (13)$$

where

$$f_i(\delta_i) = P_i [\sin(\delta_i + \delta_i^F - \alpha_i) - \sin(\delta_i^F - \alpha_i)],$$

$$N_i(\delta) = \sum_{j=1, j \neq i}^l \overline{N}_{ij}(\delta_1, \dots, \delta_l) = \sum_{j=1, j \neq i}^l \Gamma_{ij}^1 [\sin(\delta_{ij} + \delta_{ij}^F) - \sin \delta_{ij}^F],$$

$$M_i(\delta) = \sum_{j=1, j \neq i}^l \overline{M}_{ij}(\delta_1, \dots, \delta_l) = \Gamma_{ij}^1 [\cos(\delta_{ij} + \delta_{ij}^F) - \cos \delta_{ij}^F],$$

$$\Gamma_{ij}^1 = P_{ij} \cos \alpha_i, \Gamma_{ij}^2 = P_{ij} \sin \alpha_i, P_{ij} = P_{ji}, \Gamma_{ij}^k = \Gamma_{ji}^k, k = 1, 2.$$

The control will be searched in the form of:

$$u_i = v_i - M_i(\delta), i = \overline{1, l}, \quad (14)$$

where  $v_i$  to be determined.

It is required to minimize the functional

$$J(v) = J(v_1, \dots, v_l) = 0.5 \sum_{i=1}^l \int_0^T (w_{si} S_i^2 + w_{vi} v_i^2) dt + \Lambda(\delta(T), S(T)), \quad (15)$$

Under the condition (13)-(14), where  $w_{si}, w_{vi}$  – positive constants of weight coefficients;

$f_i(\delta_i) - 2\pi$  continuously differentiable periodic function;  $N_i(S) \sim 2mm$ - continuously differentiable periodic function towards  $\delta_{ij}$ ; for  $N_i(\delta)$  the condition of the integrability of the type (3) is accomplished;  $T$  - the duration of the transition process is considered as given. In addition, the initial conditions have been given:

$$\delta_i(0) = \delta_{i0}, S_i(0) = S_{i0}, i = \overline{1, l}, \quad (16)$$

Final value of the system status  $\delta_i(T)$ ,  $S_i(T)$  is not known in advance, they should be determined by solving optimal control problem (13) - (16).

To solve this problem for electric power systems used Krotov theorem on sufficient optimality conditions [1,2]. As a result, we obtain the following theorem.

**Lemma 2.** In order to manage  $v_i^0 = -\frac{S_i}{w_{vi}}$ ,  $i = \overline{1, l}$  and the relevant decision  $\{\delta^0, S^0\}$  systems (13) to be optimum, it is also necessary and enough that

$$\Lambda(\delta(T), S(T)) = -\varphi(\delta(T), S(T)), w_{si} = 2D_i + \frac{1}{w_{vi}} > 0, i = \overline{1, l},$$

$$\varphi(\delta, S) = 0.5 \sum_{i=1}^l \left[ H_i S_i^2 + \int_0^{\delta_i} f_i(\delta_i) d\delta_i \right] + \sum_{i=1}^l \int_0^{\delta_i} N_i(\delta_1, \dots, \delta_{i-1}, \xi_i, \delta_{i+1}, \dots, \delta_l) d\xi_i$$

where  $\varphi$  – Bellman-Krotov's function, besides,

$$J(v^0) = \min_v J(v) = -\varphi(\delta^0, S^0)$$

In conditions of lemma 2 assumptions (8) from lemma 1 take the form of:

$$\varphi_{\delta_i} S_i = \frac{\varphi S_i}{H_i} [f_i(\delta_i) + N_i(\delta)], \quad \text{т. е. } \varphi_{S_i} = H_i S_i, \varphi_{\delta_i} = f_i(\delta_i) + N_i(\delta), i = \overline{1, l}.$$

### 3 Numerical example. The optimal motion control of two-unit electric power system.

In the system (10) we take  $i = 1, 2$ , and assume that the mechanical damping is not available, i.e. the coefficients  $D_1, D_2$  are equal to zero. According to the values (10)-(16), the optimal control problem takes the form of [3]:

$$J(u) = J(u_1, u_2) = 0.5 \sum_{i=1}^2 \int_0^T (10S_i^2 + 0.1v_i^2) dt + 0.5(\delta^2(T), S^2(T)), \quad (17)$$

$$\frac{d\delta_i}{dt} = S_i, \frac{dS_i}{dt} = \frac{1}{H_i} [-f_i(\delta_i) - N_i(\delta) + v_i], i = 1, 2 \quad (18)$$

where  $f_i(\delta_i) = P_i [\sin(\delta_i + \delta_i^F - \alpha_i) - \sin(\delta_i^F - \alpha_i)], i = 1, 2,$

$$N_1(\delta) = \Gamma_1 [\sin(\delta_{12} + \delta_{12}^F) - \sin \delta_{12}^F],$$

$$M_1(\delta) = \Gamma_2 [\cos(\delta_{12} + \delta_{12}^F) - \cos \delta_{12}^F],$$

$$\delta_{12}^F = \delta_1^F - \delta_2^F, \quad \Gamma_1 = P_{12} \cos \alpha_{12},$$

$$\Gamma_2 = P_{12} \sin \alpha_{12}, \quad \delta_{12} = \delta_1 - \delta_2, \delta_{21} = -\delta_{12}$$

Numerics of the system (30):

$\alpha_1$	$\alpha_2$	$H_1$	$H_2$	$P_1$	$P_2$	$P_{12}$	$\delta_1^F$	$\delta_2^F$	$\alpha_{12}$
-0,052	-0,104	2135	1256	0,85	0,69	0,9	0,827	0,828	-0,078

and initial data:

$$\delta_1(0) = 0.18; \delta_2(0) = 0.1; S_1(0) = 0.001; S_2(0) = 0.002$$

The results are shown in figures 1 and 2. Herewith, the value of a functional (17) has been reduced to the value  $\approx 0,006865$ .

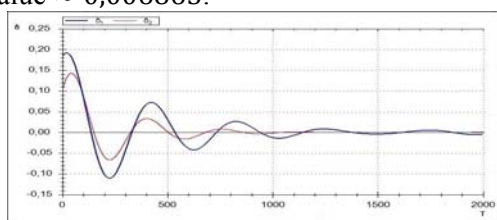


Figure 1 - Functions  $\delta_1, \delta_2$  with control,

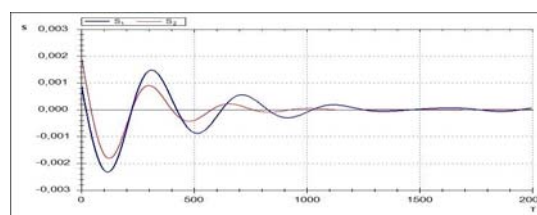


Figure 2 - Functions  $S_1, S_2$  with control

We use the 4-th order Adams-Bashford, Adams-Moulton and Runge-Kutta methods for more accurate results.

Adams-Bashford (A-B) method:

$$y_{n+4} = y_{n+3} + \frac{h}{24}(55 f(t_{n+3}, y_{n+3}) - 59 f(t_{n+2}, y_{n+2}) + 37 f(t_{n+1}, y_{n+1}) - 9 f(t_n, y_n)), \frac{251}{720} h^5(\eta)$$

Adams-Moulton (A-M) method:

$$y_{n+4} = y_{n+3} + \frac{h}{24}(9 f(t_{n+4}, y_{n+4}) + 19 f(t_{n+3}, y_{n+3}) - 5 f(t_{n+2}, y_{n+2}) + f(t_{n+1}, y_{n+1})), - \frac{19}{720} h^5(\eta).$$

Runge-Kutta (R-K) method:

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$

Comparison of the used methods is shown below:

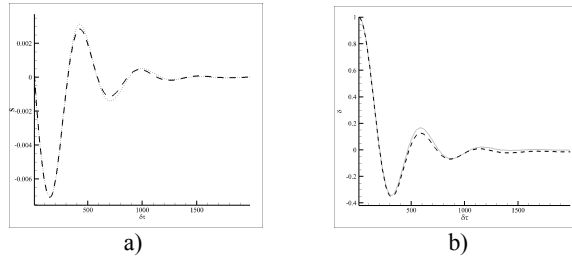


Figure 3 – methods of A-B (line - - -), A-M (line · · ·) of the 4-th order  
a) time change of  $S$ ; b) time change of  $\delta$

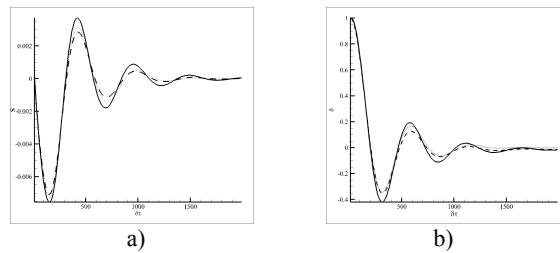


Figure 4 – methods of A-B (line- - -), A-M (line · · ·) and Euler (line -) of the 4-th order  
a) time change of  $S$ ; b) time change of  $\delta$

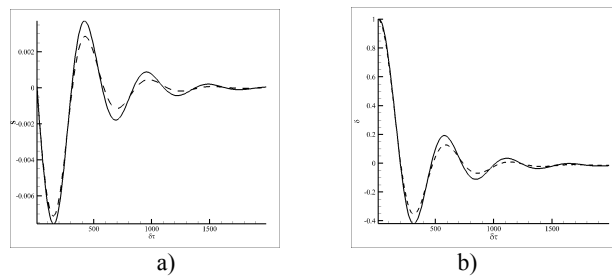


Figure 5 – methods of R-K (line - - -) and Euler (line -) of the 4-th order  
a) time change of  $S$ ; b) time change of  $\delta$

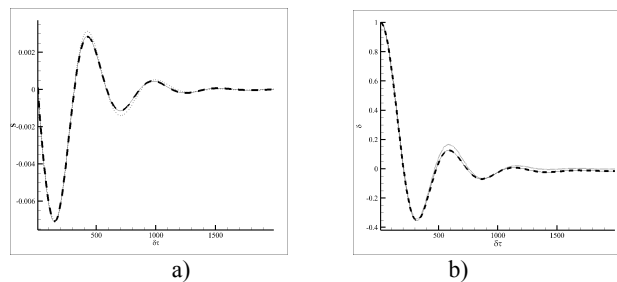


Figure 6 – methods of A-B (line - - -), A-M (line · · ·) and R-K (line-) of the 4-th order  
a) time change of  $S$ ; b) time change of  $\delta$

According to the obtained results it is clear that the increase of more than 4 is not necessary, as they equally converge to zero.

For this task the Adams-Bashford and Runge-Kutta methods converge to zero faster than when using the method of Adams-Moulton. It allows to reduce time and speed up the process of determining emergency situation. Since the Adams-Moulton method is implicit and requires the solution of the "historical" values, which takes computation time.

### **Conclusions**

The paper deals with solution of optimal control of nonlinear system of ordinary differential equations in two different cases. The studied model, in particular, describes management processes in electric power systems. The proposed methods for solving hold to the extreme tasks expansion principle, based on sufficient optimality conditions of V.F. Krotov. The numerical experiments have shown sufficient efficacy implemented algorithms.

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### **ЭЛЕКТР ЭНЕРГЕТИКАЛЫҚ ЖҮЙЕЛЕРДІ ОҢТАЙЛЫ БАСҚАРУ**

**Аннотация.** Зерттеудің өзектілігі күрделі электр-энергетикалық кешендердің жаңартылуына және динамикасын зерттеуге негізделген. Индустриалды қоғамның заманауи дамуы қуатты электр-энергетикалық кешендердің құрылуын айқындап, электр-энергиясының тұрақты және үзіліссіз артуын қамтамасыз етеді.

Турбогенераторлардың басым көпшілігі және жиі байланысты энергетикалық объектілерді қамтитын күрделі электр-энергетикалық жүйелердің жұмысының орнықтылығы мен қауіпсіздігін қамтамасыз ету және тиімді басқару мәселелерін зерттеудің өзектілігі мен практикалық құндылығының маңызы зор.

Турбогенераторлар мен күрделі түрде тығыз байланысты энергетикалық блоктардан тұратын заманауи электр-энергетикалық кешеннің математикалық моделі қарапайым дифференциалдық теңдеулердің сызықсыз жүйесін құрайды.

Тиімді басқару теориясы Л.С.Портнягиннің максимум принципі мен Р. Беллманның динамикалық бағдарламалау әдісіне негізделеді. Максимум принципі экстремальды есепті қарапайым дифференциалдық теңдеулердің арнайы жүйесінің шешіміне әкелсе, ал динамикалық бағдарламалау әдісі жеке туындылық есептің шешімін табатыны белгілі. Көптеген жағдайларда бұл есептердің нақты шешімін табу жеткілікті деңгейде қиын.

Бұл мақалада электр энергетикалық жүйелерді оңтайлы басқару мәселелерін зерттеу қарастырылған. Итерациялық алгоритмді қолдана отырып, күрделі электр энергетикалық жүйелерін басқарудың оңтайлы есептерінің сандық шешімі көрсетілген. Екі түрлі жағдайда қарапайым дифференциалдық теңдеулердің сызықты емес жүйесін оңтайлы басқаруды шешу мәселелері қарастырылады. Ұсынылған шешу әдістері В.Ф. Кротовтың оңтайлы болуы үшін жеткілікті жағдайларға негізделген экстремалды мәселелерді жалғастыру қағидасын ұстанады. Оңтайлы басқару мәселелерінің ерекше жағдайы қарастырылады. Сандық тәжірибелер орындалған алгоритмдердің тиімділігін көрсетті. Ұсынылған сандық мысалда екі жүйелі электр энергиясының оңтайлы қозғалысын басқару мәселесі графикалық түрде көрсетілген.

**Түйін сөздер:** оңтайлы басқару, электр энергетикалық жүйелер, итерациялық алгоритм.

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### **ОПТИМАЛЬНОЕ УПРАВЛЕНИЕ ЭЛЕКТРОЭНЕРГЕТИЧЕСКИМИ СИСТЕМАМИ**

**Аннотация.** Актуальность исследования обусловлена необходимостью модернизации и исследования динамики сложных электроэнергетических комплексов. Современное развитие индустриального общества требует постоянного и непрерывного роста производства электроэнергии, для обеспечения которой создаются мощные электроэнергетические комплексы.

Особую актуальность и практическое значение представляют исследования оптимального управления, обеспечения безопасности и устойчивости работы сложных электроэнергетических систем, состоящих из большого числа турбогенераторов и многосвязных энергетических объектов.

Математическая модель современной энергетической установки, состоящей из турбогенераторов и сложно связанных энергоблоков, образует нелинейную систему простых дифференциальных уравнений.

Теория эффективного управления основана на принципе максимума Л. С. Понтрягина и метода динамического программирования Беллмана. Известно, что принцип максимума приводит к решению экстремальной задачи к специальной системе простых дифференциальных уравнений, а метод динамического программирования приводит к решению индивидуальной задачи о продукте. Во многих случаях довольно сложно найти точное решение этих проблем.

В данной статье рассматриваются вопросы исследования оптимального управления электроэнергетическими системами. Показано численное решение задач оптимального управления сложными электроэнергетическими системами с использованием итерационного алгоритма. Также рассматриваются вопросы решения оптимального управления нелинейной системой обыкновенных дифференциальных уравнений в двух разных случаях. Предложенные методы решения следуют принципу продолжения экстремальных задач на основе достаточных условий оптимальности В. Ф. Кротова. Рассмотрен частный случай задач оптимального управления. Численные эксперименты показали достаточную эффективность реализованных алгоритмов. Задача оптимального управления движением двухсистемной электроэнергетической системы графически проиллюстрирована на предлагаемом численном примере.

**Ключевые слова:** оптимальное управление, электроэнергетические системы, итерационный алгоритм.

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## NEWS

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## INVESTIGATION OF PASSIVATION OF SURFACE STATES OF SINGLE CRYSTALLINE SILICON IN HETEROSTRUCTURES WITH AN INTEGRATED THIN AMORPHOUS LAYER

**Abstract.** This work presented in this publication is a continuation of works [1] and [2]. We have shown that in solar cells of the HIT structure, when the thickness of the embedded amorphous layer increases, the efficiency of the solar cell increases, but to a certain point of maximum. The increase in characteristics is due to an increase in the lifetime of non-basic charge carriers in a crystalline silicon substrate. This behaviour of the system can be explained by better passivation of surface States on the silicon substrate. The decrease in the efficiency of the solar cell after reaching the maximum is due to the increase of the resistance structure, which with a point contributes more to the characteristics of the solar cell than the contribution from the passivation of surface states.

Using the PECVD method, amorphous silicon films with a thickness of 7, 30 and 50 nm were grown on a single-crystal commercial silicon substrate on both sides. Then, the lifetime of non-basic charge carriers was measured using the contactless Sinton method on these structures. Further, the samples were examined by photoluminescence at room temperature. Using this method, it was possible to obtain the dependence of the concentration of non-equilibrium charge carriers on the intensity of photoluminescence radiation. These dependences showed that the intensity is lower on samples with a lower thickness of amorphous silicon than in samples with a higher thickness of the amorphous layer.

The detailed results of this work were first presented by the corresponding author in his dissertation [3], the theses of the work were presented at the conference of young scientists of KazNU [4] and only now the results are published as a full-fledged article in the journal.

**Key words:** Solar cell, amorphous silicon, HIT, hetero-junction, surface defects.

**Introduction.** In contrast to the traditional process of doping impurities of the same type of conductivity into a substrate having the opposite type of conductivity, the heterojunction technology is based on plasmochemical deposition of thin films of amorphous silicon of the required type of conductivity with a thickness of several tens of nanometers. This significantly increases the control over the created barrier semiconductor structure, providing sharp boundaries between regions of different types of conductivity if necessary. At the same time, however, high requirements remain for the quality of the semiconductor surface on which thin films are deposited. The state of the surface of a crystalline silicon substrate is determined by three factors: 1) corrugation or relief texturing of the surface to reduce the reflection coefficient / increase the amount of light absorbed by the solar cell, 2) effective surface cleaning from foreign contamination, 3) passivation of the silicon surface to "heal" broken bonds and reduce the surface density of defects. The influence of the latter two factors on the efficiency of a solar cell is determined by the effective lifetime of non-primary carriers, which should be as long as possible to achieve high efficiency, while:

$$1/\tau_{\text{eff}} = 1/\tau_b + 1/\tau_s,$$

where  $\tau_{eff}$  – the effective lifetime of carriers,  $\tau_b$  – the volume lifetime,  $\tau_s$  – the surface lifetime. Thus, measuring the effective lifetime of carriers in a plate is the main tool for controlling the quality of its surface. A complementary type of measurement to identify the nature of surface states associated with defects is infrared spectroscopy, including Raman spectroscopy.

HIT solar cells are the market leader in solar energy in terms of "price/quality". Laboratory samples achieve an efficiency of up to 26.7% [5-9]. The record results were the result of optimization and research of the structure of the HIT solar cell [10,11]. Optimization can be carried out in three directions: improving materials and production methods, optimizing the internal parameters of the solar cell (thickness, layer concentration) and changing the design [12].

Early studies have demonstrated the effectiveness of self-conducting amorphous silicon in surface passivation of crystalline silicon wafers. The built-in layer of amorphous conduction silicon is used in the HIT photocell for passivation of heterojunction defects, however, the presence of this layer leads to undesirable consequences for the photocell, namely, due to the qualities of the amorphous material itself, there is a drop in the diffusion length of charge carriers in the section and, as a result, a decrease in the efficiency of the photocell. One of the possibilities for further improving the efficiency of a heterojunction silicon solar cell is to increase the length of the diffusion path of charge carriers in the region of the heterojunction [1,12,13,14].

**Experiment and results.** At the first stage of the work, the thickness of the embedded layer of amorphous silicon of its own type of conductivity in a heterojunction silicon solar cell of the HIT structure is optimized by computer modeling. The model of the photocell was optimized using the software package FORS-HET [1,2]. The software package is designed for modeling heterojunction solar cells based on crystalline and amorphous silicon, and includes a database of parameters of these materials. The detailed principle of operation of the software package FORS-HET is described in [15-18]. In addition to the numerical calculation, real samples of solar cells were grown and their optimization was performed experimentally based on the thickness of the embedded layer of its own conductivity. Comparison of the obtained data with the results of the experiment showed that the optimal value of the experimental thickness of the amorphous layer (7 nm) does not coincide with the numerical calculation data (1 nm). However, further research has shown that passivation of surface states on a crystalline silicon substrate is better the thicker the thickness of the amorphous layer. It is shown in [19] that the lifetime of non-basic charge carriers increases with increasing thickness of the amorphous silicon layer.

At the second stage of the work, using the PECVD method, amorphous silicon films with a thickness of 7, 30 and 50 nm were grown on a single-crystal commercial silicon substrate on both sides. Then, the lifetime of non-basic charge carriers was measured using the contactless Sinton method on these structures, which confirmed the qualitative dependence of the results [19, 20]. Further, the samples were examined by photoluminescence at room temperature. Using this method, it was possible to obtain the dependence of the concentration of non-equilibrium charge carriers on the intensity of photoluminescence radiation. These dependences showed that the intensity is lower on samples with a lower thickness of amorphous silicon than in samples with a higher thickness of the amorphous layer. These results are in good agreement with the previous ones.

One of the most important factors affecting the efficiency of the solar cell of the HIT structure is the passivation of surface States on the plate of the crystal substrate. To study the effect of passivation of crystalline silicon by an amorphous silicon film using the PECVD method, amorphous silicon films with a thickness of 7, 30 and 50 nm were grown on a single-crystal commercial silicon substrate on both sides. Then a non-contact method of Synton measurement was used on the manufactured samples to determine the lifetime values of non-basic charge carriers. The results of measuring the characteristics of samples are shown in the table 1.

Table -Results of measurements of the life time of the NNZ by non-contact method of single-crystal silicon wafers with a resistance of  $R = 1.5 \text{ Ohms}\cdot\text{cm}$ , covered with an amorphous film with a thickness of 7 nm (for samples kz\_12 – kz\_15), 30 nm (for samples kz\_02 – kz\_05) and 50 nm (for samples kz\_06 – kz\_11)



The name of the sample	The life time ( $\tau$ ), microsecond
kz_02	1063
kz_03	1020
kz_04	1278
kz_05	1070
kz_06	2295
kz_07	1890
kz_08	1621
kz_09	2016
kz_10	2461
kz_11	2009
kz_12	924
kz_13	630
kz_14	1053
kz_15	655

As can be seen from the table, the greater the thickness of the amorphous film deposited on the crystal substrate, the greater the value of the lifetime of non-basic charge carriers. The results of the study confirmed the qualitative dependence of the results of other authors' work [7].

Figure 1 shows the dependence of the lifetime of non-primary charge carriers on their concentration when illuminated by light, during measurement. It is obvious that the best value of the lifetime is shown by the functional corresponding to the greater thickness of the amorphous passivating layer, equal to 50 nm

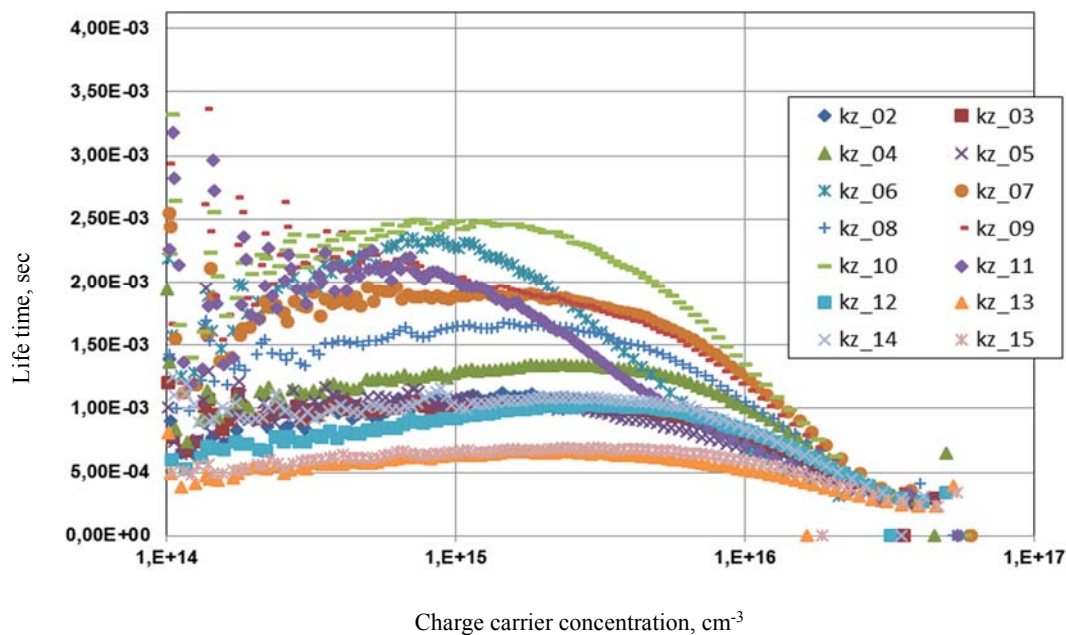


Figure 1 - dependence of the lifetime of non-basic charge carriers on their concentration at different thickness of the amorphous film

Further, the samples were examined by photoluminescence at room temperature (figure 2-4). In figures 2-4, images with higher brightness correspond to a higher concentration of non-equilibrium charge carriers. Using this method, it was possible to obtain the dependence of the concentration of non-equilibrium charge carriers on the intensity of photoluminescence radiation. These dependences showed that the intensity is lower on samples with a lower thickness of amorphous silicon than in samples with a

higher thickness of the amorphous layer. The distribution of radiation intensity is consistent with the data of measurement of the lifetime of non-basic charge carriers in the studied structures.

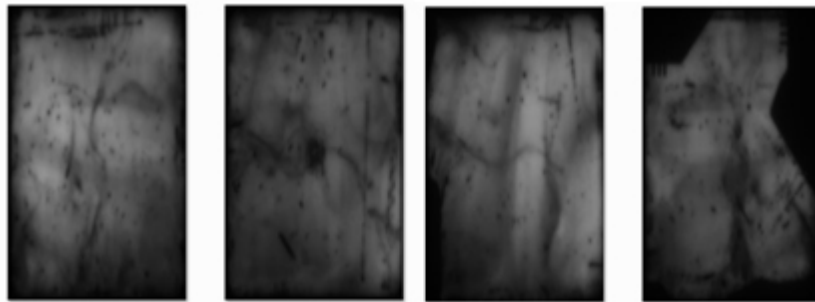


Figure 2-Photoluminescence of silicon wafer samples at 7 nm thick amorphous silicon film deposited on them at room temperature

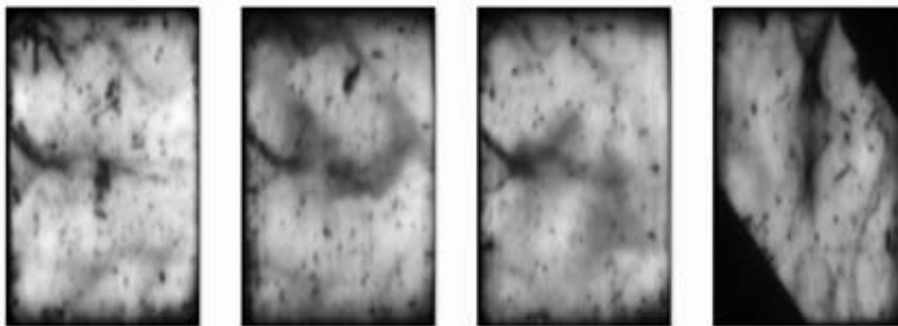


Figure 3-Photoluminescence of silicon wafer samples at 30 nm thick amorphous silicon film deposited on them at room temperature

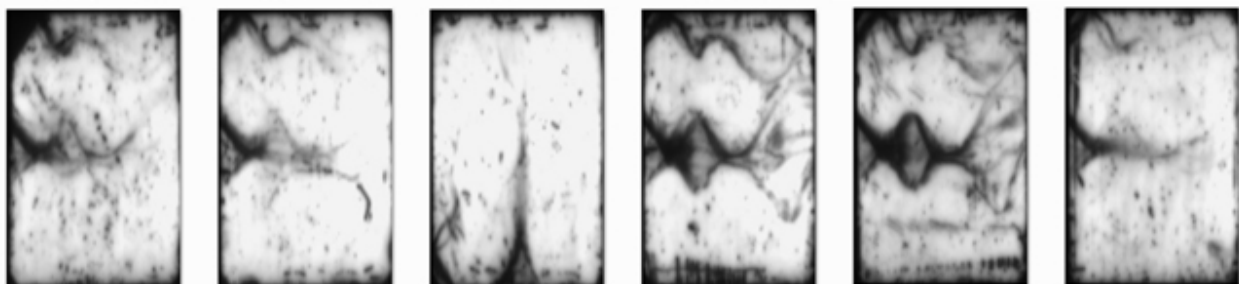


Figure 4-Photoluminescence of silicon wafer samples at 50 nm thick amorphous silicon film deposited on them at room temperature

Consequently of experiments, it show that in HIT structure solar cells, the efficiency of the solar cell increases with increasing thickness of the built-in native layer, but up to a certain point of maximum [2]. The increase in characteristics is due to an increase in the lifetime of non-basic charge carriers in a crystalline silicon substrate. This can be explained by better passivation of surface states on a silicon substrate. The decrease in the efficiency of the solar cell after reaching the extreme point explained by the fact that with increasing thickness, the consistent resistance of the structure increases, which from a certain point on makes a greater contribution to the characteristics of the solar cell than the contribution from passivation of surface states.

However, the results of this work cause to new questions about how exactly the passivation of the surface states of crystalline silicon occurs, as long as the formation of new bonds presumably takes place at distances of no more than 1 nm. Nevertheless, the growth of positive electrical properties for the operation of the solar cell continues to the thickness of the passivating film of a dozen nanometers. In search of this answer, we plan to conduct a cathodoluminescent analysis that will show the energy of

charge carrier transitions between levels and sublevels on the zone diagram of the "single-crystal silicon-amorphous silicon" heterojunction. It is also planning to conduct optical analyses of the structure in the infrared region of the spectrum. It is planning to repeat this experiment, but with films obtained by a different method (PVD method).

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#### **ҚҰРЫЛЫМЫНДА ЖҰҚА АМОΡФТЫ ҚАБАТЫ БАР ГЕТЕРОҚҰРЫЛЫМДАРДАҒЫ МОНОКРИСТАЛДЫ КРЕМНИЙДІҢ БЕТКІ КҮЙІНІҢ ПАССИВТЕЛУІН ЗЕРТТЕУ**

**Аннотация.** Мақалада [1] айналысып жатқан жұмыстардың жалғасы [2] берілді. Біз НІТ құрылымы фотоэлементтерінде кіріктірілген аморфты қабат қалыңдығының артуы негізінде белгілі бір максималды нүктеге дейін фотоэлемент тиімділігі артады. Сипаттамалардың жоғарылауы кристалды кремний субстратындағы негізгі емес тасымалдаушы өмірінің ұзаруына байланысты. Жүйенің бұл әрекетін кремний субстратындағы беткі күйдің үздік пассивациясы негізінде түсіндіруге болады. Максималды нүктеге жеткеннен кейін фотоэлемент тиімділігінің азаюы құрылым кедергісінің жоғарылауына байланысты, ол белгілі бір сәттен бастап беттік күй пассивациясынан гөрі фотоэлемент сипаттамаларына көбірек үлес қосады.

PECVD әдісімен қалыңдығы 7, 30 және 50 нм аморфты кремний пленкалары екі жағынан да монокристалды коммерциялық кремний субстраттарында өсірілді. Содан кейін бұл құрылымдардағы байланыссыз синтон әдісімен негізгі емес заряд тасымалдаушының өмір сүру уақыты өлшенді. Әрі қарай үлгілер бөлме температурасында фотолюминесценция әдісімен зерттелді. Осы әдісті қолдана отырып, тепе-тең емес заряд тасымалдаушы концентрация фотолюминесценцияның сәулелену қарқындылығына тәуелділігін алуға болады. Бұл тәуелділік аморфты кремнийдің қалыңдығы аз кездесетін үлгілерде қарқындылығы аморфты қабаттың алдыңғыға қарағанда аз болатындығын көрсетті.

Жұмыстың егжей-тегжейлі нәтижелерін бірінші кезекте автор-корреспондент өзінің диссертациясында ұсынды [3], жұмыс тезистері ҚазҰУ-дың жас ғалымдарының конференциясында ұсынылды [4] және оның нәтижелері журналда толыққанды мақала ретінде жариялануда.

**Түйін сөздер:** күн элементі, аморфты кремний, НІТ, гетероткел, беткі ақаулар.

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#### **ИССЛЕДОВАНИЕ ПАССИВАЦИИ ПОВЕРХНОСТНЫХ СОСТОЯНИЙ МОНОКРИСТАЛЛИЧЕСКОГО КРЕМНИЯ В ГЕТЕРОСТРУКТУРАХ СО ВСТРОЕННЫМ ТОНКИМ АМОΡФНЫМ СЛОЕМ**

**Аннотация.** Настоящая работа, представленная в данной публикации, является продолжением работ [1] и [2]. Мы показали, что в фотоэлементах структуры НІТ при увеличении толщины встроенного аморфного слоя растёт эффективность фотоэлемента, но до определённой точки максимума. Рост характеристик обусловлен увеличением времени жизни неосновных носителей заряда в подложке кристаллического кремния. Такое поведение системы можно объяснить лучшей пассивацией поверхностных состояний на

подложке кремния. Падение эффективности фотоэлемента после достижения точки максимума связано с увеличением сопротивления структуры, которое с определённого момента вносит больший вклад в характеристики фотоэлемента, чем вклад от пассивации поверхностных состояний.

Методом PECVD на монокристаллической коммерческой подложке кремния с обеих сторон были выращены плёнки аморфного кремния толщиной 7, 30 и 50 нм. Затем бесконтактным методом Синтона на этих структурах было измерено время жизни неосновных носителей заряда. Далее образцы были исследованы методом фотolumинесценции при комнатной температуре. С помощью данного метода удалось получить зависимость концентрации неравновесных носителей заряда от интенсивности излучения фотolumинесценции. Данные зависимости показали, что на образцах с меньшей толщиной аморфного кремния интенсивность меньше, нежели в образцах с большей толщиной аморфного слоя.

Подробные результаты настоящей работы в первую очередь были представлены автором-корреспондентом в своей диссертации [3], тезисы работы были представлены на конференции молодых учёных КазНУ [4] и только сейчас результаты публикуются в виде полноценной статьи в журнале.

**Ключевые слова:** солнечный элемент, аморфный кремний, HIT, гетеропереход, поверхностные дефекты.

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ON THE SEARCH FOR HADRON MODES  
 OF HIDDEN CHARM MESONS DECAY

**Abstract.** From the results obtained in publication [1] as well percentage between decay modes of excited states of mesons, cited in reference book “Particle Data Group” became possible to evaluate the number of hadron final states on which the meson with *hidden charm* decay. In the paper is shown a rough estimate of final states number for hadron decays of mesons with *hidden charm*. The most statistically secured are the final states  $KK\pi$  for  $\eta_c(1S)$ ,  $2(\pi^+\pi^-)$  and  $\pi^+\pi^-K^+K^-$  for  $\chi_{c0}(1P)$  mesons.

**Key words:** hadron, meson, mode decay, hidden charm, final state.

Since hadron decay modes of mesons with *hidden charm* are strongly suppressed due to the Okubo - Zweig - Iizuka (OZI) rule. Then in connection with the publication of work on the study of inelastic photoproduction of  $J/\psi(1S)$  and  $\psi(2S)$  mesons [1], it became possible to estimate the number of mesons with *hidden charm* which decay on hadron final states. All statistics  $ep$  interaction accumulated during the period 1997 - 2007 were used. This is  $\sim 350$  million interactions. The number of  $J/\psi(1S)$  and  $\psi(2S)$  mesons with lepton decay to  $\mu^+\mu^-$  is obtained equal to 11 295 and 448 respectively.

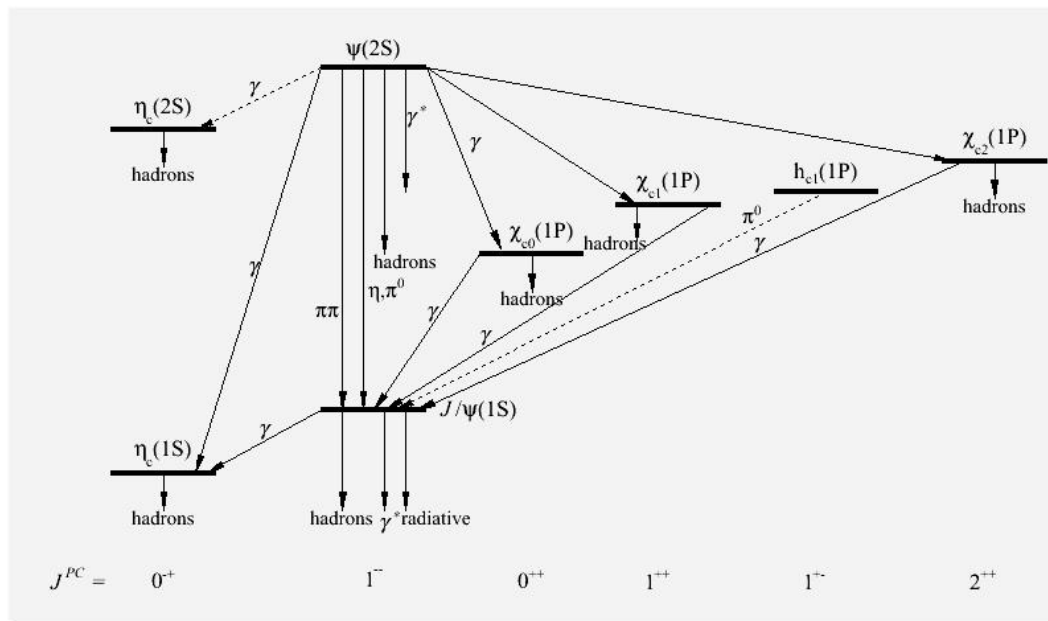


Figure 1 - Scheme of radiative decays of  $J/\psi(1S)$  and  $\psi(2S)$  mesons

Since the percentage of events which decay on this mode is known from the "Particle Data Group" (PDG) [2, 3] tables, we can estimate the total number of generated  $J/\psi$  (1 S) and  $\psi$  (2 S) mesons which is 167 044 for  $J/\psi$  (1 S) mesons and 56 172 for  $\psi$  (2 S) mesons, respectively. And from this number, it is possible to calculate the number of mesons decaying on the hadron mode.

The number of mesons with *hidden charm*  $\eta_c$  (1S) and mesons  $\chi_{c0}$ (1P),  $\chi_{c1}$ (1P) and  $\chi_{c2}$  (1P) can also be estimated from the radiative decay data of  $J/\psi$  (1S) and  $\psi$  (2S) mesons using percentage from the "Particle Data Group" tables.

Table 1 - Estimation of the number of mesons with *hidden charm*  $\eta_c$  (1S),  $\chi_{c0}$  (1P),  $\chi_{c1}$  (1P),  $\chi_{c2}$  (1P) from the decays of  $J/\psi$ (1S) and  $\psi$  (2S) mesons decaying on the radiative mode

Radiative decay modes	Share in %	Quantity
$J/\psi(1S) \rightarrow \gamma \eta_c(1S)$	1.7 %	2840
$\psi(2S) \rightarrow \gamma \eta_c(1S)$	0.34 %	191
$\psi(2S) \rightarrow \gamma \chi_{c0}(1P)$	9.84 %	5527
$\psi(2S) \rightarrow \gamma \chi_{c1}(1P)$	9.3 %	5224
$\psi(2S) \rightarrow \gamma \chi_{c2}(1P)$	8.7 %	4887

Table 2 - Estimation of the number of mesons with *hidden charm* that decay on different hadron modes

Meson	Final state	Share in %	Quantity
$\eta_c$ (1 S)	$2 (\pi^+ \pi^-)$	0.9 %	31
$\eta_c$ (1 S)	$2 (K^+ K^-)$	0.15 %	5
$\eta_c$ (1 S)	$\pi^+ \pi^- K^+ K^-$	0.69 %	21
$\eta_c$ (1 S)	$KK \pi$	7.3%	221
$\chi_{c0}$ (1P)	$2 (\pi^+ \pi^-)$	2.25 %	124
$\chi_{c0}$ (1P)	$2 (K^+ K^-)$	0.28 %	15
$\chi_{c0}$ (1P)	$\pi^+ \pi^- K^+ K^-$	2.77 %	98
$\chi_{c1}$ (1P)	$2 (\pi^+ \pi^-)$	0.76 %	40
$\chi_{c1}$ (1P)	$2 (K^+ K^-)$	0.056 %	3
$\chi_{c1}$ (1P)	$\pi^+ \pi^- K^+ K^-$	0.45 %	23
$\chi_{c2}$ (1P)	$2 (\pi^+ \pi^-)$	1.1 %	54
$\chi_{c2}$ (1P)	$2 (K^+ K^-)$	0.0178 %	0
$\chi_{c2}$ (1P)	$\pi^+ \pi^- K^+ K^-$	0.091 %	4

Knowing the total number of mesons with *hidden charm*  $\eta_c$  (1S) and  $\chi_{c0}$  (1P),  $\chi_{c1}$  (1P),  $\chi_{c2}$  (1P), we can roughly estimate their number for a specific hadron mode of their decay.

From the obtained estimates, we can conclude that the most favorable (in terms of the quantity of mesons decaying into hadron final states) for searching and selecting mesons with *hidden charm* are the following final states -  $KK\pi$ ,  $\pi^+ \pi^- K^+ K^-$  and  $2 (\pi^+ \pi^-)$ .

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### ЖАСЫРЫН ТАРТЫМДЫЛЫҒЫ БАР МЕЗОН АДРОНДАРЫНЫҢ ЫДЫРАУ МОДАСЫН ІЗДЕУ МӘСЕЛЕСІ ЖӨНІНДЕ

**Аннотация.** Жұмыс барысында алынған нәтижелерге [1] және «Particle Data Group» анықтамасында келтірілген мезондардың қозған күйінің ыдырау модалары арасындағы пайыздық қатынасқа сүйене отырып, жасырын тартымдылығы бар мезондардың ыдырайтын соңғы күйінің санын бағалау мүмкіндігі туды. Осы аталған зерттеу жұмысында *жасырын тартымдылығы* бар мезондардың адронды ыдырауының соңғы күйіне қатаң сандық бағалау жүргізіледі. Біршама статистикалық қамтылған мезондар  $\eta_c$  (1 S),  $2(\pi^+\pi^-)$  үшін  $KK\pi$  және  $\chi_{c0}$  (1P) үшін  $\pi^+\pi^- K^+K^-$  шамасының соңғы күйі есептеледі.

Жасырын *тартымдылығы* бар мезон адрондарының ыдырау модасы Okubo - Zweig - Iizuka (OZI) ережесі бойынша қатты басылады. Кейінірек,  $J/\psi(1S)$  және  $\psi(2S)$  мезондардың серпімді емес күйін зерттеу жұмысында [1] адрондық соңғы күйге ыдырайтын жасырын тартымдылығы бар мезондардың санын бағалау мүмкіндігі пайда болды. 1997-2007 жылдар аралығында жинақталған өзара әрекеттесудің барлық статистикасы пайдаланылды, ол шамамен 350 млн жуық. Лептонның  $\mu^+\mu^-$ -ге ыдырауы негізінде  $J/\psi(1S)$  және  $\psi(2S)$  мезондар санын аламыз. Осы режимде ыдырайтын оқиғалардың «Particle Data Group» анықтамасында келтірілген мезондардың қозған күйінің ыдырау модалары арасындағы пайыздық қатынасқа сүйене отырып, жасалған мезондардың жалпы санын бағалай аламыз. Деректерге сүйене отырып, адрондық модада ыдырайтын мезондардың санын есептеуге болады.

*Жасырын тартымдылығы* бар мезондар мен  $\chi_{c0}$  (1P),  $\chi_{c1}$  (1P) және  $\chi_{c2}$  (1P), мезондарының санын  $J/\psi$  мезон (1s) және мезондарының радиациялық ыдырауы бойынша да бағалауға болады.

Жасырын тартымдылығы бар мезондардың  $\eta_c$  (1S) және де  $\chi_{c0}$  (1P),  $\chi_{c1}$  (1P),  $\chi_{c2}$  (1P) жалпы санын біле отырып, олардың ыдыраудағы белгілі бір адронды мода үшін мөлшерін шамамен бағалай аламыз.

Мақалада жасырын тартымдылығы бар мезондарды іздеу және таңдау үшін ең қолайлы (мезон адрондарының соңғы күйіне ыдырайтын саны бойынша) келесі соңғы күй екендігі анықталды: states -  $KK\pi$ ,  $\pi^+\pi^- K^+K^-$  және  $2(\pi^+\pi^-)$ .

Мақалада ҚР Мемлекет бюджетінен гранттық қаржыландырылатын AP05131547 « $e^+e^-$  – әрекеттестіктерде пайда болған ғажап бариондардың және жасырын ғажаптығы бар мезондардың ыдырауын зерттеу» жобасының аясында атқарылған жұмыс нәтижелері берілді.

**Түйін сөздер:** адрон, мезон, ыдырау модасы, жасырын тартымдылық, соңғы күй.

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### К ВОПРОСУ ПОИСКА АДРОННЫХ МОД РАСПАДА МЕЗОНОВ СО СКРЫТЫМ ОЧАРОВАНИЕМ

**Аннотация.** Исходя из результатов, полученных в работе [1], а также из процентных соотношений между модами распада возбужденных состояний мезонов, приведенных в справочнике “Particle Data Group”, стало возможным оценить число конечных состояний, на которые распадаются мезоны со *скрытым очарованием*. В настоящей работе приводится грубая оценка числа конечных состояний адронных распадов мезонов со *скрытым очарованием*. Наиболее статистически обеспеченными являются конечные состояния  $KK\pi$  для  $\eta_c(1S)$ ,  $2(\pi^+\pi^-)$  и  $\pi^+\pi^- K^+K^-$  для  $\chi_{c0}(1P)$  мезонов.

Мода распада адронов мезонов со скрытым очарованием сильно подавлена по правилу Okubo – Zweig – Iizuka (OZI). Позднее, в работе по изучению неупругого состояния  $J/\psi(1S)$  и  $\psi(2S)$  – мезонов [1] появилась возможность оценить количество мезонов со скрытым очарованием, распадающихся на адронные конечные состояния. Была использована вся статистика взаимодействия, накопленная за период 1997-2007гг. Это около ~350 миллионов взаимодействий. Получаем число мезонов  $J/\psi(1S)$  и  $\psi(2S)$  с распадом лептона на  $\mu^+\mu^-$ . Поскольку процент событий, которые распадаются в этом режиме известен по таблице мы можем оценить общее количество сгенерированных мезонов. Исходя из данных, можно вычислить количество мезонов, распадающихся на адронной моде.

Число мезонов со скрытым очарованием и мезонов  $\chi_{c0}(1P)$ ,  $\chi_{c1}(1P)$  и  $\chi_{c2}(1P)$  также можно оценить по данным радиационного распада мезонов  $J/\psi(1S)$  и  $\psi(2S)$ .



Зная общее количество мезонов со скрытым очарованием  $\eta_c$  (1S) и  $\chi_{c0}$  (1P),  $\chi_{c1}$  (1P),  $\chi_{c2}$  (1P), мы можем приблизительно оценить их количество для конкретной адронной моды их распада.

В данной статье приведен вывод, что наиболее благоприятными (по количеству распадающихся в конечные состояния адронов мезонов) для поиска и отбора мезонов со скрытым очарованием являются следующие конечные состояния states -  $KK\pi$ ,  $\pi^+\pi^-K^+K^-$  и  $2(\pi^+\pi^-)$ .

В статье использованы результаты, полученные в рамках проекта AP05131547 «Исследование распадов очарованных барионов и мезонов со скрытым очарованием, образованных в  $e^+e^-$  взаимодействиях» грантового финансирования из РБ РК.

**Ключевые слова:** адрон, мезон, мода распада, скрытое очарование, конечное состояние.

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## INFLATION IN $F(R, T)$ GRAVITATION WITH $f$ -ESSENCE

**Abstract.** Modified theories of gravity have become a kind of paradigm in modern physics because they seem to solve several shortcomings of the standard General Theory of Relativity (GTR) related to cosmology, astrophysics and quantum field theory. The most famous modified theories of gravity are  $F(R)$  and  $F(T)$  theories of gravity. A generalization of these two modified theories and gravitations, which was first proposed by Myrzakulov Ratbay. In this paper, we study an inhomogeneous isotropic cosmological model with a fermion field  $f$ -essence whose action has the form  $S_{RT} = \int d^4x \sqrt{-g} [F(R, T) + L_m]$ , where  $R$  is the scalar of curvature, and  $T$  is the torsion scalar, and  $L_m$  is the Lagrangian  $f$ -essence. A particular case  $F(R, T) = \alpha R + \beta T$  is studied in detail when parameters are obtained that describe the current accelerated expansion of the Universe. The type of Lagrangian  $f$ -essence of this model is determined. The presented results show that gravity with  $f$ -essence can describe inflation in the early evolution of the Universe. A modified  $F(R, T)$  gravity with  $f$ -essence is considered. Equations of motion were obtained and the inflationary period of the early Universe was considered. To describe the inflationary period, the form of the Hubble parameter and the slow-roll parameter were determined.

**Key words:** Universe, accelerated expansion, inflation, cosmology,  $f$ -essence, slow-roll parameter, Friedmann-Robertson-Walker metric,  $F(R, T)$ -gravity.

**Introduction.** The idea of expanding Einstein's theory of gravity is productive and economical in terms of several attempts that try to solve problems by adding new and, in most cases, unjustified new components to give a self-consistent picture of dynamics. The accelerated expansion of the Hubble flow observed today and the missing material of astrophysical large-scale structures are primarily included in these considerations. Both problems can be solved by changing the gravity sector, i.e. field equations. Philosophy is an alternative for adding new cosmic fluids (new components in the field equations), which should lead to cluster structures (dark matter) or to accelerated dynamics (dark energy) due to exotic equations of state. In particular, weakening the hypothesis that the gravitational Lagrangian should be a linear function of the scalar of Ricci curvature  $R$ , as in the Hilbert-Einstein formulation, one can take into account the effective action as a minimum extension when the gravitational Lagrangian is a general function.

The main and well-known examples of such modified theories of gravity are  $F(R)$  theory of gravity and  $F(T)$  theory of gravity. In this direction, the generalization of the theories of gravity  $F(R)$  and  $F(T)$  promises to be a promising and interesting version of modified theories of gravity. One of these generalizations of the  $F(R)$  and  $F(T)$  theories was first presented in [1] (see also [2-7]). There was shown that the torsion scalar, as well as the curvature scalar without a source of matter, leads to the formation of this union, the expansion of the Universe, inflation, and so on. This generalization includes all the properties of the  $F(R)$  and  $F(T)$  theory of gravity.

Astronomical observations suggest the universe can be viewed as homogeneous and isotropic for cosmological purposes that at very large scales, and so this is well described by the Friedmann-Robertson-Walker (FRW) metric. In this paper, we investigate the dynamics of the fermionic field of the  $f$ -essence in the framework of  $F(R, T)$  gravity described by the FRW metric. The special case of  $F(R, T) = \alpha R + \beta T$ , where  $\alpha$  and  $\beta$  are real constants, is investigated in detail. The necessary cosmological parameters have been obtained.

The expansion in the early time of space-time in the evolution of the Universe, namely inflation, is introduced to solve some cosmological problems. The discovery of a gravitational wave from a heavy celestial body and the observation of a black hole (BH) is a great success of Einstein's general relativity. However, from a cosmological point of view, some problems remain, such as flatness, horizon, monopole problems.

The theory of inflation is one of the elegant and simple solutions for these problems. An alternative approach is to modify Einstein's gravity.

The standard method for determining observable inflation is to perform detailed disturbance analysis [8]. However, the approach can be simplified by introducing slow-roll parameters [9], either when inflation occurs due to modification of standard gravitational actions, or due to scalar fields.

### **$F(R, T)$ gravity**

The action has the following form [1] (see also [2-17]):

$$S_{RT} = \int d^4x \sqrt{-g} [F(R, T) + L_m], \quad (1)$$

where  $L_m$  is the matter Lagrangian,  $R$  is the curvature scalar,  $T$  is the torsion scalar

$$R = g^{\mu\nu} R_{\mu\nu}, \quad T = S_{\rho}^{\mu\nu} T^{\rho}_{\mu\nu}. \quad (2)$$

This generalization was first proposed in [1], and these properties were investigated in [2-17]. In general,  $G^{\lambda}_{\mu\nu}$  has the form

$$G^{\lambda}_{\mu\nu} = e_i^{\lambda} \partial_{\mu} e^i_{\nu} + e_j^{\lambda} e^i_{\nu} \omega^j_{i\mu} = \Gamma^{\lambda}_{\mu\nu} + K^{\lambda}_{\mu\nu}, \quad (3)$$

where  $\Gamma^{\lambda}_{\mu\nu}$  - Levi-Civita connection, and  $K^{\lambda}_{\mu\nu}$  - contorsion tensor. We write the metric in the standard form as

$$ds^2 = g_{ij} dx^i dx^j. \quad (4)$$

Orthonormal tetrad components  $e_i(x^{\mu})$  are related to the metric tensor as follows

$$g_{\mu\nu} = \eta_{ij} e^i_{\mu} e^j_{\nu}. \quad (5)$$

The orthonormalized condition is given as

$$\eta_{ij} = g_{\mu\nu} e^{\mu}_i e^{\nu}_j. \quad (6)$$

Here's  $\eta_{ij} = \text{diag}(-1, 1, 1, 1)$  where we used the relation

$$e^i_{\mu} e^{\mu}_j = \delta^i_j. \quad (7)$$

### **Basics of $f$ -essence**

Briefly describe some of the basics fermion model  $f$ -essences indicated in [7]. The action of the model has the form [7]

$$S_f = \int d^4x \sqrt{-g} [R + 2K(Y, \psi, \bar{\psi})], \quad (8)$$

where  $\psi$  and  $\bar{\psi} = \psi^{\dagger} \gamma^0$  denote the fermionic field and its conjugation, respectively, the cross means complex conjugation and  $R$  is the Ricci scalar.  $K$  is the density of the Lagrangian of the fermionic field, the canonical kinetic term of which has the form

$$Y = \frac{1}{2}i[\bar{\psi}\Gamma^\mu D_\mu\psi - (D_\mu\bar{\psi})\Gamma^\mu\psi]. \quad (9)$$

Here  $\Gamma^\mu = e_a^\mu \gamma^a$  is the generalized Dirac-Pauli matrices satisfying the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad (10)$$

where the brackets denote the anti-commutation relation.  $e_a^\mu$  denotes a tetrad, while the covariant derivative is given by

$$D_\mu\psi = \partial_\mu\psi - \Omega_\mu\psi, \quad D_\mu\bar{\psi} = \partial_\mu\bar{\psi} + \bar{\psi}\Omega_\mu. \quad (11)$$

The fermionic coupling  $\Omega_\mu$  is determined by the formula

$$\Omega_\mu = -\frac{1}{4}g_{\rho\sigma}[\Gamma_{\mu\delta}^\rho - e_b^\rho\partial_\mu e_\delta^b]\Gamma^\sigma\Gamma^\delta, \quad (12)$$

where  $\Gamma_{\mu\delta}^\rho$  Christoffel symbols.

Fermionic fields are considered here as classically commuting fields. By the classical fermionic field, we mean a set of four complex space-time functions that transform in accordance with the spinor representation of the Lorentz group. The existence of such fields have of decisive importance in our work, because, despite the fact that fermions are described by quantized fermion fields which that do not have a classical limit, we assume that such classical fields exist and use them as matter fields associated with gravity.

### **$F(R,T)$ gravity with $f$ -essence**

Now let's consider  $F(R,T)$  gravity with  $f$ -essence. Its action is given as

$$S = \int d^4x \sqrt{-g}[F(R,T) + 2K(Y,\psi,\bar{\psi})], \quad (13)$$

where  $R$  and  $T$  are curvature and torsion scalars, respectively, expressed as

$$R = g^{\mu\nu}R_{\mu\nu}, \quad T = S_\rho^{\mu\nu}T^\rho_{\mu\nu}. \quad (14)$$

### **FRW metric case**

To simplify, this section will make the next two to simplify the problem, namely, we assume that:

1)  $F$  is of the form

$$F = \alpha R + \beta T, \quad (15)$$

where  $\alpha$  and  $\beta$  are some positive constants.

2) consider only the FRW space-time

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2). \quad (16)$$

Then action (13) takes the form

$$S = \int dx^4 \sqrt{-g}[\alpha R + \beta T + 2K(Y,\psi,\bar{\psi})]. \quad (17)$$

Next important suggestion is that for the FRW metric,  $R$  and  $T$  take the forms

$$R = u + 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = u + 6(\dot{H} + 2H^2) = u + R_s, \quad (18)$$

$$T = v - 6\frac{\dot{a}^2}{a^2} = v - 6H^2 = v - T_s,$$

where  $u$  and  $v$  are some functions from  $a$ ,  $a$ ,  $\frac{d^j a}{dt^j}$ ,  $R, T$  and so on. Here

$$R_s = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = 6(\dot{H} + 2H^2), \quad (19)$$

$$T_s = -6\frac{\dot{a}^2}{a^2} = -6H^2.$$

Next step for our model can be rewritten as

$$S = \int dx^4 \sqrt{-g} [\alpha R_r + \beta T_w + \bar{L}_m], \tag{20}$$

where

$$\bar{L}_m = \alpha u + \beta v + L_m = \alpha u + \beta v + 2K(Y, \psi, \bar{\psi}) = 2\bar{K}(Y, \psi, \bar{\psi}). \tag{21}$$

For the FRW metric, the Dirac matrices of the curved space-time  $\Gamma^\mu$  have the form

$$\Gamma^0 = \gamma^0, \Gamma^j = a^{-1}\gamma^j, \Gamma^5 = -i\sqrt{-g}\Gamma^0\Gamma^1\Gamma^2\Gamma^3 = \gamma^5, \Gamma_0 = \gamma^0, \Gamma_j = a\gamma^j (i=1,2,3). \tag{22}$$

Hence we get

$$\Omega_0 = 0, \Omega_j = \frac{1}{2}\dot{a}\gamma^j\gamma^0 \tag{23}$$

and

$$Y = \frac{1}{2}i(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi) \tag{24}$$

Finally, note that the gamma matrices that we write in the Dirac basis are of the form

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \tag{25}$$

where  $I = diag(1,1)$  and  $\sigma^k$  are Pauli matrices having the following forms

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{26}$$

We rewrite action (20) in the following form

$$S = \int dt \sqrt{-g} L, \tag{27}$$

where  $L$  is the point-like Lagrangian. To find the equations of motion of the model, we use the Euler-Lagrange equations. In our case, the Euler-Lagrange equations are defined as follows

$$\frac{\partial L}{\partial a} - \frac{d}{dt} \frac{\partial L}{\partial \dot{a}} = 0, \frac{\partial L}{\partial R} - \frac{d}{dt} \frac{\partial L}{\partial \dot{R}} = 0, \frac{\partial L}{\partial T} - \frac{d}{dt} \frac{\partial L}{\partial \dot{T}} = 0, \frac{\partial L}{\partial \psi} - \frac{d}{dt} \frac{\partial L}{\partial \dot{\psi}} = 0, \frac{\partial L}{\partial \bar{\psi}} - \frac{d}{dt} \frac{\partial L}{\partial \dot{\bar{\psi}}} = 0. \tag{28}$$

Accordingly, the condition is given as

$$E_L = \frac{\partial L}{\partial \dot{a}} \dot{a} + \frac{\partial L}{\partial \dot{R}} \dot{R} + \frac{\partial L}{\partial \dot{T}} \dot{T} + \frac{\partial L}{\partial \dot{\psi}} \dot{\psi} + \frac{\partial L}{\partial \dot{\bar{\psi}}} \dot{\bar{\psi}} - L = 0. \tag{29}$$

Finally, the equations of motion for our model take the form

$$\begin{aligned} 3H^2 - \bar{\rho} &= 0, \\ 2\dot{H} + 3H^2 + \bar{p} &= 0, \\ \bar{K}_Y \dot{\psi} + 0.5(3H\bar{K}_Y + \dot{\bar{K}}_Y)\psi - i\gamma^0 \bar{K}_{\bar{\psi}} &= 0, \\ \bar{K}_Y \dot{\bar{\psi}} + 0.5(3H\bar{K}_Y + \dot{\bar{K}}_Y)\bar{\psi} + i\bar{K}_{\psi} \gamma^0 &= 0, \end{aligned} \tag{30}$$

where

$$\bar{\rho} = Y\bar{K}_Y - \bar{K}, \quad \bar{p} = \bar{K}. \tag{31}$$

where  $\dot{H} = \dot{a}/a$  is the Hubble parameter,  $a(t)$  is the scale factor, and  $\dot{a}(t)$  is cosmic time derivative  $t$ .

From the second equation of system (30) can be seen that, according to the rules of differentiation, if expressions from the spirit of the sides of equality depend on different arguments, then this equality will be constant

$$2\dot{H}(a) + 3H(a)^2 = -\bar{K}(Y, \psi, \bar{\psi}) = const.$$

To study inflation is enough to determine the form of the Hubble parameter, which has the form:

$$H = \frac{1}{6} \frac{\left( \left( 12\sqrt{3} \sqrt{\frac{4C_1^3 + 27e^{2t}C_2}{C_2}} + 108e^t \right) C_2^2 \right)^{1/3}}{C_2} \frac{2C_1}{\left( \left( 12\sqrt{3} \sqrt{\frac{4C_1^3 + 27e^{2t}C_2}{C_2}} + 108e^t \right) C_2^2 \right)^{1/3}} \quad (32)$$

**Inflation.** Let us consider an inflationary model with a minimum kinetic term, where the behavior of the system is described by the FRW and Dirac equations (30). To describe the evolution of the background is convenient to introduce the functions of the Hubble flow  $\varepsilon_n$ , which are defined by the formula

$$\varepsilon_{n+1} \equiv -\frac{d \ln |\varepsilon_n|}{dN}, \quad n \geq 0, \quad (33)$$

where  $\varepsilon_0 \equiv H_{ini}/H$  and  $N \equiv \ln(a_{ini}/a)$  are the amount of e-folding. By definition, inflation is a phase of accelerated expansion  $\ddot{a}/a > 0$  or, equivalently  $\varepsilon_1 < 1$ . As a consequence, the end of inflation is determined by the condition  $\varepsilon_1 = 1$ . On the other hand, the conditions of slow-rolling down (or the slow-rolling approximation) refer to the situation when all  $\varepsilon_n$  satisfy  $\varepsilon_n \ll 1$ . For our case, the inflation solution evolves with the (positive) Hubble flow functions

$$\varepsilon_1 = -\frac{\dot{H}}{H^2}, \quad (34)$$

$$\varepsilon_2 = -\frac{2\dot{H}}{H^2} + \frac{\ddot{H}}{HH}, \quad (35)$$

where  $C_1$  and  $C_2$  are integration constants. Figure 1 shows the typical behavior of the slow-roll parameter for different  $C_1$  and  $C_2$  values.

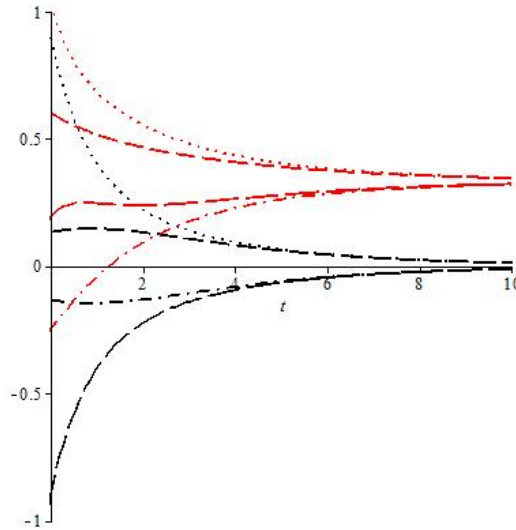


Figure 1 - Graphical analysis of  $\varepsilon_1$  (black lines) and  $\varepsilon_2$  (red lines). Dotted line:  $C_1 > 0$  and  $C_2 > 0$ ; dotted line:  $C_1 < 0$  and  $C_2 > 0$ ; long dotted line:  $C_1 < 0$  and  $C_2 < 0$ ; dot-dotted line:  $C_1 > 0$  and  $C_2 < 0$

**Conclusion.** In this work we have considered a modified  $F(R, T)$  gravity with  $f$ -essence. Equations of motion were obtained and the inflationary period of the early Universe was considered. To describe the inflationary period, we determined the form of the Hubble parameter and the slow-roll parameter. As can be seen from the graph for our model, the slow-roll parameter does not satisfy the inflation conditions given in [9]. Thus, the given  $F(R, T)$  gravity model with  $f$ -essence cannot describe viable inflation.

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### *f*-ЭССЕНЦИЯСЫ БАР $F(R,T)$ ГРАВИТАЦИЯСЫНДАҒЫ ИНФЛЯЦИЯ

**Аннотация.** Жаңартылған гравитациялық теориялары қазіргі физикада өзгеше парадигманың түріне айналды, өйткені олар космология, астрофизика және кванттық өріс теориясымен байланысты стандартты жалпы салыстырмалылық теориясының бірнеше кемшіліктерін шешкен болуы мүмкін. Гравитациялықтың ең танымал модификацияланған теориялары –  $F(R)$  және  $F(T)$ . Осы екі теорияның жалпыланған түрін алғаш рет Мырзақулов Ратбай ұсынған. Бұл жұмыста біз *f*-эссенция фермиондық өрісі бар біртекті емес изотропты космологиялық модельді зерттейміз, әсерін былай көрсетеміз:  $S_{RT} = \int d^4x \sqrt{-g} [F(R,T) + L_m]$ , мұнда  $R$  – қисық скаляры және  $T$  – ширату скаляры,  $L_m$  – *f*-эссенция лагранжианы. Әлемнің үдетілген кеңею жағдайын сипаттайтын параметрлер алынған. Нақты жағдай  $F(R,T) = \alpha R + \beta T$  қарастырылып, толықтай зерттеледі. Бұл модельдің *f*-эссенция лагранжианың түрі анықталған. Ұсынылған нәтижелер *f*-эссенциясы бар  $F(R,T)$  гравитациясы әлемнің ерте инфляциялық эволюциясын сипаттайтындығын көрсетеді.

Жұмыста фермиондық өрісі бар және канондық емес кинетикалық мүшесі (*f*-эссенция) бар космологиялық модель қарастырылған. Модельдің кейбір нақты шешімдері табылған және біз осындай гравитациялық-фермиондық өзара әрекеттесудің әлемнің бақыланатын үдемелі ұлғаю әсерін тексереміз. Әрі қарай, осы модельдердің космологиялық қосымшалары талқыланады. Сондай-ақ, *f*-эссенция моделінде Эйнштейннің статикалық әлемі секілді шешім жоқ. *f*-эссенциясы бар модификацияланған  $F(R,T)$  гравитациясы қарастырылған. Қозғалыс теңдеулері алынды және алғашқы Әлемнің инфляциялық кезеңі қарастырылған. Инфляциялық кезеңді сипаттау үшін Хаббл параметрінің формасы және баяу оралу параметрі анықталған.

**Түйін сөздер:** әлем, үдемелі ұлғаю, инфляция, космология, *f*-эссенция, баяу сырғу параметрі, Фридман-Робертсон-Уокер метрикасы,  $F(R,T)$ -гравитациясы.

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### ИНФЛЯЦИЯ В $F(R,T)$ ГРАВИТАЦИИ С *f*-ЭССЕНЦИЕЙ

**Аннотация.** Модифицированные теории гравитации стали своего рода парадигмой в современной физике, поскольку они, кажется, решают несколько недостатков стандартной Общей теории относительности (ОТО), связанной с космологией, астрофизикой и квантовой теорией поля. Наиболее известные модифицированные теории гравитации являются  $F(R)$  и  $F(T)$  теории гравитации. Обобщение этих двух модифицированных теорий  $F(R)$  и  $F(T)$  гравитации впервые было предложено Мырзақуловым Ратбаем. В данной работе исследована неоднородная изотропная космологическая модель с фермионным полем *f*-эссенцией, действие которой имеет вид  $S_{RT} = \int d^4x \sqrt{-g} [F(R,T) + L_m]$ , где  $R$  – скаляр кривизны, а  $T$  – скаляр кручения,  $L_m$  – лагранжиан *f*-эссенции. Детально изучается частный случай при  $F(R,T) = \alpha R + \beta T$ , получены параметры, описывающие текущее ускоренное расширение Вселенной. Определён вид лагранжиана *f*-эссенции данной модели. Представленные результаты показывают, что  $F(R,T)$  гравитация с *f*-эссенцией может описывать инфляцию в ранней эволюции Вселенной.

В этой работе рассмотрена космологическая модель с фермионным полем и с неканоническим кинетическим членом (*f*-эссенция). Найдены некоторые точные решения модели и проверяем влияние такого гравитационно-фермионного взаимодействия на наблюдаемое ускоренное расширение Вселенной. Далее, обсуждаются космологические применения точных моделей. Также модель *f*-эссенции не имеет решения типа статической Вселенной Эйнштейна. Рассмотрено модифицированная  $F(R,T)$  гравитация с *f*-эссенцией. Было получены уравнения движения и рассмотрен инфляционный период ранней Вселенной. Для описания инфляционного периода, были определены вид параметра Хаббла и параметр медленного скатывания.

**Ключевые слова:** Вселенная, ускоренное расширение, инфляция, космология, *f*-эссенция, параметр медленного скатывания, метрика Фридмана-Робертсона-Уокера,  $F(R,T)$  - гравитация.

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E-mail: [n\\_pokrovskiy@satbayev.university](mailto:n_pokrovskiy@satbayev.university)**TO THE QUESTION OF THE EXISTENCE  
OF CENTAURS - EVENTS IN COSMIC RAYS**

**Abstract.** One of the first exotic phenomena observed in the 80s of the last century was an event registered by a calorimetric type installation. In this event, an anomalous relationship was observed between charged and neutral hadrons, arising from the collision of a high-energy particle with a carbon nucleus. In accordance with the principle of isotopic invariance, the number of neutral pions should be equal to the number of charged pions. The event, which was recorded by Japanese physicists, contained only charged pions with no neutral [1]. This event was named "Centaur".

Until now, such events have not been observed in accelerator experiments. The article presents the results of searching for such events in experiments at the DESY HERA collider. For the candidates for Centaur events and for all other events, constructed distributions by the type of fitability of charged tracks in CTD.

Since the absence of neutral particles in events is explained by methodological reasons, "Centaur" events were not detected in electron-proton interactions at the ZEUS facility.

**Key words:** centaur events, charged particles, plurality, ZEUS, anomalous event, cosmic rays.

In  $e \pm p$  interactions obtained at the ZEUS facility (Hamburg, Germany), analysed the distributions over the multiplicity of charged particles, registered in the central track detector (CTD), and neutral, registered in the uranium calorimeter (CAL). An anomalously large number of events was detected in the range of charged particle multiplicity from 40 to 100, where the neutral component is factually absent. Figures 1-2 show the distributions of the multiplicity of neutral particles from the multiplicity of charged particles for exposure 07p, where this anomaly is clearly appears (shown by an arrow). A similar picture is observed for all other exposures (03, 04, 05, 06e and 06p).

A similar phenomenon has already been observed in cosmic rays. Thus, long-term measurements of the processes occurring with cosmic particles in the Earth's atmosphere, performed by various experimental methods, led to the discovery of a number of exotic phenomena that do not fit into the modern concept of interactions at high and ultrahigh energies. One of the first exotic phenomena observed in the 80s of the last century was an event recorded by a calorimetric type installation. In this event, an anomalous relationship observed between charged and neutral hadrons, arising from the collision of a high-energy particle with a carbon nucleus. In accordance with the principle of isotopic invariance, the number of neutral pions should be equal to the number of charged pions. The event, which was recorded by Japanese physicists, contained only charged pions with no neutral [1]. This event was named "Centaur". Further experiments with a similar technique performed in different collaborations (collaboration "Pamir" [2], Japanese-Brazilian collaboration [1], etc.), were aimed at finding such events. Table 1 shows the statistics of Centaur events accumulated over all these years.

Statistics of Centaur events accumulated in different experiments [1-3]

Laboratories	Height m (g/cm <sup>2</sup> )	Absorbers above cameras	Exposition m <sup>2</sup> per year	Amount Centaur events
Chacaltaya (Brazil-Japan)	5200 (540)	2-layer carbon	300	8
Pamir (USSR-Poland)	4300 (600) or 4900	carbon	500	3
Pamir (Russia-Japan)	4300	Carbon or thick plumbum	530	2

The table shows that the number of Centaur events is small. Thus, the discovery of this phenomenon remains questionable due to poor statistical availability. Until now, such events have not been observed in accelerator experiments. Nevertheless, it is planned to search for such events in experiments at the DESY HERA collider.

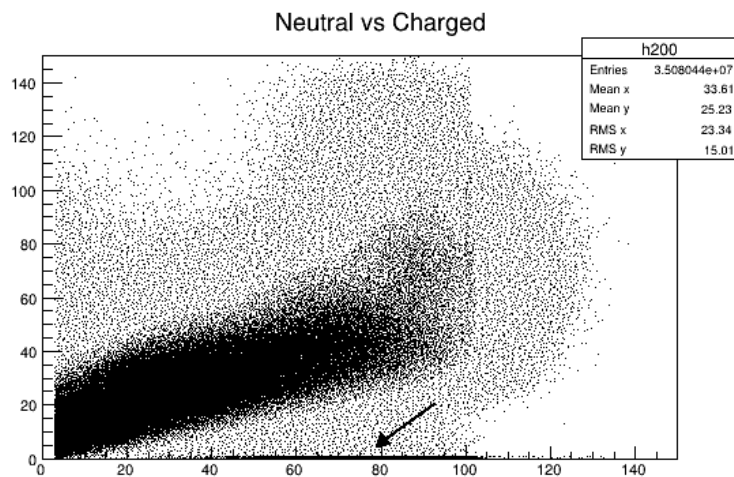


Figure 1 - Distribution of the multiplicity of neutral particles from the multiplicity of charged

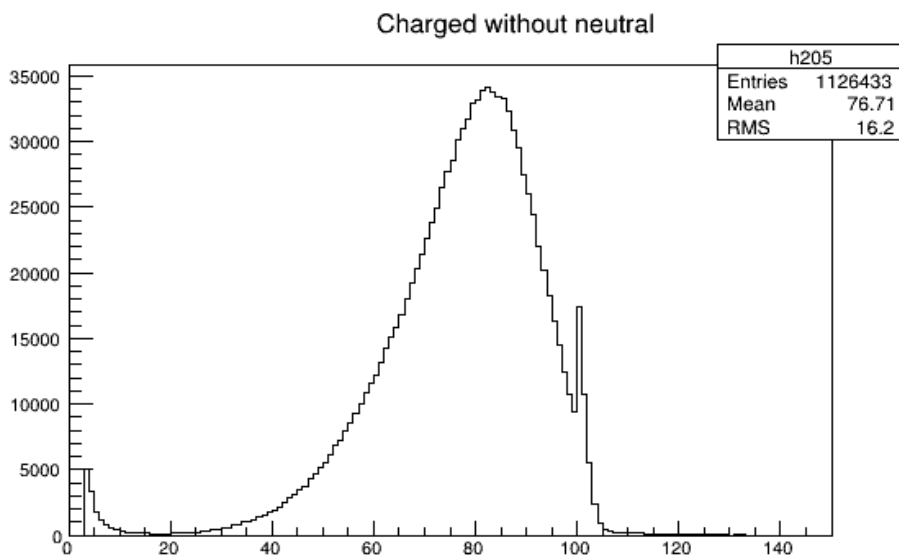


Figure 2 - Distribution of the multiplicity of charged particles for events without a neutral component

To check whether the appearance of Centaur events in  $e \pm p$  interactions is connected with any methodological factors in the ZEUS setup, for candidates for Centaur events and for all other events, distributions were constructed by the type of fitability of charged tracks in CTD.

All charged tracks are divided into three categories: tracks fitted to the primary interaction vertex, tracks fitted to secondary vertices, and unfitted tracks. Figures 3 - 4 show the distributions by fit type for the Centaur event candidates and for all others. It can be seen that in the Centaur events, in comparison with the rest of the events, unfitted tracks predominate and a small proportion of tracks originating from the primary summit. From an analysis of the distributions over ionization losses  $dE / dx$  for unfitted tracks it follows, that the main source of their appearance should be considered secondary interactions in the materials of the detector components, since they exhibit an increased content of protons and even a significant amount of deuterons.

About half of the Centaur event candidates have a primary apex located 80 cm from the central region of the detector, which makes it difficult to reliably register all particles, especially neutral ones in CAL.

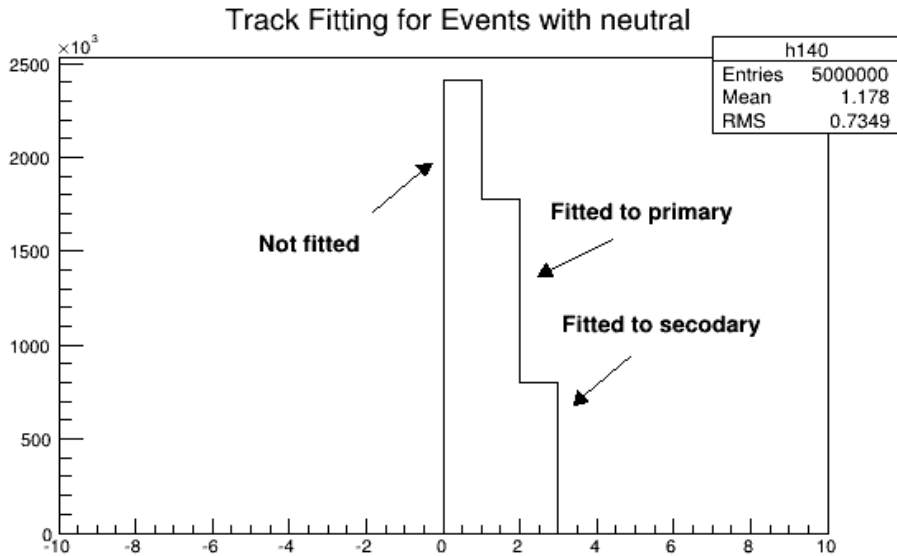


Figure 3 - Distributions by fit type of tracks for events with a neutral component

To neutralize methodological factors that may affect the appearance of Centaur events in  $e \pm p$  interactions for analyzed events were subject to the following conditions:

- in all events unfit tracks removed;
- were taken into account only the events with a primary vertex in the region  $|Z_{up}| < 50$  cm.

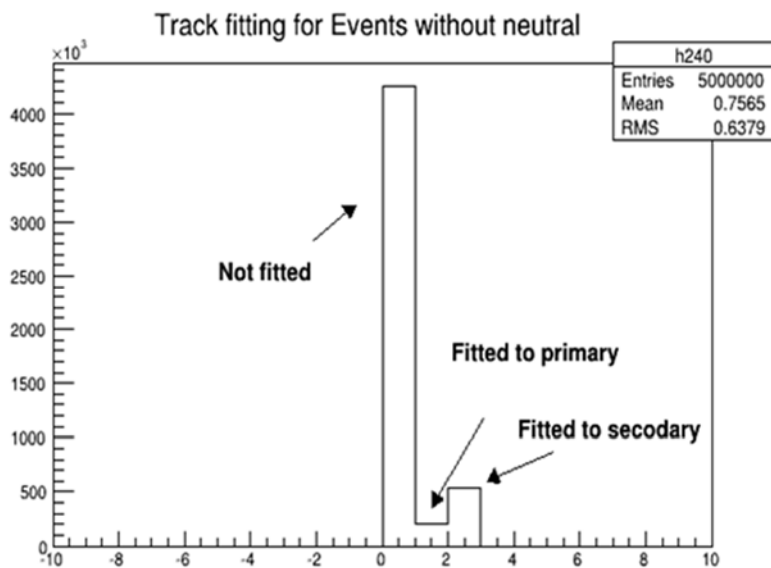


Figure 4 - Distributions by the type of track fit for candidates in Centaur Events

Figures 5 - 6 show the distribution by the number of such events without a neutral component from the multiplicity of charged particles in the usual and logarithmic scales.

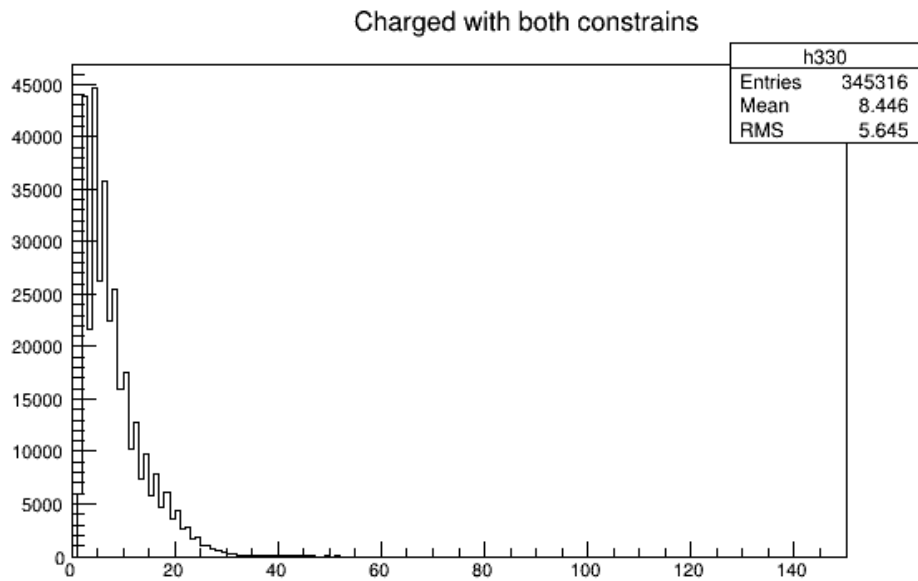


Figure 5 - Distribution of charged particles for events without neutral component

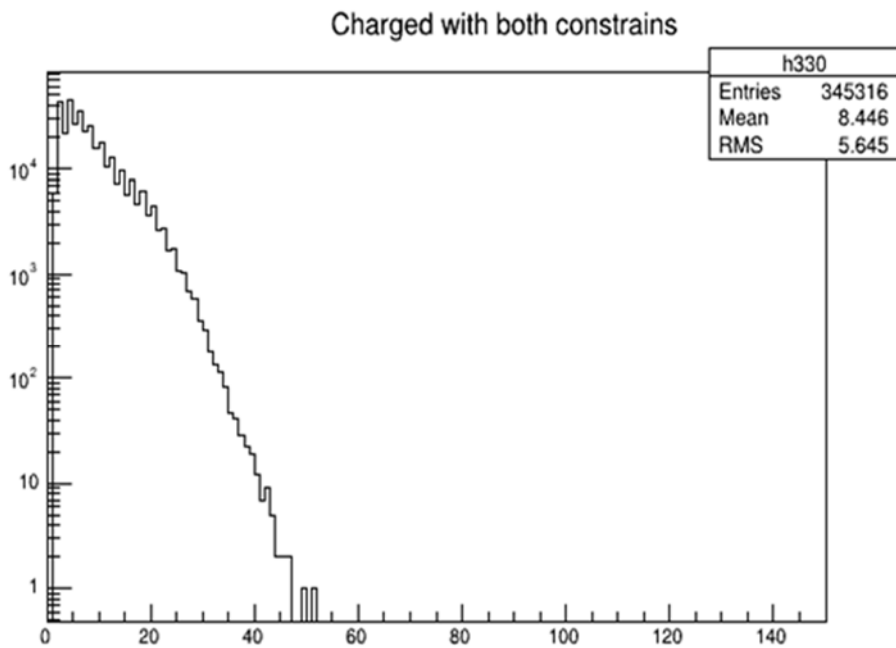


Figure 6 - Distribution of charged particles for events without neutral component

**Conclusion.** With increasing multiplicity, there is a natural exponential decline in the number of events and no anomalies appear. Thus, no events detected in electron-proton interactions at the ZEUS "Centaur", since the absence of neutral particles in events is explained by methodological reasons.

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### ҒАРЫШ СӘУЛЕСІНДЕ КЕНТАВР – ОҚИҒАЛАРДЫҢ БОЛУ МӘСЕЛЕСІ

**Аннотация.** Өткен ғасырдың 80-жылдары байқалған алғашқы экзотикалық құбылыстардың бірі – калориметрлік типтегі кондырғымен тіркелген оқиға еді. Бұл оқиғада жоғары энергия бөлшегінің көміртегі ядросымен соқтығысуынан пайда болған зарядталған және бейтарап адрондар арасындағы қалыпты емес қатынас байқалды. Изотоптық инвариант принципіне сәйкес бейтарап пиондар саны зарядталған пиондар санына тең болуы керек. Жапон физиктері тіркеген оқиға бейтарап болмаған кезде тек зарядталған пиондарды қамтыды [1]. Бұл оқиға «Кентавр» деп аталды.

Кентавр саны – аз. Осылайша бұл құбылыстың ашылу жағдайы нашар қолжетімді статистикаға байланысты болғандықтан күмәнді. Осы уақытқа дейін мұндай оқиғалар үдеткіштегі эксперименттерде байқалмады. Дегенмен LHC коллаидеріндегі эксперименттерде осындай оқиғаларды іздеу жоспарлануда.

Кентавр оқиғаларының пайда болуы ZEUS кондырғысындағы кез-келген әдістемелік факторлармен  $e^+p$  өзара қатынасына байланысты еместігін тексеру үшін Кентавр оқиғаларына үміткерлер мен барлық басқа оқиғаларға CTD-де зарядталған тректердің фитуляциялық түрі бойынша үлестірілді. Барлық зарядталған трек үш санатқа бөлінеді: өзара қатынастың бастапқы шыңына сілтеме жасалған тректер, екінші шыңға бекітілген және жетілмеген тректер. Иондандырылмайтын жолға  $dE/dx$  иондану шығыны бойынша бөлуді талдаудан олардың пайда болуының негізгі көзі детектор компоненттері материалдарындағы қайталама өзара қатынасты қарастырған жөн, өйткені олардан көп мөлшердегі протондар және дейтондардың да едәуір мөлшері байқалады.

Кентавр оқиғаларына үміткерлердің жартысына жуығы детектордың орталық аймағынан 80 см қашықтықта орналасқан бастапқы шыңға ие әрі бұл жағдай барлық бөлшекті, әсіресе, бейтарап бөлшекті CAL-да сенімді тіркеуді күрделендіреді. Кентавр оқиғаларының пайда болуына әсер етуі ықтимал әдістемелік факторларды теңестіру үшін  $e^+p$  өзара әрекеттесу талданған оқиғаларға келесі шарттар қойылды:

- барлық оқиғада жарамсыз тректер алынып тасталды;
- аумақта бастапқы шыңы бар оқиғалар ғана ескерілді /  $Z_{\text{верш}} / < 50$  см.

Көптіктің жоғарылауы арқылы оқиға саны табиғи экспоненциалды төмендеген және ауытқау байқалмайды. Осылайша, ZEUS «Кентавр» кондырғысындағы электрон-протонның өзара қатынасындағы оқиғалар айқындалмады, себебі оқиғаларда бейтарап бөлшектердің болмауы әдістемелік себептерге байланысты.

**Түйін сөздер:** Кентавр-оқиғалар, зарядталған бөлшектер, еселік, ZEUS, аномальді оқиға, ғарыш сәулесі.

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### К ВОПРОСУ СУЩЕСТВОВАНИЯ КЕНТАВР – СОБЫТИЙ В КОСМИЧЕСКИХ ЛУЧАХ

**Аннотация.** Одним из первых экзотических явлений, наблюдаемым в 80-х годах прошлого века, было событие, зарегистрированное установкой калориметрического типа. В этом событии наблюдалось аномальное соотношение между заряженными и нейтральными адронами, возникшими при соударении частицы высокой энергии с ядром углерода. В соответствии с принципом изотопической инвариантности количество нейтральных пионов должно равняться количеству заряженных пионов. Событие, которое было зарегистрировано японскими физиками, содержало только заряженные пионы при полном отсутствии нейтральных [1]. Это событие было названо «Кентавром».

Количество Кентавр-событий невелико. Таким образом, открытие данного явления остается под вопросом из-за плохой статистической обеспеченности. До настоящего времени такие события не наблюдались в экспериментах на ускорителях. Тем не менее, планируется поиск таких событий в экспериментах на коллайдере LHC.

Чтобы проверить, не связано ли появление Кентавр-событий в  $e^+p$  взаимодействиях с какими-либо методическими факторами в установке ZEUS, для кандидатов в Кентавр-события и для всех остальных событий были построены распределения по типу фитируемости заряженных треков в CTD. Все заряженные треки делятся на три категории: треки, фитируемые к первичной вершине взаимодействия, треки, фитируемые к вторичным вершинам и нефитируемые треки. Из анализа распределений по ионизационным

потерям  $dE/dx$  для нефитируемых треков следует, что основным источником их появления следует считать вторичные взаимодействия в материалах компонент-детектора, так как в них наблюдается повышенное содержание протонов и даже значительное количество дейтонов.

Примерно половина кандидатов в Кентавр-событиях имеют первичную вершину, расположенную на расстоянии 80 см от центральной области детектора, что затрудняет надежную регистрацию всех частиц, особенно нейтральных в *CAL*. Чтобы нивелировать методические факторы, которые могут повлиять на появление Кентавр-событий в  $e^+p$  взаимодействиях на анализируемые события были наложены следующие условия:

- во всех событиях были удалены нефитируемые треки;
- учитывались только события, имеющие первичную вершину в области  $|Z_{\text{верш}}| < 50$  см.

С увеличением множественности наблюдается естественный экспоненциальный спад числа событий и никаких аномалий не проявляется. Таким образом, в электрон-протонных взаимодействиях на установке ZEUS «Кентавр» события не обнаружены, поскольку отсутствие нейтральных частиц в событиях объясняется методическими причинами.

**Ключевые слова:** кентавровые события, заряженные частицы, множественность, ZEUS, аномальное событие, космические лучи.

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